PHYSICS

XI

KINEMATICS

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SCALAR AND VECTORS

SCALARS

DEFINITION

Physical quantities that are completely specified by their magnitude (a number with suitable unit) are called scalars.

EXAMPLES

Some of the physical quantities that are scalars are:

1 Length	2. Distance.	3. Time	4. Speed	5. Volume
6. Density	7 Work	8. Mass	9. Frequency	10 Energy
11. Temperature	12. Wavelength	13 Current	14. entropy	

VECTORS

DEFINITION

Physical quantities that have both magnitude and direction and that follow the laws vector addition are called vectors.

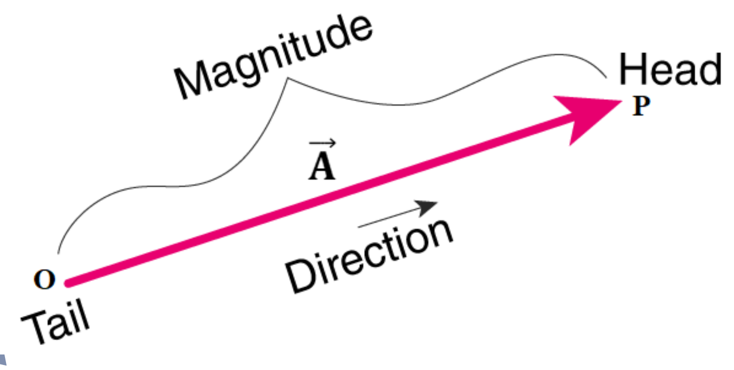
EXAMPLES

Some physical quantities that are vectors are

1. Displacement	2. Velocity	3. Acceleration	4. Momentum
5. Force	6. Angular Velocity.	7. Weight	8. Torque
9. Electric Field	10. Magnetic Field	11.Tension	

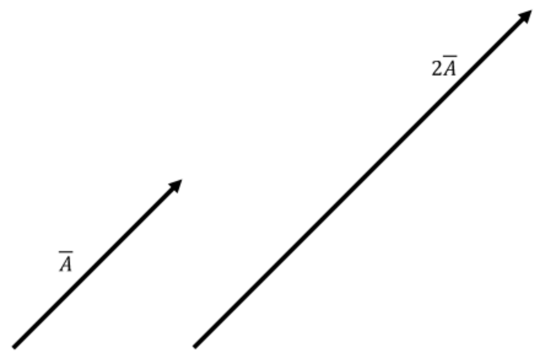
REPRESENTATION OF VECTORS

Vectors can be represented graphically as arrows. The length of the arrow indicates the magnitude of the vector. The direction of the arrow indicates the direction of the vector.

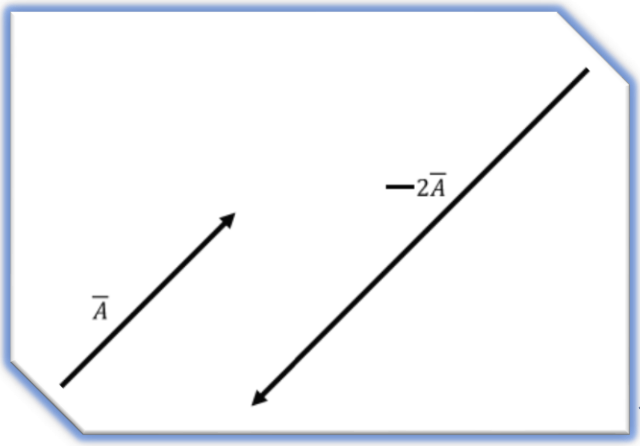


MULTIPLICATION OF A VECTOR BY A NUMBER:- (n ≠ 0):-

when a vector is multiplied by a positive number, its magnitude changes but not its direction. For example, if vector $\vec{A}$ is multiplied by 2, its magnitude is doubled.



when a vector is multiplied by a negative number (n < 0), The direction of this vector will be opposite to the direction of vector $\vec{A}$ and its length will be twice the length of $\vec{A}$.



The multiplication of a vector by a scalar obeys following rules.

1. $m\vec{A} = \vec{A}m$: Commutative law
2. $m(n\vec{A}) = mn\vec{A}$: Associative law
3. $(m+n)\vec{A} = m\vec{A} + n\vec{A}$: Distributive law
4. $m(\vec{A}+\vec{B}) = m\vec{A} + m\vec{B}$: Distributive law

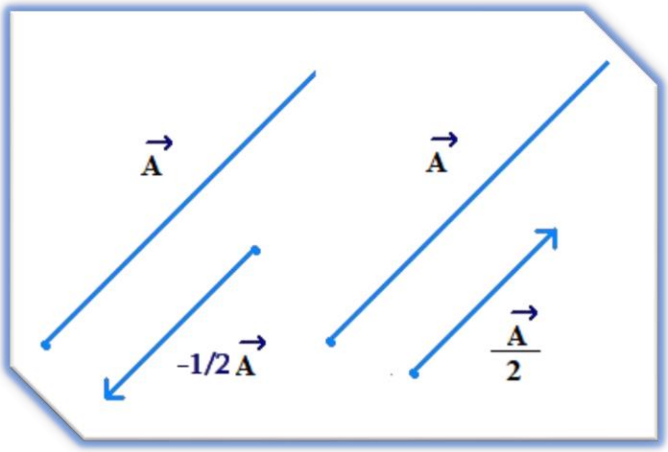
DIVISION OF A VECTOR BY A NUMBER:- (n ≠ 0)

When a vector $\vec{A}$ is divided by a pure number n (non-zero), this operation is equivalent to the multiplication of the vector $\vec{A}$ by the reciprocal of scalar number(1/n).

$$\therefore \quad \frac{\vec{A}}{n} = \left(\frac{1}{n}\right)\vec{A}$$

The vector $-\frac{\vec{A}}{2}$ will have a direction opposite to that of $\vec{A}$ and will have a length equal to half the length of $\vec{A}$

The vector $\frac{\vec{A}}{2}$ will have a direction same to that of $\vec{A}$ and will have a length equal to half the length of $\vec{A}$



KINDS OF VECTOR

UNIT VECTOR

A vector of magnitude equal to unity in a given direction is known as a unit vector.

NOTATION: -

In handwritten notation unit vectors are represented by small letters with a circumflex or 'hat' such as $\hat{n}, \hat{a}, \hat{u}$ **and** $\hat{b}$.

FORMULA: -

A unit vector along $\vec{A}$ **can be obtained by dividing the vector** $\vec{A}$ **with its magnitude i.e.**

$$\vec{A} = |\vec{A}|\ \hat{a}$$

$$\hat{a} = \frac{\vec{A}}{|\vec{A}|}$$

RECTANGULAR UNIT VECTOR: -

In the rectangular coordinate system the special symbols $\hat{i}$, $\hat{j}$ **and** $\hat{k}$ **are used for unit vectors in the positive x,y and z directions respectively. These are called rectangular unit vectors.**

Now a vector $\vec{A}$ **in these dimensions is given as:**

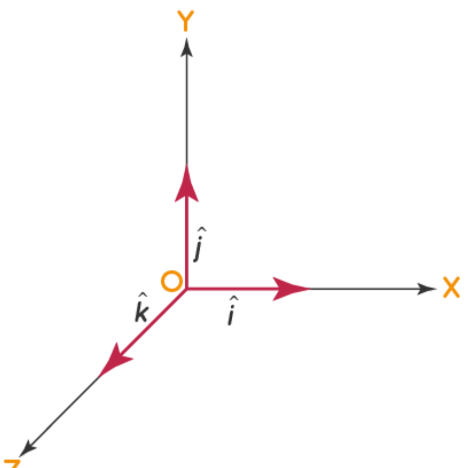
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

its magnitude

$$|\vec{A}| = \sqrt{A^2{}_x + A^2{}_y + A^2{}_z}$$

Thus,

$$\hat{a} = \frac{A_x}{|\vec{A}|}\ \hat{i} + \frac{A_y}{|\vec{A}|}\ \hat{j} + \frac{A_z}{|\vec{A}|}\ \hat{k}$$



