

PHYSICS XI

UNIT 3

DYNAMICS

PROF:IMRAN HASHMI



DYNAMICS

Dynamics has its origins in the Greek word *dynamis*, "force, power." In physics, Dynamics is the branch of classical mechanics that is concerned with the study of forces and their effects on motion

FORCE

Force is that agent which produce motion or tends to produce motion or stops motion or tends to stop motion.

OR

Force may also be defined as it is that agent which changes or tends to change a body's state of rest or of uniform motion in a straight line.

OR

The rate of change of linear momentum of a body is called Force.

NEWTON'S FIRST LAW OF MOTION

In the first law of motion, Newton states that,

If the body is at rest, it will remain at rest. If the body is moving, it will continue to move forever at the same velocity (same speed and direction) unless it is acted upon by an external force.

EXPLANATION

This law consists of two parts i.e. "if a body is at rest it will remain at rest until it is acted upon an external force."

FOR EXAMPLE

A book lying on a table or a table in a room will not change its position until an external force acts on it.

The other part of the law is "if a body is in uniform motion in a straight line. it will continue its state until an external force acts upon it."

For example

When we roll a ball on a surface it will come to rest due to friction and air resistance if these forces will not oppose the motion of the ball it will continue its uniform motion.

From the above discussion we conclude two very important concepts.

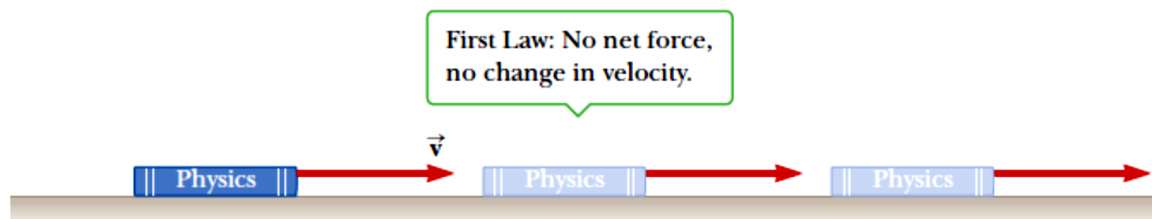
- Force may be defined as it is that changes or tends to change a body's state of rest or uniform motion in a straight line**
- THE INERTIA OF A BODY**

All material objects possess the property of opposing any change in their state of rest or of uniform motion in a straight line. So, we conclude that if there is no external force acting on a body it will be in an inertial frame.

In other words, “Inertia is the tendency of an object to resist a change in its state”. Therefore, the first law of motion is also called the “Law of Inertia”.

EXPLANATION

A block of stone which is large can hardly be pushed along the ground while a book can easily be pushed along the ground. The mass of the stone or book is a measure of inertia.



EXAMPLE

- 1 A person riding a bicycle along a level road does not comes to rest immediately he stops pedaling. The bicycle continues to move forward due to inertia.
- 2 A small coin is put on a card and placed over the mouth of a glass. When the card if flicked away with the finger horizontally, the coin drops neatly in to the glass due to inertia.

NEWTON'S SECOND LAW

This law provides the most fundamental equation of mechanics and stated as follows:

STATEMENT

When a net force acts on an object, it will cause the object to accelerate in the direction of the force. The amount of acceleration of an object is directly proportional to the net force and inversely proportional to its mass

$$a \propto \frac{\Sigma F}{m}$$

MATHEMATICAL FORM

Let m = mass of a body

ΣF = vector sum of all forces acting on the resultant force on the body

a = acceleration produced.

Now by definition

$$\Sigma F = m a$$

EXPLANATION

The second law gives the quantitative definition of force and thus provides a means to calculate it

1- When $m = \text{constant}$ then $a \propto F \dots\dots\dots(i)$

i.e., the greater the force applied on a fixed mass the larger will be the acceleration produced.

2- When $F = \text{constant}$ then $a \propto \frac{1}{m}$ (ii)

i.e., the greater the mass of a body for a constant force to be applied the less will be the acceleration.

On combining equation (i) and (ii), we get

$$a \propto \frac{F}{m}$$

$$a = k \frac{F}{m} \dots \dots \dots (iii)$$

Where, $k = \text{constant of proportionality}$

1 Newton force is defined as the mass of 1kg object produces the acceleration of 1m/s^2

$$1 = k \frac{1}{1}$$

$$1 = K$$

\therefore by putting the value of $k=1$ in equation (iii)

$$F = ma$$

UNIT OF FORCE

The SI unit of force is Newton.

The unit of force is as follows

$$F = m \times a$$

Putting $m = 1 \text{ kg}$ and $a = 1 \text{ m/s}^2$, F becomes 1 newton.

So,

$$1 \text{ Newton} = 1\text{kg} \times 1 \text{ m/s}^2$$

So, the SI unit of force is kg.m/s^2 which is represented by N (Newton).

NEWTON

One Newton Force is defined as follows

“One Newton is that amount of force which produces an acceleration of one meter per second per second in a mass of one kilogram.”

$$1\text{N} = 1 \text{ kg. } 1 \text{ m/s}^2$$

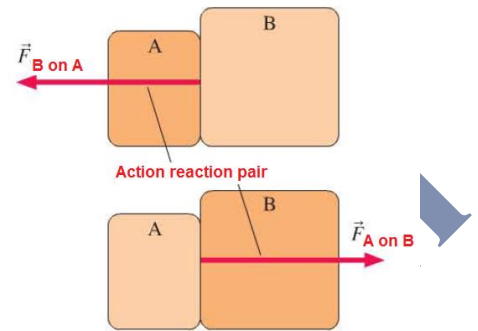
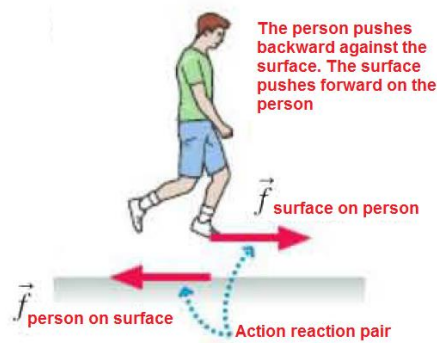
NEWTON'S THIRD LAW

STATEMENT

If two objects interact, the force F_{AB} exerted by object A on object B is equal in magnitude but opposite in direction to the force F_{BA} exerted by object B on object A

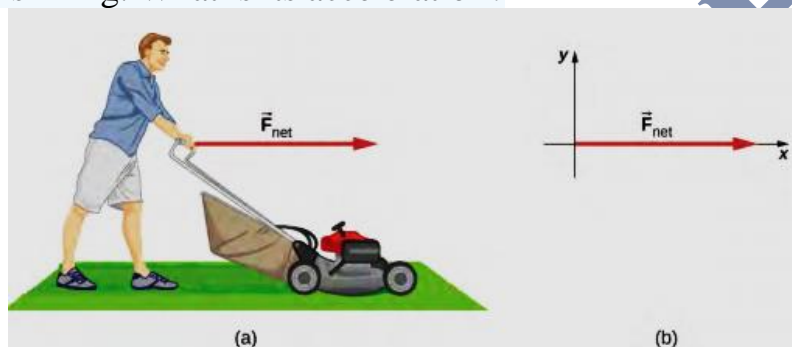
OR

“The forces that two interacting bodies exert on each other are always equal in magnitude and opposite in direction.”.



WORKED EXAMPLE -1

A man exerted the net external force on a lawn mower is 51 N parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?



SOLUTIONS

$$F = m a$$

$$a = \frac{F}{m}$$

$$a = \frac{51}{24}$$

$$a = 2.1 \text{ m/s}^2$$

LINEAR MOMENTUM

DEFINITION

The linear momentum of an object of mass **m** moving with a velocity **v** is defined as **the product of mass of a body and its velocity is called momentum**. It is a vector quantity.

FORMULA

Let m = mass of a body
 v = linear velocity
and P = linear momentum
Now by definition we can write

$$P = mv$$

UNIT

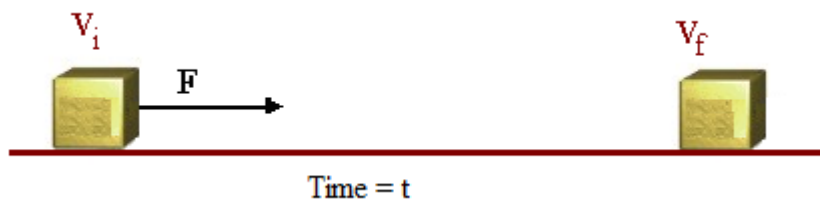
The unit of momentum are Kg m/s
(Ns)Newton sec is the unit of momentum in SI system

DIMENSIONS

$$\begin{aligned} P &= mv \\ [P] &= MLT^{-1} \end{aligned}$$

NEWTONS SECOND LAW AND LINEAR MOMENTUM

Consider a mass “m” moving with velocity V_i and force “F” changes its velocity “ V_f ” in time “t”.



According to Newton’s second law of motion, the net force acting on an accelerating body is given by

$$F = ma \text{ ----- (i)}$$

The acceleration of body of mass m is given by

$$a = \frac{V_f - V_i}{t} \text{ ----- (ii)}$$

Substituting the expression of acceleration from equation (ii) in equation (i)

$$F = m \left(\frac{V_f - V_i}{t} \right)$$

$$F = \left(\frac{m V_f - m V_i}{t} \right)$$

$$F = \left(\frac{\text{Change in momentum}}{\text{time interval}} \right)$$

Net force = rate of change of momentum

IMPULSE

Impulse is defined as “*The sudden force acting on an object for a short interval of time*”. It is denoted by “J”.