Unit vector

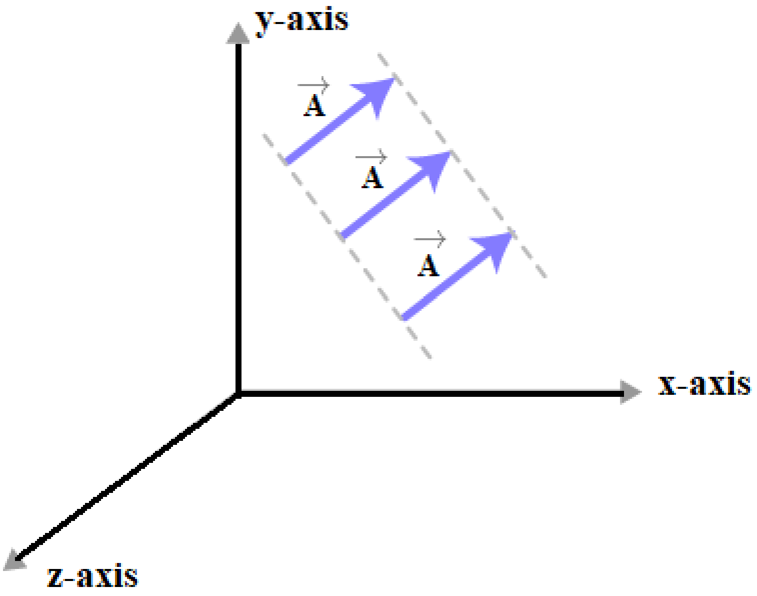
FREE VECTOR: -A free vector can be moved or translated without changing its essential characteristics, such as magnitude and direction. It is represented by an arrow and is not attached to any specific point in space.

PROPERTIES:

(i) free vector can be shifted and drawn parallel to itself.
(ii) It can be applied at any point.

EXAMPLE OF FREE VECTOR

(i) Force is a free vector. Its initial point can be situated anywhere in the line of action of the force.
(ii) couple is a free vector.



POSITION VECTOR

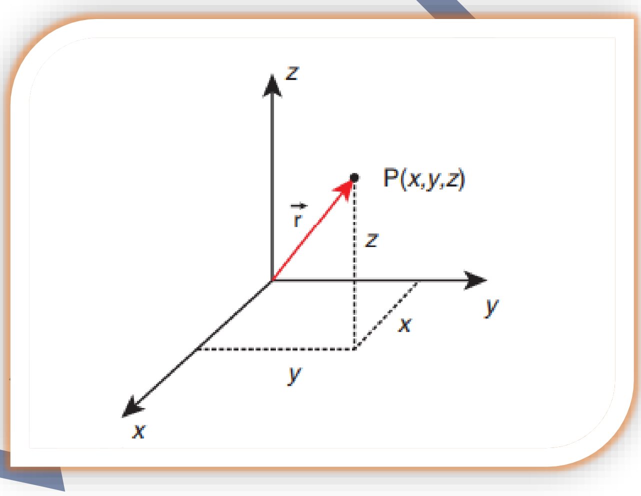
The vector which locates a point with respect to the origin of a rectangular coordinate system i.e whose initial point (tail) is attached to the origin is called position vector.

PROPERTIES: -

(i) it is localized vector since its initial point (tail) is specified.
(ii) it cannot be shifted.

EXPLANATION: -

Position vector is usually represented by $\vec{r}$. Consider a point ‘P’ in three-dimensional space. Let $\vec{r}$ be the position vector of this point. The component x, y, z of position vector $\vec{r}$ are called the coordinates of ‘P’.



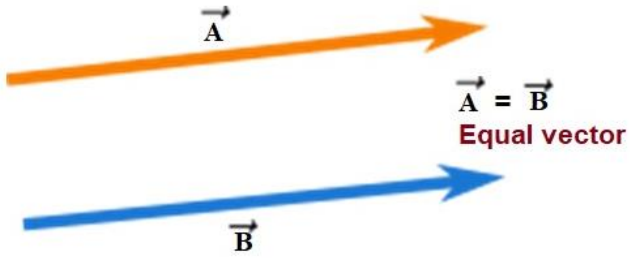
$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

and its magnitude is given by

$$|\vec{r}| = \sqrt{x^2+y^2+z^2}$$

EQUAL VECTOR

Two vectors which are equal both in magnitude and direction are called equal vectors

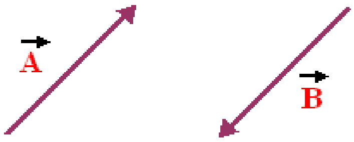


NULL VECTOR

A null vector is a vector having magnitude equal to zero. A null vector has no direction or it may have any direction. Generally a null vector is either equal to resultant of two equal vectors acting in opposite directions

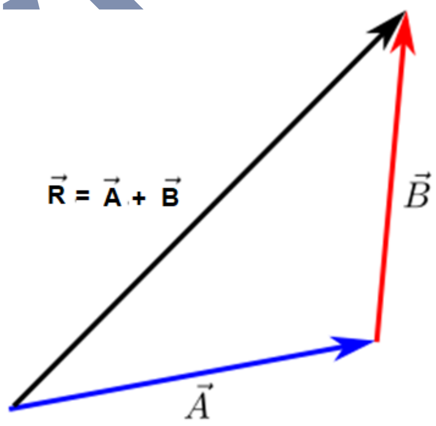
Here $\vec{A} = -\vec{B}$ $\qquad |\vec{A}| = |\vec{B}|$

Now $\vec{A} - \vec{A} = \vec{0}$



HEAD-TO-TAIL

The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.



PROPERTIES OF VECTOR ADDITION

1. COMMUTATIVE LAW: -

"In addition of two vectors $\vec{A}$ and $\vec{B}$ the resultant is independent of the order of addition".

i.e. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

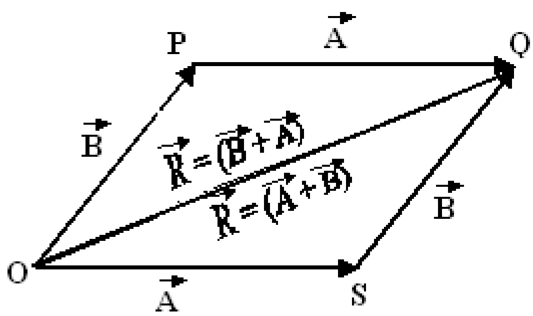
PROOF:-

Consider two vectors forming two adjacent sides of a parallelogram as shown.

From Figure. Using head to tail rule we have

$$\overline{OS} + \overline{SQ} = \vec{OQ}$$

Or $\vec{A} + \vec{B} = \vec{R}$ _______ **(1)**



Again in the same figure.

$$\overline{OP} + \overline{PQ} = \vec{OQ}$$

Or $\vec{B} + \vec{A} = \vec{R}$ _______ **(2)**

Comparing equations (1) & (2)

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

ASSOCIATIVE LAW:-

The resultant of time vectors $\vec{A}$, $\vec{B}$ & $\vec{C}$ remains same irrespective of any order or grouping of vectors.

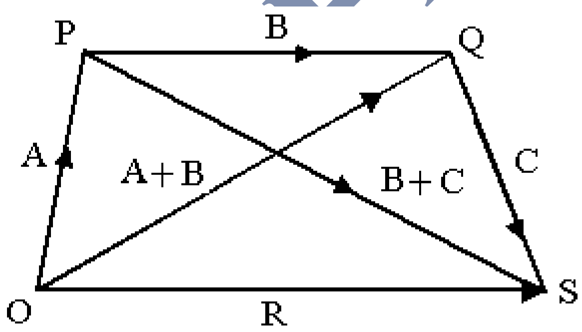
i.e. $$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

PROOF: -

Consider these vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as shown in figure. Using head to tail rule we can write.

$$\vec{OP} + \vec{PQ} = \vec{OQ}$$

Or $$\vec{A} + \vec{B} = \vec{OQ}$$



Again using head to tail rule.

$$\vec{PQ} + \vec{QS} = \vec{PS}$$

$$\left(\vec{B} + \vec{C}\right) = \vec{PS}$$

Now,

$$\vec{OP} + \vec{PS} = \vec{OS}$$

$$\vec{A} + \left(\vec{B} + \vec{C}\right) = \vec{R} \quad \text{-------(1)}$$

Similarly,

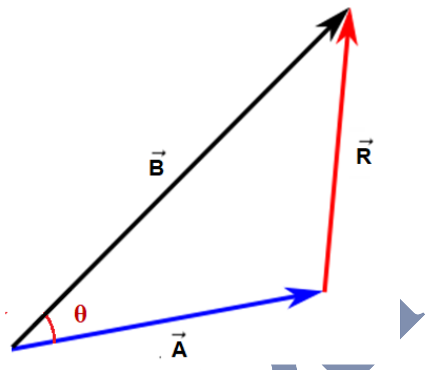
$$\vec{OQ} + \vec{QS} = \vec{OS}$$

$$\left(\vec{A} + \vec{B}\right) + \vec{C} = \vec{R} \quad \text{----------(2)}$$