FORMULA

$$\text{Impulse} = \text{Force} \times (\text{final time} - \text{initial time})$$

$$\text{Impulse} = \text{Force} \times \Delta t$$

$$J = F \times \Delta t$$

UNIT

The SI unit of impulse is N s

The unit of impulse is kg m/s

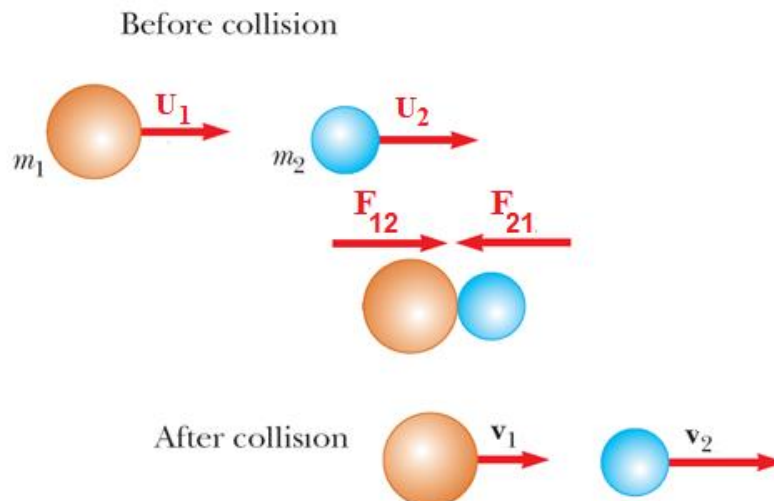
LAW OF CONSERVATION OF MOMENTUM

STATEMENT

In an isolated system of two interacting bodies, The total momentum of the system remains constant.

PROOF

Consider an isolated system of two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 respectively. after collision their velocities becomes v_1 and v_2 .



When two bodies collide with each other, they come in contact for a time interval t . During this interval let the average force exerted by body m_1 on body m_2 be F_{12}

Change in momentum of ' m_2 ' after collision = $m_2 v_2 - m_2 u_2$

$$\text{Rate of change of momentum of body A} = \frac{m_2 v_2 - m_2 u_2}{t}$$

According to Newton's second law of motion;

Net Force acting on the object = The time rate of change of momentum

Therefore,

$$F_{12} = \frac{m_2 v_2 - m_2 u_2}{t}$$

Similarly for body ' m_1 ' we can write

$$F_{21} = \frac{m_1 v_1 - m_1 u_1}{t}$$

According Newton's third law of motion these forces will be equal but opposite.

$$\begin{aligned} F_{12} &= -F_{21} \\ \frac{m_2 v_2 - m_2 u_2}{t} &= -\left(\frac{m_1 v_1 - m_1 u_1}{t}\right) \\ m_2 v_2 - m_2 u_2 &= -m_1 v_1 + m_1 u_1 \end{aligned}$$

$$\therefore m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\left(\begin{array}{c} \text{Total momentum} \\ \text{before collision} \end{array}\right) = \left(\begin{array}{c} \text{Total momentum} \\ \text{after collision} \end{array}\right)$$

This result is known as the **conservation of momentum**.

ELASTIC COLLISION

A collision in which both the momentum and kinetic energy of the system before and after collision remain constant is called Elastic Collision.

EXAMPLES

1. The collision between the atoms is also an example of elastic collision.
2. The collision between two billiard balls is an example of elastic collision.

INELASTIC COLLISION

A collision in which the momentum of the system before and after collision remains conserved but the kinetic energy does not, is called Inelastic Collision.

TYPES OF INELASTIC COLLISION

There are two types of inelastic collision

1. Perfectly inelastic collision
2. partially inelastic collision

PERFECTLY INELASTIC COLLISION

A perfectly inelastic collision can happen when the maximum amount of energy in the system is lost.

Examples:

An arrow hitting a target board.

In a perfectly inelastic collision, the colliding particles stick together

PARTIALLY INELASTIC COLLISIONS

In the real world, partially inelastic collisions are the most common type of collision. The objects that are involved in the collision do not stick, but some energy is lost. [Friction](#), sound, and heat are some of the ways that energy can be lost in a partial inelastic collision

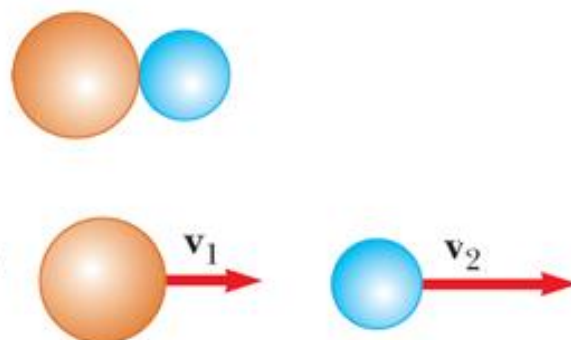
ELASTIC COLLISION IN ONE DIMENSION

Consider two non-rotating spheres of masses m_1 and m_2 moving along the line joining their centers with initial velocities u_1 and u_2 respectively. Where $u_1 > u_2$

Before collision



After collision



When they collide with each other, their velocities become v_1 and v_2 as shown.

Now,

Momentum of the
system before collision = $m_1 u_1 + m_2 u_2$

Momentum of the system after collision = $m_1 v_1 + m_2 v_2$

Using the law of conservation of momentum.

$$\begin{aligned} \left(\begin{array}{c} \text{Total momentum} \\ \text{before collision} \end{array} \right) &= \left(\begin{array}{c} \text{Total momentum} \\ \text{after collision} \end{array} \right) \\ m_1 u_1 + m_2 u_2 &= m_1 v_1 + m_2 v_2 \quad \text{-----(1)} \end{aligned}$$

$$\begin{aligned} m_1 u_1 - m_1 v_1 &= m_2 v_2 - m_2 u_2 \\ m_1(u_1 - v_1) &= m_2(v_2 - u_2) \quad \text{-----(2)} \end{aligned}$$

$$\text{K.E. of the system before collision} = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$\text{K.E. of the system after collision} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Since the collision is elastic therefore K.E. before and after collision must also be equal.

$$\begin{aligned} \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\ m_1 u_1^2 + m_2 u_2^2 &= m_1 v_1^2 + m_2 v_2^2 \\ m_1 u_1^2 - m_1 v_1^2 &= m_2 v_2^2 - m_2 u_2^2 \\ m_1(u_1 + v_1)(u_1 - v_1) &= m_2(v_2 + u_2)(v_2 - u_2) \quad \text{-----(3)} \end{aligned}$$

Divide equation (3) by equation (2).

$$\begin{aligned} \frac{m_1(u_1 + v_1)(u_1 - v_1)}{m_1(u_1 - v_1)} &= \frac{m_2(v_2 + u_2)(v_2 - u_2)}{m_2(v_2 - u_2)} \\ u_1 + v_1 &= v_2 + u_2 \quad \text{-----(4)} \end{aligned}$$

FORMULA FOR V_1 :

We can write the equation (4) in the following form

$$v_2 = v_1 + u_1 - u_2$$

Substitute this value of V_2 in equation (1)

$$\begin{aligned} m_1 u_1 + m_2 u_2 &= m_1 v_1 + m_2(u_1 + v_1 - u_2) \\ m_1 u_1 + m_2 u_2 - m_2 u_1 + m_2 u_2 &= m_1 v_1 + m_2 u_1 + m_2 v_1 - m_2 u_2 \end{aligned}$$

$$m_1 u_1 - m_2 u_1 + 2m_2 u_2 = m_1 v_1 + m_2 v_1$$

$$(m_1 - m_2)u_1 + 2m_2 u_2 = (m_1 + m_2)v_1$$

$$\frac{(m_1 - m_2)u_1 + 2m_2 u_2}{(m_1 + m_2)} = v_1$$

or

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) u_1 + \left(\frac{2m_2}{m_1 + m_2} \right) u_2 \text{ -----(5)}$$

FORMULA FOR V₂:

We can write the equation (4) in the following From

$$v_1 = v_2 + u_2 - u_1$$

Substitute this value of V₁ in equation. (1)

$$m_1 u_1 + m_2 u_2 = m_1 (v_2 + u_2 - u_1) + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v_2 + m_1 u_2 - m_1 u_1 + m_2 v_2$$

$$2m_1 u_1 + m_2 u_2 - m_1 u_2 = m_1 v_2 + m_2 v_2$$

$$2m_1 u_1 + (m_2 - m_1) u_2 = (m_1 + m_2) v_2$$

$$\frac{2m_1 u_1 + (m_2 - m_1) u_2}{(m_1 + m_2)} = v_2$$

or

$$v_2 = \left(\frac{2m_1}{m_1 + m_2} \right) u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) u_2 \text{ -----(6)}$$

FRICTION

Friction is the force that resists motion when the surface of one object comes in contact with the surface of another.

There are two main kinds of friction

1. CONTACT FRICTION

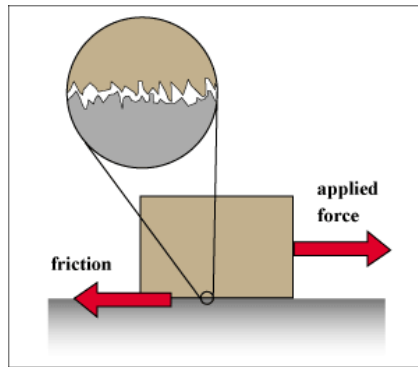
Contact friction arises when one solid object is sliding over another solid object.