Comparing equations (1) and (2)

$$\vec{A} + \left(\vec{B} + \vec{C}\right) = \left(\vec{A} + \vec{B}\right) + \vec{C}$$

LAW OF COSINE:-



If two vectors $\vec{P}$ and $\vec{Q}$ act such that their tails coincide, then magnitude is given By the "*LAW OF COSINE* "

$R^2 = A^2 + B^2 - 2\,A\,B\,COS\,(180 - \theta)$ [COS (180- θ) = - COS θ]

$R^2 = A^2 + B^2 - 2\,A\,B\,\{-COS\theta\}$

$R^2 = A^2 + B^2 + 2\,A\,B\;COS\theta$

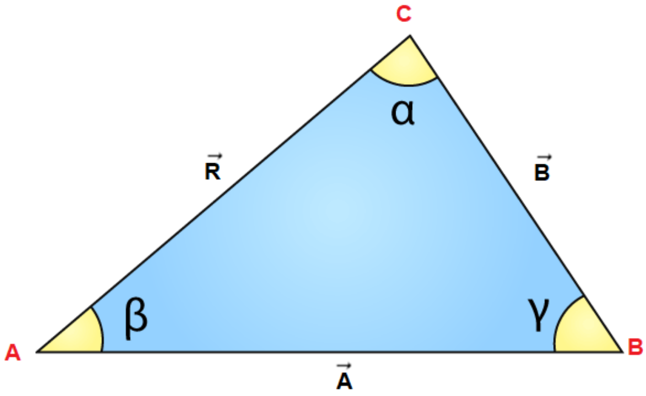
Magnitude of $\vec{R}$

$$R = \sqrt{A^2 + B^2 + 2\,A\,B\cos\theta}$$

Sine Law

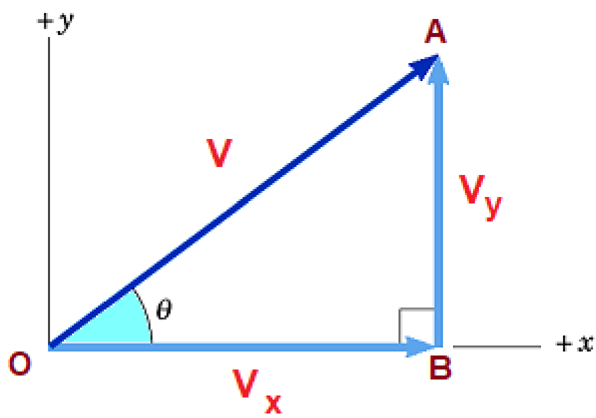
Use of sine formula in Triangle law of vector addition ie magnitude of vector $\vec{A}$ by sin α is to magnitude of vector $\vec{B}$ by sin β is to magnitude of vector $\vec{R}$ by sin γ

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{R}$$



RESOLUTION OF A VECTOR:-

Splitting of a vector into its components with respect to a particular coordinates system is called resolution of a vector.



EXPLANATION:-

Consider a vector $\vec{V}$ making angle θ with x-axis as shown in figure. Draw perpendicular lines AB on x-axis from the head of $\vec{V}$. Here,

$\overline{OB} = V_x$ = X – Components of $\vec{V}$.

$\overline{AB} = V_y$ = Y–Components of $\vec{V}$

The magnitude of those Components can be determined as follows.

X-Component	Y-Component
In Δ OPQ:	In Δ OPQ:
$\text{Cos}\,\theta = \frac{Base}{Hyp}$	$\text{Sin}\,\theta = \frac{Perp}{Hyp}$
$\text{Cos}\theta = \frac{OB}{OA}$	$\text{Sin}\,\theta = \frac{AB}{OA}$
$\text{Cos}\theta = \frac{Vx}{V}$	$\text{Sin}\,\theta = \frac{Vy}{V}$
Or $Vx = V\,\text{Cos}\theta$	$Vy = V\,\text{Sin}\theta$

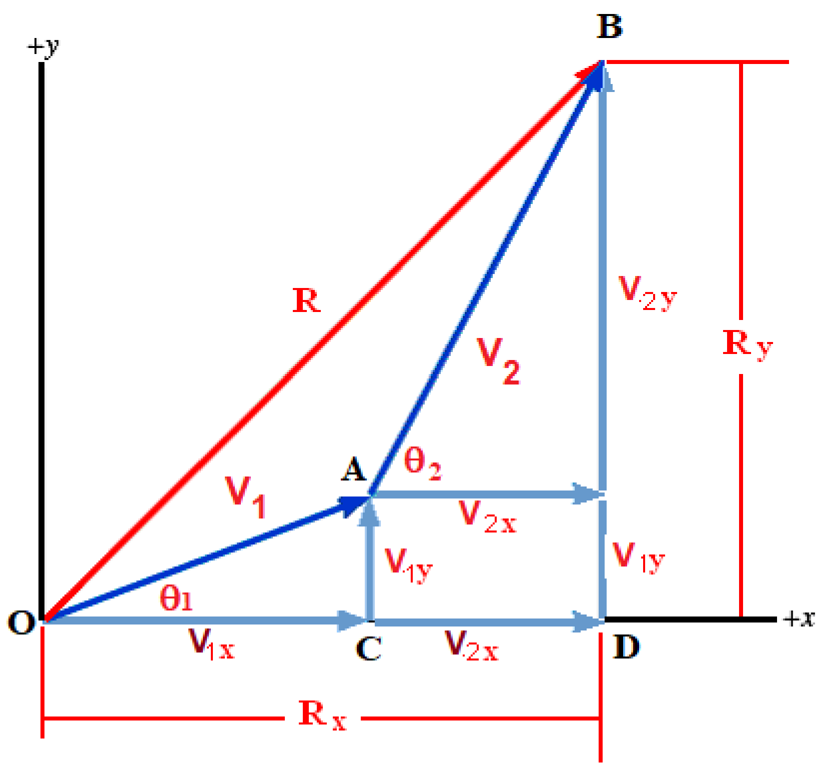
Since Vx and Vy are called rectangular components.

ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

The way of adding vectors with the help of their rectangular components is called edition of vectors by rectangular component method.

supposed two vectors $\vec{V}_1$ and $\vec{V}_2$ making angle θ_1 and θ_2 respectively are to be added.

For this purpose we first adopt head to tail rue and then we draw perpendicular from their heads on x and y axes to get their rectangular components as shown in figure



STEP I:-

Resolve vector $\vec{V}_1$ into its rectangular components i.e. x- components and y-component.

$$\vec{V}_{1x} = V_1 \cos\theta_1 \hat{i} \qquad ; \qquad \vec{V}_{1y} = V_1 \sin\theta_1 \hat{j}$$

Magnitude form of these components are

$$V_{1x} = V_1 \cos\theta_1 \;; \qquad V_{1y} = V_1 \sin\theta_1$$

STEP 2:-

Resolve vector $\vec{V}_2$ into its rectangular components i.e. x- components and y-component.

$$\vec{V}_{2x} = V_2 \cos\theta_2 \hat{i} \qquad ; \qquad \vec{V}_{2y} = V_2 \sin\theta_2 \hat{j}$$

Magnitude form of these components are

$$V_{2x} = V_2 \cos\theta_2 \;; \qquad V_{2y} = V_2 \sin\theta_2$$

STEP 3:-

Resultants vector along x-axis can be determine as

$$\vec{R}_X = \vec{V}_{1X} + \vec{V}_{2X}$$

$$R_x \hat{i} = V_{1x} \hat{i} + V_{2x} \hat{i}$$

$$R_x \hat{i} = (V_{1x} + V_{2x}) \hat{i}$$

Equating the coefficient

$$R_x = V_{1x} + V_{2x}$$

$$R_x = V_1 \cos\theta_1 + V_2 \cos\theta_2$$

STEP 4:-

Resultants vector along x-axis can be determine as

$$\vec{R}_y = \vec{V}_{1y} + \vec{V}_{2y}$$

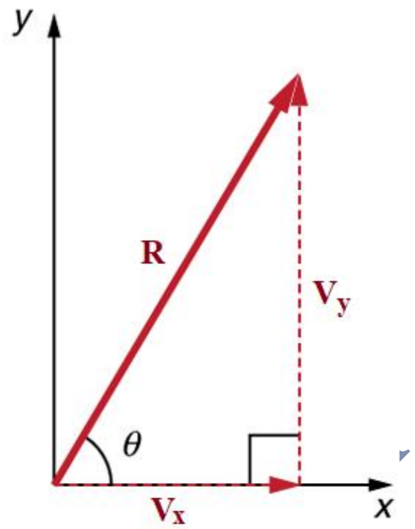
$$R_y \hat{j} = V_{1y} \hat{j} + V_{2y} \hat{j}$$

$$R_y \hat{j} = (V_{1y} + V_{2y}) \hat{j}$$

Equating the coefficient

$$R_y = V_{1y} + V_{2y}$$

$$R_y = V_1 \sin\theta_1 + V_2 \sin\theta_2$$



STEP 5:-

The magnitude of resultant vector is determined by

The formula

$$(Hyp)^2 = (Base)^2 + (Perp)^2$$

$$R^2 = V_x^2 + V_y^2$$

Or

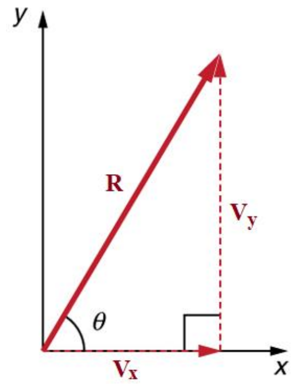
$$R = \sqrt{V_X{}^2 + V_Y{}^2}$$

$$R = \sqrt{(V_1\cos\theta_1 + V_2\cos\theta_2)^2 + (V_1\sin\theta_1 + V_2\sin\theta_2)^2}$$

STEP 6:-

The direction of resultant vector is given by the angle θ which its make with positive x-axis.

From figure



$$\text{Tan}\,\theta = \frac{V_Y}{V_X}$$

$$\theta = \text{Tan}^{-1}\left(\frac{Vy}{Vx}\right)$$

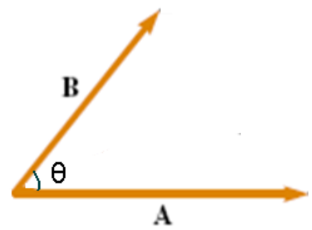
SCALAR PRODUCT OR DOT PRODUCT: -

DEFINITION: -

When two vectors are multiplied, their resultant quantity will be a scalar quantity this is called a scalar product.". A dot (.) is placed the multiplying vectors. Hence it is called "dot product"

FORMULA: -

Consider two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them as shown. Their scalar product is defined as "The product of the magnitude of the vectors and the cosine of the angle between them".



$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\,\text{Cos}\theta$$

EXAMPLES: -

1. **Work = (Force) . (displacement)**

$$W = \vec{F} \cdot \vec{S}$$

2. **power = (Force) . (velocity)**

$$P = \vec{F} \cdot \vec{V}$$

3. **Electric flux = (electric field) . (vector area)**

$$\Phi = \vec{E} \cdot \vec{A}$$

PROPERTIES OF SCALAR PRODUCT

1. COMMUTATIVE LAW IS OBEYED

the dot product unaltered by changing the order of multiplying vectors.

$$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$$

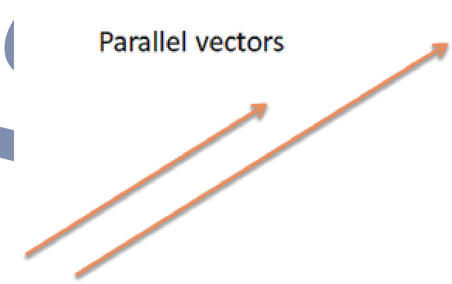
2. DISTRIBUTIVE LAW IS OBEYED

Scalar product obeys distributive law.

$$\vec{A}\cdot\left(\vec{B}+\vec{C}\right)=\vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}$$

3. PARALLEL VECTORS

When two vectors $\vec{A}$ and $\vec{B}$ are parallel, the angle between them is 0º $[\theta=0]$



$$\vec{A}.\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\mathrm{Cos}0^0 \qquad \because \mathrm{Cos}0^o=1$$

$$\vec{A}.\vec{B}=\left|\vec{A}\right|\ \left|\vec{B}\right| \qquad \text{Then }\left(\vec{A}=A\right),\left(\vec{B}=B\right)$$

$$\vec{A}.\vec{B}=A\ \ B$$

4. CONDITION OF PERPENDICULARITY

If two vectors are orthogonal (perpendicular), then their dot product is zero.

$$\vec{A}.\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\mathrm{Cos}\theta$$

$$\vec{A}.\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\mathrm{Cos}90^0 \qquad \because \mathrm{Cos}90^0=0$$

$$\vec{A}.\vec{B}=0$$

CARTESIAN UNIT VECTOR

$(i)\ \ \hat{\imath}.\hat{\jmath}=0 \qquad (ii)\ \ \hat{\imath}.\hat{k}=0 \qquad (iii)\ \ \hat{\jmath}.\hat{k}=0$

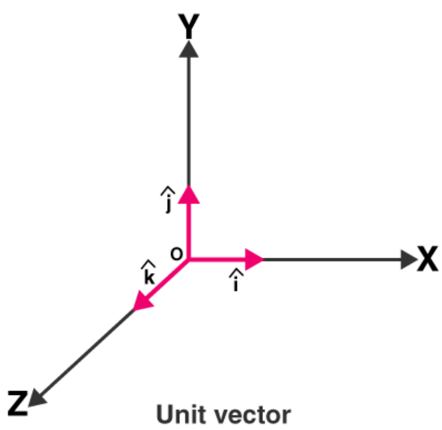
PROOF

$(i)\ \ \hat{\imath}.\hat{\jmath}=0$

$$\vec{A}.\vec{B}=\left|\vec{A}\right|\left|\vec{B}\right|\mathrm{Cos}\theta$$

$$\hat{\imath}.\hat{\jmath}=|\hat{\imath}|\ |\hat{\jmath}|\ \mathrm{Cos}90^0$$

$$\hat{\imath}.\hat{\jmath}=1\times1\times0=0$$



PROOF

(ii) $\hat{i}.\hat{k} = 0$

$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\,Cos\theta$$

$$\hat{i}.\hat{k} = |\hat{i}|\,|\hat{k}|\,Cos90^0$$

$$\hat{i}.\hat{k} = 1 \times 1 \times 0 = 0$$

PROOF

(iii) $\hat{j}.\hat{k} = 0$

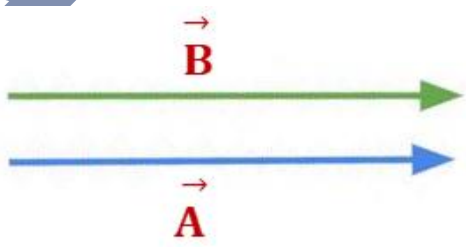
$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\,Cos\theta$$

$$\hat{j}.\hat{k} = |\hat{j}|\,|\hat{k}|\,Cos90^0$$

$$\hat{j}.\hat{k} = 1 \times 1 \times 0 = 0$$

5. EQUAL VECTORS

When two vectors $\vec{A}$ and $\vec{B}$ are parallel, the angle between them is 0º $[\theta = 0]$



$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\,Cos0^0$ $\because Cos0^o = 1$

$\vec{A}.\vec{B} = |\vec{A}|\;|\vec{B}|$ Then $(\vec{A} = A), (\vec{B} = B)$

$\vec{A}.\vec{B} = A\;\;B$ $B = A$

$$\vec{A}.\vec{B} = A\;\;A$$

$$\vec{A}.\vec{B} = A^2$$

IN THE CASE OF CARTESIAN UNIT VECTOR

(i) $\hat{i}.\hat{i} = 1$ **(ii)** $\hat{j}.\hat{j} = 1$ **(iii)** $\hat{k}.\hat{k} = 1$