2. FLUID FRICTION

Fluid friction arises when a solid body moves through a fluid (gas or liquid).

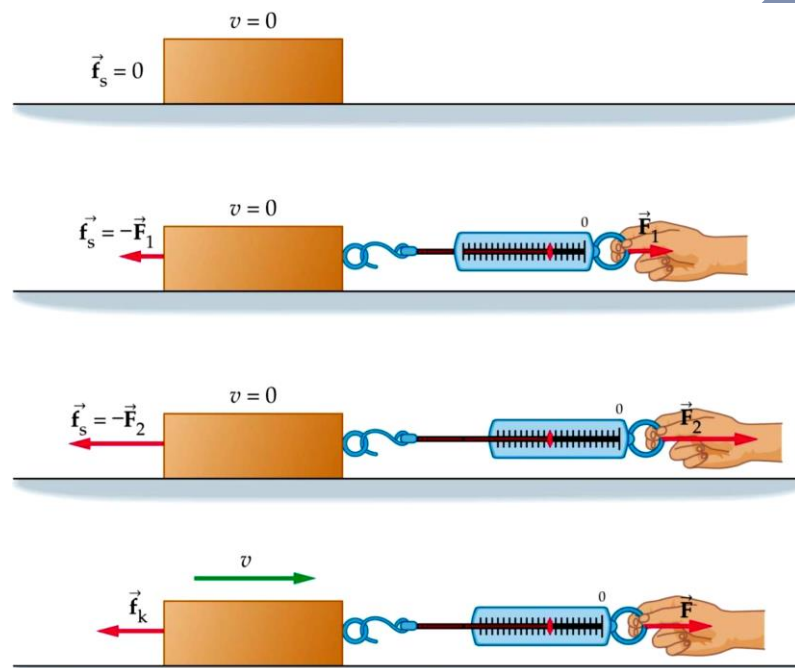
CAUSES OF FRICTION

Ordinary surfaces have projections and depressions, when such surfaces of two bodies are in physical contact, the gross interlocking of projections and depressions take place which opposes the relative motion.



EXPLANATION

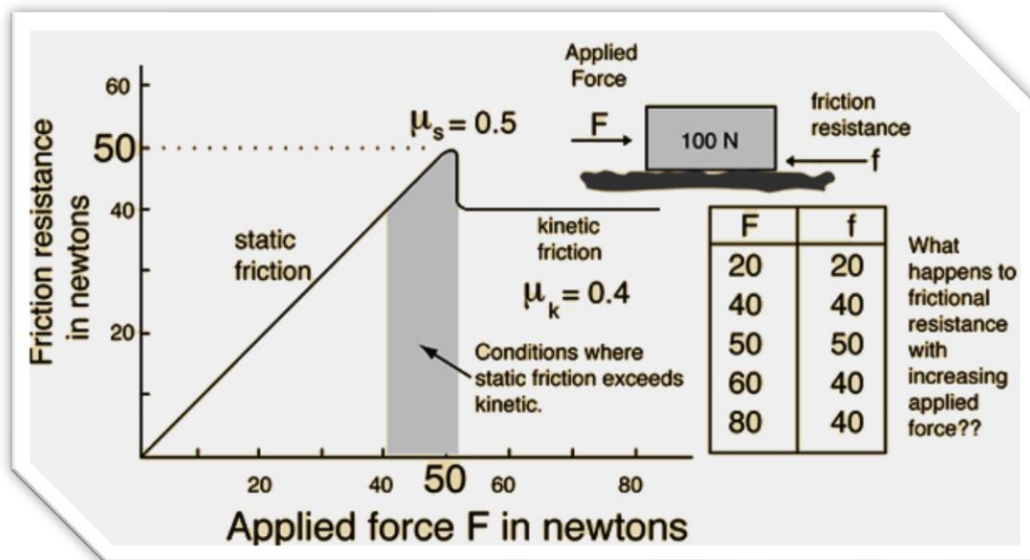
Consider a wooden block placed on a horizontal surface and is attached to one end of a cord. While the other end of the cord is connected to a spring balance.



If we apply force “F” on the block horizontally we observe that initially it will not move, while if we increase the applied force from its frictional force, the block will move. This gives an idea that the frictional force increases with the increase of applied force. It is so-called self-adjusting force. But if the body is on the verge of motion then frictional force is maximum and is called limiting friction (f).

In the above case, applied force is equal to the force of friction. While if we increase the applied force from the limiting friction, the block will slide on the surface. In this case the applied force is greater than frictional force.

We can better explain the whole procedure by the following table and a graph.



It is observed that the fractional force (limiting) is directly proportional to the Normal Reaction (R) of the surface which is opposite to the weight of the block.