PROOF

(i) $\hat{i}.\hat{i} = 1$

$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\,Cos\theta$$

$$\hat{i}.\hat{j} = |\hat{i}|\,|\hat{i}|\,Cos0^0$$

$$\hat{i}.\hat{i} = 1 \times 1 \times 1 = 1$$

PROOF

$(ii)\quad \hat{\mathbf{j}}.\hat{\mathbf{j}} \;=\; \mathbf{1}$

$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\,\mathrm{Cos}\theta$$

$$\hat{j}.\hat{j} = |\hat{j}|\,|\hat{j}|\,\mathrm{Cos}0^{0}$$

$$\hat{i}.\hat{k} = 1 \times 1 \times 1 = 1$$

PROOF

$(iii)\quad \hat{\mathbf{k}}.\hat{\mathbf{k}} \;=\; \mathbf{1}$

$$\vec{A}.\vec{B} = |\vec{A}|\,|\vec{B}|\,\mathrm{Cos}\theta$$

$$\hat{k}.\hat{k} = |\hat{k}|\,|\hat{k}|\,\mathrm{Cos}0^{0}$$

$$\hat{j}.\hat{k} = 1 \times 1 \times 1 = 1$$

6. the scalar product of two vectors in terms of rectangular components form: -

For two vectors $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_x\hat{k}$ **and** $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_x\hat{k}$

The dot product is given as

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

PROVE THAT $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Consider two vectors A and B in rectangular components form i.e.

$$\vec{A} = A_x\,\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\,\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Now take the dot product of these two vectors.

$$\vec{A}.\vec{B} = (A_x\,\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\,\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x B_x (\hat{i}.\hat{i}) + A_x B_y (\hat{i}.\hat{j}) + A_x B_z (\hat{i}.\hat{k})$$

$$+ A_y B_x (\hat{j}.\hat{i}) + A_y B_y (\hat{j}.\hat{j}) + A_z B_z (\hat{j}.\hat{k})$$

$$+ A_z B_z (\hat{k}.\hat{i}) + A_z B_y (\hat{k}.\hat{j}) + A_x B_z (\hat{k}.\hat{k}).$$

But $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ **&** $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

Substituting these values in above equation.

$$\vec{A} \cdot \vec{B} = A_x b_x (1) + A_x B_y (o) + A_x B_z (o)$$
$$+A_y B_x (o) + A_y B_y (1) + A_y B_z (o)$$
$$+A_z B_x (o) + A_z B_y (o) + A_z B_z (1).$$

Finally, we are left with

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

DEFINITION

"The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is independent of the order of vectors to be multiplied".

i.e., $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

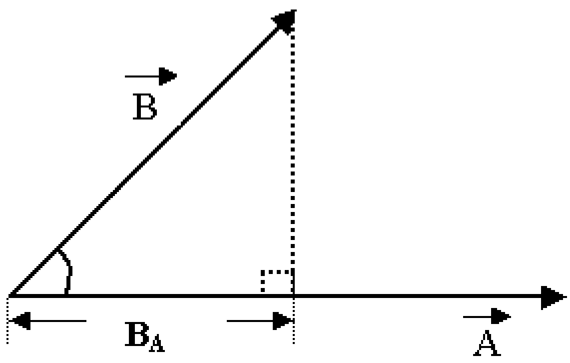
PROOF

The scalar product of two vectors $\vec{A}$ and $\vec{B}$ making angle θ with each other is defined as "The product of magnitude of $\vec{A}$ and the projection of $\vec{B}$ onto $\vec{A}$"

$$\vec{A} \cdot \vec{B} = A B_A$$

$$\vec{A} \cdot \vec{B} = A(B \cos\theta) \qquad B_A = B\cos\theta$$

$$\vec{A} \cdot \vec{B} = AB\cos\theta \quad \text{------ (1)}$$



Similarly we can write $\vec{B} \cdot \vec{A}$ as follows

$$\vec{B} \cdot \vec{A} = BA_B$$

Here A_B is the projection of $\vec{A}$ onto $\vec{B}$

$$\vec{B} \cdot \vec{A} = B(A \cos\theta) \qquad A_B = A\cos\theta$$

$$\vec{B} \cdot \vec{A} = AB\cos\theta \quad \text{-------- (2)}$$

Comparing equation (1) and (2), we have

$$AB\cos\theta = BA\cos\theta$$

And $AB_A = BA_B$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

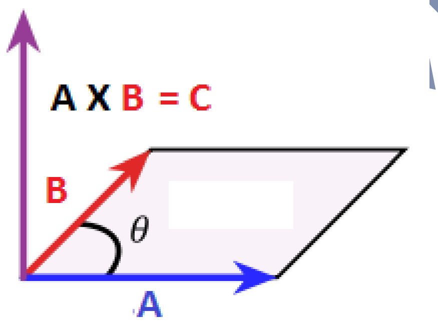
VECTOR PRODUCT

DEFINITION

"Product of two vectors which results in a vector quantity is called vector product".

FORMULA

Consider two vectors $\vec{A}$ and $\vec{B}$ making angle θ with each other as shown. The vector product of $\vec{A}$ and $\vec{B}$ is written as



$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{C} = |\vec{A}|\,|\vec{B}|\,\text{Sin}\,\theta\,\hat{n}$$

$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|\,\text{Sin}\,\theta\,\hat{n}$$

and $$|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}|\,\text{Sin}\,\theta$$

The vector product of two vectors is denoted by a cross between the two vectors. Because of this notation, $\vec{A} . \vec{B}$ is also called the cross product.

(i) **The vector $\vec{C} = \vec{A} \times \vec{B}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$**

(ii) $\hat{n}$ **is a unit vector in the direction of $\vec{C}$.**

RECTANGULAR COMPONENT FORM:-

For two vectors $\vec{A}$ = Axi + Ayj + Azk and

$\vec{B}$ = Bxi + Byj + Bzk.

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$$

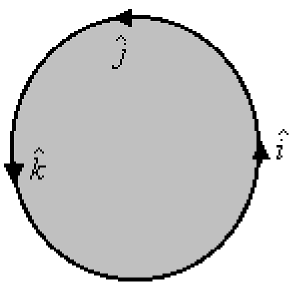
EXAMPLES

1- **Torque** = $\vec{\tau}$ = $\vec{r} \times \vec{F}$

2- **Velocity** = $\vec{V}$ = $\vec{r} \times \vec{\omega}$ $\quad$ $\vec{\omega}$ **is angular velocity**

3- **acceleration** = $\vec{a}$ = $\vec{r} \times \vec{\alpha}$ $\quad$ $\vec{\alpha}$ **is angular acceleration**

PROPERTIES OF VECTOR PRODUCT



The following are some important properties of vector products. Vector product.

1. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
2. $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
3. $(\vec{A} + \vec{B}) \times \vec{C} = \vec{A} \times \vec{C} + B \times \vec{C}$
4. $\vec{A} \neq o, \vec{B} \neq o$ and $\vec{A} \times \vec{B} = o$ then $\vec{A}$ is parallel to $\vec{B}$.
5. $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = o$
6. $\hat{\imath} \times \hat{\jmath} - \hat{k}$

   $\hat{\jmath} \times \hat{k} = \hat{\imath}$

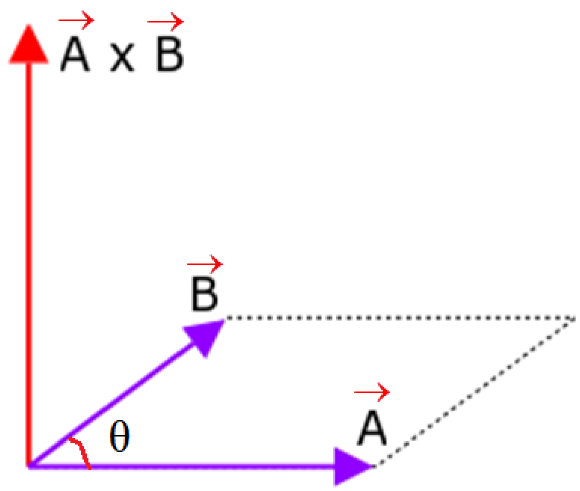
   $\hat{k} \times \hat{\imath} = \hat{\jmath}$

PROVE THAT: $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Consider two vectors $\vec{A}$ and $\vec{B}$ making angle θ with each other. Their vector product is given as:

$$\vec{C} = \vec{A} \times \vec{B}$$

$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|\,\text{Sin}\theta\hat{n} \text{------------- (1)}$$



The direction of $\hat{n}$ is out of the page, the vector $\vec{C}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$. the direction of $\vec{C}$ is determined by right hand rule,

Similarly the vector product $\vec{B} \times \vec{A}$ is given by:

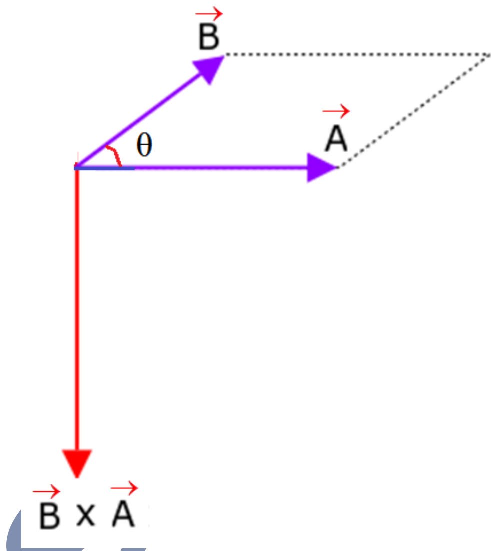
$$\vec{D} = \vec{B} \times \vec{A} = |\vec{B}|\,|\vec{A}|\,\text{Sin}\theta\hat{n}$$

The direction of $\hat{n}$ is out of the paper

$$\vec{D} = \vec{B} \times \vec{A} = |\vec{B}|\,|\vec{A}|\,\mathrm{Sin}\theta\,(-\hat{n})$$

$$\vec{D} = -\vec{B} \times \vec{A} = |\vec{B}|\,|\vec{A}|\,\mathrm{Sin}\theta\hat{n}$$

$$-\vec{D} = |\vec{A}|\,|\vec{B}|\,\sin\theta\,\hat{n} \quad \ldots\ldots\ldots..(2)$$



OR

Comparing equations (1)and (2).

$$\vec{C} = -\vec{D}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

SPEED

Speed is defined as the rate of change in distance. In other words, speed is the distance moved per unit time. It tells us how fast or slow an object is moving.

FORMULA

$$\text{speed} = \frac{\text{distance}}{\text{time taken}}$$

$$v = \frac{d}{t}$$

UNITS OF SPEED

The SI unit of the speed is m/s or ms^{-1}

speed is a scalar quantity

DIMENSIONS

Dimensions of speed are LT^{-1}

AVERAGE SPEED

The average speed is defined as the total distance traveled divided by the total time is takes to travel that distance.

It is denoted by $\langle V \rangle$ or V_{av}

$$\text{Average speed} = \frac{\text{total distance (d) covered}}{\text{total time (t)}}$$

$$v_{av} = \frac{d}{t}$$

BELOW ARE SOME COMMON AVERAGE SPEEDS

The Moving object	Approximate Speed in ms^{-1}	Speed in $km\ h^{-1}$
A man walking	2.5	9.0
Car	12.5	45
A cheetah	30.5	110
Sprinter	10	36

UNIFORM SPEED:

If an object covers an equal distance in an equal interval of time it is moving with a uniform speed.

VARIABLE SPEED:

If the speed of a body is changing with respect to the time it is moving with a variable speed. It is also known as non-uniform speed.

INSTANTANEOUS SPEED:

The instantaneous speed is the speed of an object at a particular moment (instant) in time.

$$v_{inst} = \lim_{\Delta t \to 0} \left(\frac{\Delta S}{\Delta t}\right)$$

VELOCITY

Velocity is defined as the rate of change of displacement. It is speed in a specified direction.

OR

The distance covered by a body in a particular direction per unit time

FORMULA

$$\text{speed} = \frac{\text{dispacement}}{\text{time taken}}$$

$$v = \frac{S}{t}$$

UNIT OF VELOCITY

the SI unit of the speed is m/s or ms^{-1}

velocity is a vector quantity

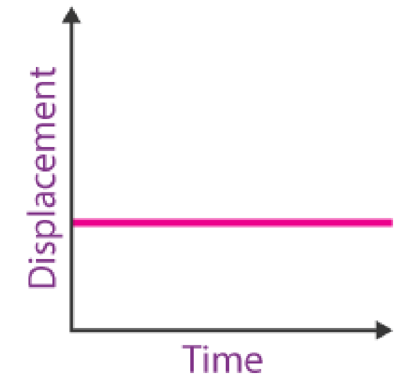
DIMENSIONS

Dimensions of velocity are LT^{-1}

THE DISPLACEMENT –TIME GRAPH

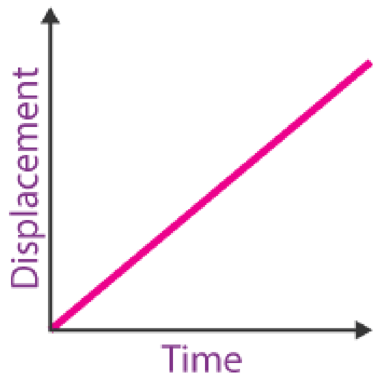
We can study the velocity of a moving object by plotting the displacement moved by the object, S, against, t. The gradient of the graph gives us the velocity of the object. Let us look at the graphs for different types of motion

<u>OBJECT AT REST (STATIONARY)</u>



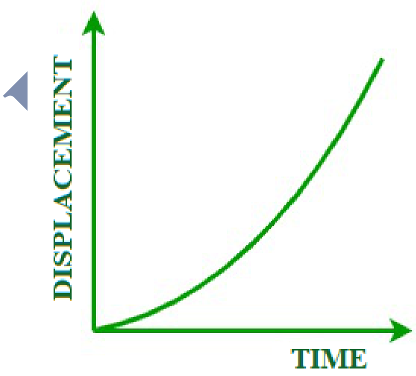
1 There is no change in the displacement over time.
2 The graph is as straight line parallel to time x-axis.
3 velocity = slope of the graph
4. velocity = This graph gives a straight line with a zero slope

<u>OBJECT MOVING WITH CONSTANT VELOCITY</u>

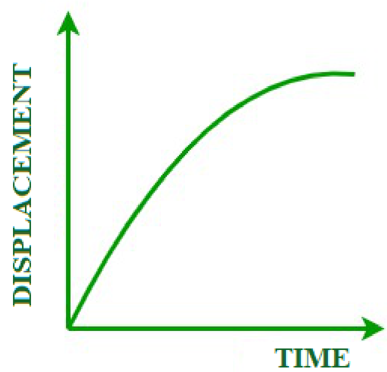


1 The displacement moved per unit time is constant.
2 The graph is as straight with constant Slope.
3 velocity = slope of the graph
4. velocity = This graph gives a straight line With a non-zero slope

OBJECT MOVING WITH ACCELERATION



1 The displacement moved per unit time is not constant.
2 The graph is curved line. The slope is not constant.
3 velocity at certain instant = slope of a graph at that instant



AVERAGE VELOCITY

Consider a body moving along a curved path as shown. Let S_1 be initial position of the body at time t_1, represented by $\vec{r}_1$ with respect to a fixed point 'O'. If the final position is S_2 represented by $\vec{r}_2$ then,

$$\vec{r}_1 + \Delta\vec{r} = \vec{r}_2 \text{ (using head to tail rule)}$$

change in position

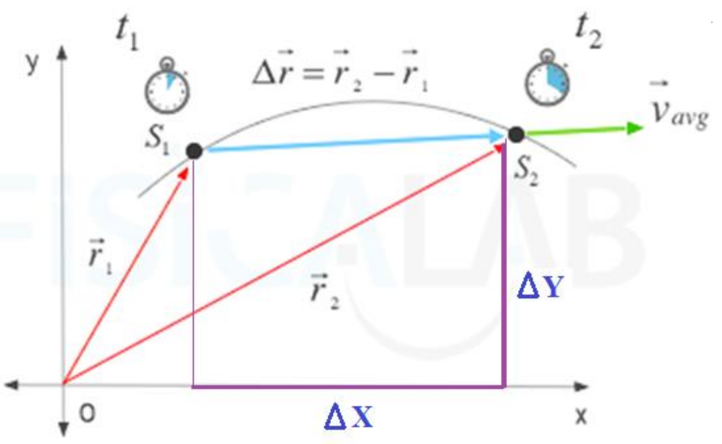
and $$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

time interval.

$$\Delta t = t_2 - t_1$$

But

$$Average\ speed = \frac{Total\ displacement}{elapsed\ time}$$



$$\therefore \quad \vec{V}_{av} = \frac{\vec{r_2} - \vec{r_1}}{t_2 - t_1}$$

$$\vec{V}_{av} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{V}_{av} = \frac{\Delta x}{\Delta t}\,\hat{i} + \frac{\Delta y}{\Delta t}\,\hat{j}$$

$$\vec{V}_{av} = v_{av\,(x)}\,\hat{i} + v_{av\,(y)}\,\hat{j}$$

INSTANTANEOUS VELOCITY

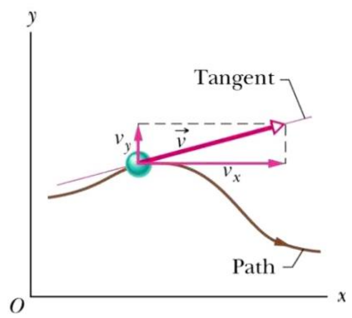
It is defined as the velocity of a body at a particular instant. It gives point to point information of the motion of a body. Mathematically instantaneous velocity can be calculated by reducing the time interval to a very small value i.e., $\Delta t \to 0$.