TYPES OF FRICTION

STATIC FRICTION

Static friction is the friction that exists between a stationary object and the surface on which it's resting

Mathematically,

$$f_s = \mu_s R$$

$$\frac{f_s}{R} = \mu_s$$

Where μ_s coefficient of static friction (μ) has no unit because it is the ratio of frictional force and normal reaction. Its value depends upon the nature of the surfaces but independent of the area of surfaces in contact.

KINETIC FRICTION

Kinetic friction is defined as a force that acts between moving surfaces. A body moving on the surface experiences a force in the opposite direction of its movement. The magnitude of the force will depend on the coefficient of kinetic friction between the two materials.

LAWS OF FRICTION

- 1 The frictional force opposes the relative motion of two surfaces.
- 2 The frictional force is parallel to the surfaces in contact.
- 3 The frictional force is proportional to the normal reaction.
- 4 The frictional force is independent of the area of contact.
- 5 The friction experienced by the object is dependent on the nature of the surface it is in contact with.
- 6 The kinetic friction is independent of the relative velocities of the surfaces

ANGLE OF FRICTION

The angle made by the resultant of the normal reaction and limiting friction with the normal reaction is called the angle of friction

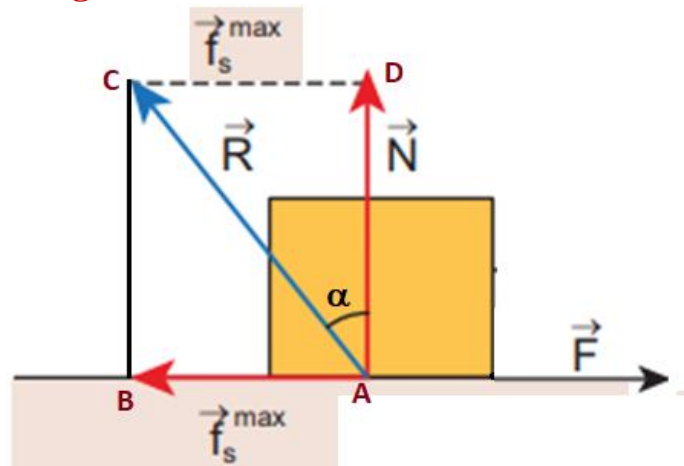
Consider a triangle ADC

$$\tan \alpha = \frac{CD}{AD}$$

$$\tan \alpha = \frac{f}{N} \quad \{ N = R \}$$

$$\tan \alpha = \frac{f}{N} \quad \{ N = \frac{f}{\mu} \}$$

$$\tan \alpha = \mu \quad \{ \mu = \frac{f}{N} \}$$



Therefore, the tangent of an angle of friction is called the coefficient of friction.

RELATION BETWEEN ANGLE OF FRICTION AND ANGLE OF REPOSE

The angle of repose is defined as the minimum angle made by an inclined plane with the horizontal such that an object placed on the inclined surface just begins to slide.

Consider the plane makes an angle θ with the horizontal, the body just begins to move. Let ' R ' be the normal reaction of the body and ' f ' be the frictional force. As shown in the figure

Here

$$W \sin \theta = f$$

$$mg \sin \theta = f \dots\dots\dots(i)$$

$$W \cos \theta = R$$

$$mg \cos \theta = R \dots\dots\dots(ii)$$

Dividing equation (i) by (ii)

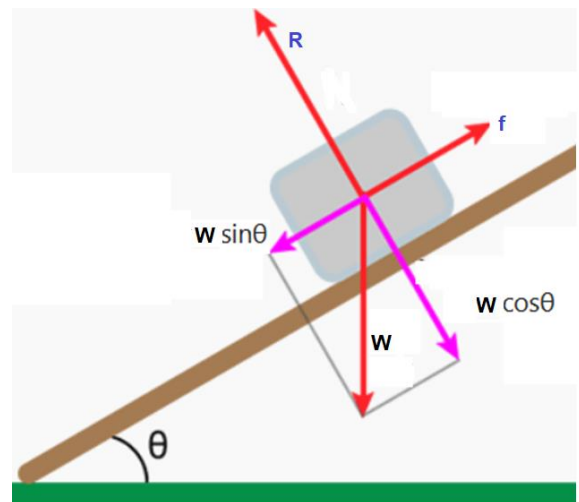
$$\frac{mg \sin \theta}{mg \cos \theta} = \frac{f}{R}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{f}{R} \quad \{ \mu = \frac{f}{R} \}$$

$$\tan \theta = \mu \quad \{ \tan \alpha = \mu \}$$

$$\tan \theta = \tan \alpha$$

$$\theta = \alpha$$



The angle of repose is equal to the angle of friction

CHAPTER = 3 DYNAMICS

SHORTS QUESTIONS (BOOK- 11)

ANSWER:

NEWTON'S SECOND LAW OF MOTION

STATEMENT:

When a force acts on an object, it produces acceleration in its own direction, which is directly proportional to the magnitude of the force and inversely proportional to the mass of the object.

EXPLANATION

Second law gives the quantitative definition of force and thus provides a means to calculate it

1- When $m = \text{constant}$ then $a \propto F$ (i)

i.e., the greater the force applied on a fixed mass the larger will be the acceleration produced.

2- When $F = \text{constant}$ then $a \propto \frac{1}{m}$ (ii)

i.e., the greater the mass of a body for a constant force to be applied the less will be the acceleration.

On combining equation (i) and (ii), we get

$$a \propto \frac{F}{m}$$

$$a = k \frac{F}{m} \dots \dots \dots (iii)$$

Where, $k = \text{constant of proportionality}$

1 Newton force is defined as the mass of 1kg object produces the acceleration of 1m/s^2

$$1 = k \frac{1}{1}$$

$$1 = K$$

\therefore by putting the value of $k=1$ in equation (iii)

$$F = ma$$

Unit:

In M.K.S system, its unit of force is N (kg.m/s²).

In C.G.S system its unit of force is Dyne(gm.cm/s²).

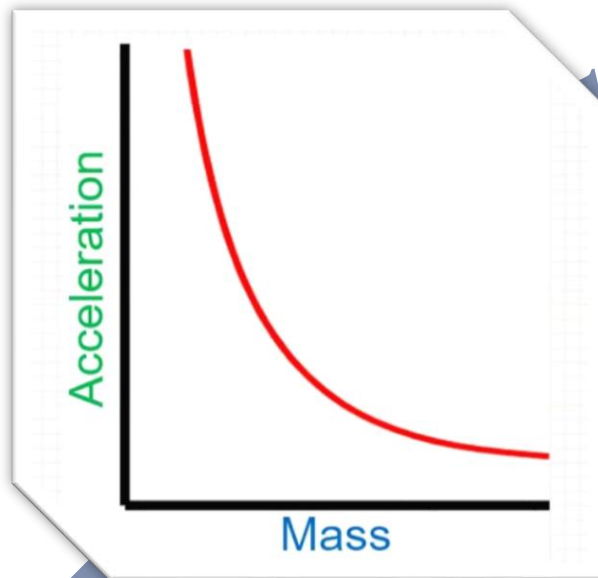
In F.P.S System its unit of force is ft-lb/s².

QUESTION 2: How does mass affect an object's acceleration?

ANSWER: Mass and acceleration are inversely proportional according to Newton's Second Law of Motion

$$a \propto 1/m$$

Acceleration is inversely proportional to mass. Thus, the resulting acceleration-mass graph is an inverse (hyperbola)



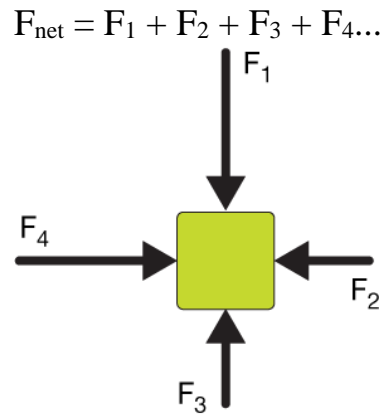
QUESTION 3: What are the different types of forces that can act on an object?

ANSWER: There are many different types of forces that can act on an object. Here are some of the most common:

- Gravitational force
- Electric force
- Magnetic force
- Frictional force
- Normal force
- Tension force
- Spring force
- Air resistance force

QUESTION 4: How do you calculate net force?

ANSWER: The net force is the vector sum of all the forces acting on an object. It is calculated using the following formula:



QUESTION 5: State the law of conservation of momentum.

ANSWER: LAW OF CONSERVATION OF LINEAR MOMENTUM

Statement:

The momentum of an isolated system always remains constant.

OR

If there is no external force applied to a system, then the total momentum of that system remains constant.

QUESTION 6: What is the difference between elastic and inelastic collisions?

ANSWER: The main difference between elastic and inelastic collisions is the conservation of kinetic energy. In an elastic collision, the kinetic energy of the system is conserved, meaning that the total kinetic energy of the objects before the collision is equal to the total kinetic energy of the objects after the collision. In an inelastic collision, some of the kinetic energy is converted into other forms of energy, such as heat or sound, so the total kinetic energy of the system is not conserved. In Elastic and inelastic collision both the total momentum will always be conserved.

QUESTION 7: How does impulse relate to force and time?

ANSWER: Impulse is defined as the product of force and the time interval over which of the force acts. It is denoted by the symbol \vec{J} .

$$\vec{J} = \vec{F} \cdot \Delta t = \Delta \vec{P}$$

$$\vec{J} = \Delta \vec{P}$$

Unit:

It is equal to the change of momentum, so it has same unit as momentum (N-s).

QUESTION 8: How does friction influence the motion of an object?

ANSWER: FRICTION

Friction is the force that opposes the relative motion or tendency toward such motion of two surfaces in contact.