Thus,

$$v_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta r}{\Delta t}\right)$$

$$v_{inst} = \left(\frac{dr}{dt}\right)$$

$$v_{inst} = \left(\frac{dx}{dt}\right)\hat{\imath} + \left(\frac{dy}{dt}\right)\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$$



UNIFORM VELOCITY

A body has a uniform velocity if it travels in a straight line and moves over equal distances in equal intervals of time, no matter how small these time intervals may be.

When velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time

ACCELERATION

The velocity of a body can be changed in the following ways.

1- **By changing the magnitude of the velocity.**
2- **By changing the direction of the velocity.**
3- **By changing both the direction and the magnitude.**

Any change in velocity of a body is termed as acceleration, which is defined as.

"Rate of change of velocity is called acceleration". It is a vector quantity.

FORMULA

$$a = \frac{\Delta V}{\Delta t} = \frac{\text{change in velocity}}{\text{Time taken}}$$

UNIT

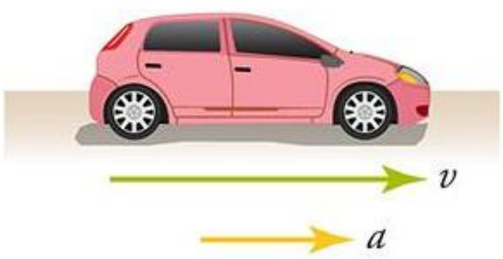
The SI units of acceleration are metres per second per second. i.e., m/sec^2.

DIMENSIONS

∴ **The dimension of acceleration = LT^{-2}**

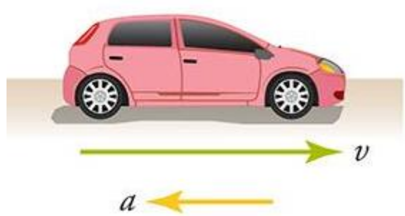
POSITIVE ACCELERATION

When an object's velocity increases while moving, the acceleration produced is called positive acceleration. A body usually has a positive acceleration when its velocity is increasing. Its direction is along the direction of motion of the body, as that is the direction in which the velocity is increasing. Thus, when a body is speeding up, its acceleration and velocity are in the same direction



NEGATIVE ACCELERATION

When an object's velocity decreases while moving, the acceleration produced is called negative acceleration**. A body usually has a negative acceleration when its velocity is decreasing. Its direction is opposite to the direction of motion of the body, as that is the direction in which the velocity is increasing. Thus, when a body is** slowing down**, its acceleration and velocity are** oppositely **directed. Negative acceleration is also called** deceleration **or** retardation.



UNIFOR ACCELERATION *intervals*

A body has a uniform acceleration if it travels in a straight line and its velocity increases or decreases by equal amounts in equal interval of time. In other words, a body has a uniform acceleration if its velocity changes at a uniform rate

NON-UNIFORMLY ACCELERATED MOTION

When an object's velocity increases or decreases by unequal amounts in equal intervals of time, the acceleration of the object is said to be non-uniform.

AVERAGE ACCELERATION

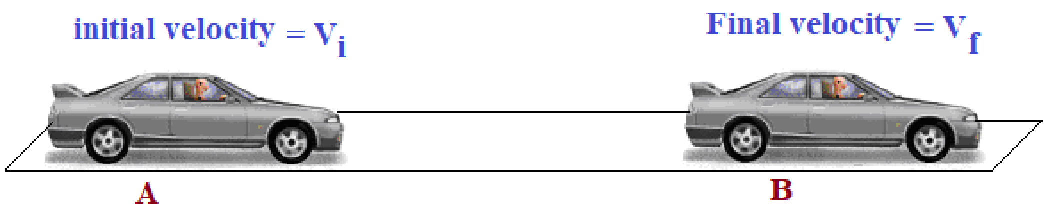
The average acceleration is defined as

"*The total change in velocity divided by the time interval during which that occurred*".

Consider a body moving with variable velocity

$\vec{V}_i$ = **initial velocity at point A**

$\vec{V}_f$ = **final velocity at point B**



$$\vec{\Delta V} = \vec{V}_f - \vec{V}_i$$

Let the time interval be Δt i.e.,

$$\Delta t = t_2 - t_1$$

Now by definition

$$\vec{a}_{av} = \frac{\vec{V}_f - \vec{V}_i}{t_2 - t_1}$$

$$\vec{a}_{av} = \frac{\vec{\Delta V}}{\Delta t}$$

$$\vec{a}_{av} = \frac{\Delta V_x}{\Delta t}\,\hat{i} + \frac{\Delta V y}{\Delta t}\,\hat{j}$$

$$\vec{a}_{av} = a_{av\,(x)}\,\hat{i} + a_{av\,(y)}\,\hat{j}$$

INSTANTANEOUS ACCELERATION

The instantaneous acceleration can be defined as the limit of the average acceleration as the time interval Δt goes to zero.

$\vec{a}_{inst}$ =

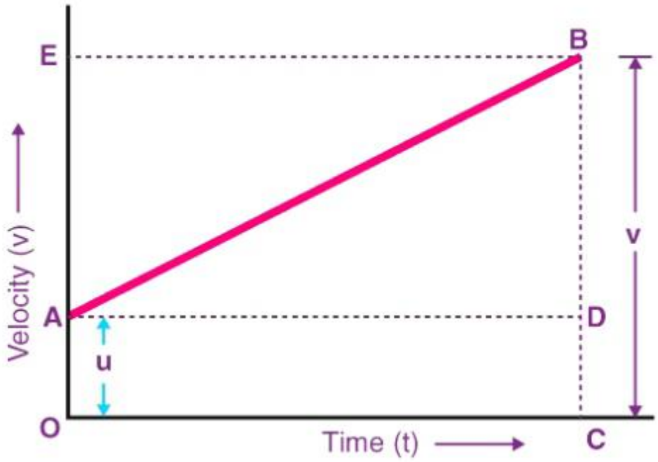
$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \left(\frac{\Delta V}{\Delta t}\right)$$

$$\vec{a}_{inst} = \left(\frac{dr}{dt}\right)$$

$$a_{inst} = \left(\frac{dV_x}{dt}\right)\hat{\imath} + \left(\frac{dV_y}{dt}\right)\hat{\jmath} = a_x\hat{\imath} + a_y\hat{\jmath}$$

DERIVATION OF FIRST EQUATION OF MOTION BY GRAPHICAL METHOD:

The first equation of motion can be derived using a velocity-time graph for a moving object with an initial velocity of u, final velocity v, and acceleration a.



The velocity of the body changes from A to B in time t at a uniform rate.
Following details are obtained from the graph: The initial velocity of the body, u = OA
The final velocity of the body, v = BC
From the graph, we know that

BC = CD + DB { DC = OC }
V = OC + DB
V = U + BD…………(i)

Now, since the slope of a velocity-time graph is equal to acceleration a,
So,

a = slope of line AB

$$a = \frac{BD}{AD}$$

Since AD = OC
AD = t the above equation becomes:

$$a = \frac{BD}{t}$$

BD = at ....(ii)

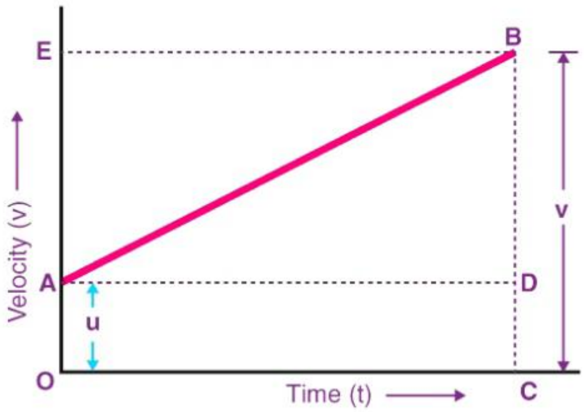
Now, combining Equation (i) & (ii),

v = u + at

DERIVATION OF SECOND EQUATION OF MOTION BY GRAPHICAL METHOD:

The second equation of motion can be derived using a velocity-time graph for a moving object with an initial velocity of u, final velocity v, and acceleration a.
From the graph, we know that

Distance = (area of rectangle) + (area of triangle)



$$S = (OA \times OC) + \frac{1}{2}\,(BD \times AD)$$

$$S = (u\,t) + \frac{1}{2}\,(BD \times t)\text{…………(i)}$$

Now, since the slope of a velocity -time graph is equal to acceleration a,
So,

$$a = \frac{BD}{AD}$$

$$a = \frac{BD}{t}$$

BD = at ....(ii)

Now, substituting the expression for BD in equation (i), we get

$$S = (u\,t) + \frac{1}{2}\,(at \times t)$$

$$S = u\,t + \frac{1}{2}\,a\,t^2$$

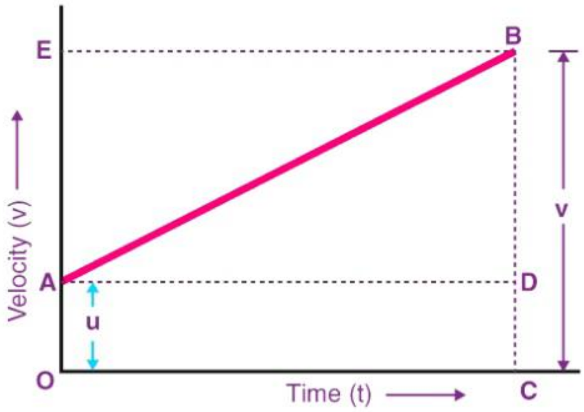
Now $u = v_i$ *, the above equation become*

$$S = v_i + \frac{1}{2}\,a\,t^2$$

DERIVATION OF SECOND EQUATION OF MOTION BY GRAPHICAL METHOD:

The third equation of motion can be derived using a velocity-time graph for a moving object with an initial velocity of u, final velocity v, and acceleration a. From the graph, we know that

Distance = area of trapezium



$$S = \frac{1}{2}\,(\text{sum of parallel sides}) \times \text{height}$$

$$S = \frac{1}{2}\,(BC + OA) \times OC$$

$$S = \frac{1}{2}\,(V + u) \times t \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Acceleration is given by

$$a = \frac{v - u}{t}$$

$$t = \frac{v - u}{a} \;\ldots\ldots\ldots\ldots\ldots\ldots. (ii)$$

Now substituting the value of t in equation (i)

$$S = \frac{1}{2}\,(v + u) \times \frac{(v - u)}{a}$$

$$S = \frac{(v + u)\,(v - u)}{2a}$$

$$2\,a\,S = v^2 - u^2$$

Now $u = v_i$, $V = v_f$ *the above equation become*

$$2\,a\,S = V_f^2 - V_i^2$$

PROJECTILE MOTION

The motion of a body is said to be projectile motion if its velocity has two components in which one component remain constant and other changes continuously throughout the motion. It is a two-dimensional motion under gravitational force and negligible frictional resistance of air this motion follows a parabolic path.

EXAMPLES:-

Some good examples of projectile motion are

1 An object thrown from a window
2 A bomb released from a bomber plane
3 A shell shot from a gun
4 A kicked or thrown ball obliquely into the air , etc

ASSUMPTIONS-

The analysis of a projectile motion becomes very simple if following assumptions are made.

1 The effect of air resistance is negligible
2 The value of 'g' is constant over the range of projectile.
3 The rotation of earth doesn't affect the motion

1. **<u>TIME TO REACH MAXIMUM HEIGHT:</u>**

The time the projectile takes to move from the point of projection to the peak point on its path is called the time to reach the maximum height.

<u>DERIVATION</u>

Let t be time taken by a projectile to reach the maximum height, then it can be found by using the formula

$$V_{fy} = V_{iy} + a_y t \quad \text{.............(i)}$$

Where $V_{iy} = V_{oy}$

$V_{fy} = 0 \qquad a_y = -g$

substituting values in equation(i)

$$O = V_{oy} + (-g)\,t$$

$$\cancel{-}V_{oy} = \cancel{-}gt$$

$$t = \frac{V_{oy}}{g}$$

$$t = \frac{V_o \text{Sin}\theta}{g} \qquad \{V_{oy} = V_o \text{Sin}\theta\}$$

2. **<u>TOTAL TIME OF FLIGHT</u>**

The time taken by a projectile to move from the point of projection to the point where it hits the ground level from which it was launched, is called the total time of flight. It is twice the time to reach the maximum height.

Let $t = \frac{V_o \text{Sin}\theta}{g}$ **= time to reach the maximum height**

$T = 2\ t$ **= total time of flight**

$$T = 2\left(\frac{V_o \text{Sin}\theta}{g}\right)$$

$$T = \frac{2V_o \text{Sin}\theta}{g}$$

2. MAXIMUM HEIGHT:

The maximum vertical distance attained by the projection is called the maximum height.

DERIVATION

Let H be the maximum height reached by the projectile, then it can be found by using the formula

$$y = v_{iy}\, t + \frac{1}{2} a_y t^2 \quad \dots\dots (i)$$

Where $y = H \qquad V_{iy} = V_{oy} = V_o \sin\theta$

$a_y = -g \qquad t = \frac{V_o \text{Sin}\theta}{g}$

substituting values in equation (i)

$$H = V_{0Y}\, t + \frac{1}{2}(-g)\, t^2$$

$$H = V_0 \text{Sin}\theta \left(\frac{V_0 \text{Sin}\theta}{g}\right) + \frac{1}{2}(-g)\left(\frac{V_0 \text{Sin}\theta}{g}\right)^2$$

$$H = \frac{V_0{}^2 \sin^2\theta}{g} + \frac{1}{2}(-g)\frac{V_0{}^2 \sin^2\theta}{g^2}$$

$$H = \frac{V_0{}^2 \sin^2\theta}{g} - \frac{1}{2}(\not{g})\frac{V_0{}^2 \sin^2\theta}{\not{g}\ g}$$

$$H = \frac{V_0{}^2 \sin^2\theta}{g} - \frac{V_0{}^2 \sin^2\theta}{2g}$$

$$H = \frac{2V_0{}^2 \sin^2\theta - V_0{}^2 \sin^2\theta}{2g}$$

$$H = \frac{V_0{}^2 \sin^2\theta}{2g}$$

4. HORIZONTAL RANGE:-

The distance between the point of projection and the point where it hits the level from which it was launched is known as its horizontal range.

DERIVATION

Let R be the range of the projectile, then it can be found by using the formula

$$x = v_{ix}\ t + \frac{1}{2} a_x\ t^2 \ldots\ldots (i)$$

Where $X = R$ $\quad V_{ix} = V_{ox} = V_o \cos\theta$

$a_y = -g$ $\quad t = T = \frac{2V_o \sin\theta}{g}$

substituting these values in equation (i)

$$R = V_0 \cos\theta \times \frac{2V_o \sin\theta}{g} + \frac{1}{2}(0)\ t^2$$

$$R = (V_o \cos\theta)\left(\frac{2V_o \sin\theta}{g}\right)$$

$$R = \frac{V_o{}^2\ 2\sin\theta\ \cos\theta}{g} \qquad \because 2\sin\theta\cos\theta = \sin 2\theta$$

$$R = \frac{V_o{}^2\ \sin 2\theta}{g}$$

5. **<u>THE MAXIMUM RANGE</u>**

Consider $R = \frac{V_o^2}{g} \operatorname{Sin} 2\theta$. **It is clear form this equation that, for a given initial velocity V_o, the horizontal range depends upon the value of Sin2θ**

Therefore, when Sin2θ = 1, range will be maximum then $R = R_{max}$

Hence, $R_{max} = \frac{V_o^2}{g}$ **For Sin2θ =1; 2θ = 90º**

$$\theta = \frac{90^o}{2}$$

$$\theta = 45^0$$

Thus, a projectile must be projected at an angle of 45º to attain the maximum range R_{max}

THE RANGE OF PROJECTILE AT COMPLEMENTARY ANGLES ARE SAME

The angles 40° and 50° are called complementary angles because they add up to 90°. Other examples of complementary pairs are: 30° and 60°; 15° and 75° etc. In other words, the range of a projectile will be the same for elevation angles of θ and 90° – θ