Coefficient of Friction:

The ratio of the force that maintains contact between an object and a surface and the frictional force that resists the motion of the object. It is denoted by μ .

$$\mu = \frac{F}{R}$$

Where,

F = limiting friction

R = normal reaction

- **Static Friction:** When an object is at rest, static friction opposes the force trying to set it in motion. The static frictional force adjusts itself to match the applied force until the point of impending motion is reached.

$$f_s \leq \mu_s n$$

- **Kinetic Friction:** Kinetic friction comes into play once the object is in motion. It acts to oppose the motion and is generally slightly less than static friction.

$$f_k = \mu_k n$$

CHAPTER = 2 DYNAMICS

ADDITIONAL SHORT QUESTIONS (BOOK- XI)

1. For what kind of motion are the instantaneous and average velocities equal?

Ans. If an object moves on a straight line with a constant velocity over a particular time interval, its instantaneous velocity equals its average velocity.

2. Is it possible for a body to have acceleration when moving with
 - i. Constant velocity
 - ii. Constant speed?

Ans. (a) As the body is moving with constant velocity neither the magnitude of the velocity nor the direction will change. So it is not possible for a body to have acceleration when moving with constant velocity.

- (b) As the body is moving with constant speed, its direction may change with time. Since the change in direction changes the velocity so the body may have acceleration.

3. when a plane accelerates down the runway during takeoff, passengers feel as if they are being pushed back into their seats, why?

Ans. The thrust(forward force) from the engine accelerates the plane for your body to have the same acceleration a force must act on your body; this is the force you feel as the seat pushes on your back.

4 Two objects are known to have the same momentum. Do these two objects necessarily have the same kinetic energy? Explain.

Ans. No, their kinetic energies are not necessarily the same. Suppose one object A has a mass 4Kg and a speed 5 m/s; a second object B has a mass 2 Kg and has a speed 10 m/s

Momentum of object A = $mv = (4)(5) = 20 \text{ N.s}$

Momentum of object B = $mv = (2)(10) = 20 \text{ N.s.}$

Kinetic energy of object A = $\frac{1}{2} m v^2 = \frac{1}{2} \cdot 4 \cdot (5)^2 = 50 \text{ J}$

Kinetic energy of object B = $\frac{1}{2} m v^2 = \frac{1}{2} \cdot 2 \cdot (10)^2 = 100 \text{ J}$

Above calculation clearly indicate that the momenta are the same, but the second object has the twice as much kinetic energy of the first.

5. Two object under goes an elastic collision. Is the kinetic energy of each object the same before and after the collision? Is the momentum of each object the same before and after the collision? Explain.

Ans. The answer to both questions is no. In an elastic collision we know that the total kinetic energy of the system is conserved, as is the total momentum of the system. There is no reason the individual kinetic energy or individual momenta should conserved- and in general they are not. Conservation applies only to system as a whole.

6. State the law of conservation of linear momentum and prove that the rate of change of linear momentum equal to applied force (Karachi board 2011-for failure and improvement of grade)

Ans. **In an isolated system of interacting bodies, The total momentum of system remains constant.**

According to newton's second law of motion the net force acting on an accelerating body is given by

$$F = ma \text{ ----- (i)}$$

The acceleration of body of mass m is given by

$$a = \frac{V_f - V_i}{t} \text{ -----(ii)}$$

Substituting the expression of acceleration form equation (ii) in equation (i)

$$F = m \left(\frac{V_f - V_i}{t} \right)$$

$$F = \left(\frac{m V_f - m V_i}{t} \right)$$

$$F = \left(\frac{\text{Change in momentum}}{\text{time interval}} \right)$$

Net force = rate of change of momentum

7. State and prove the law of conservation of linear momentum

LAW OF CONSERVATION OF MOMENTUM

STATEMENT

In an isolated system of interacting bodies, The total momentum of system remains constant.

According to law of conservation of momentum.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

PROOF

When two bodies collide with each other, they come in contact for a time interval t . During this interval, they exert equal and opposite forces on each other. Then

$$\text{Change in momentum of 'A' after collision} = m_1 v_1 - m_1 u_1$$

$$\text{Rate of change of momentum of body A} = \frac{m_1 v_1 - m_1 u_1}{t}$$

According to Newton's second law of motion;

$$\text{Net Force acting on the object} = \text{The time rate of change of momentum}$$

Therefore,

$$F_{AB} = \frac{m_1 v_1 - m_1 u_1}{t}$$

Similarly for body 'B' we can write

$$F_{BA} = \frac{m_2 v_2 - m_2 u_2}{t}$$

But

$$-F_{BA} = F_{AB}$$
$$-\frac{m_2 v_2 - m_2 u_2}{t} = \frac{m_1 v_1 - m_1 u_1}{t}$$

$$-m_2 v_2 + m_2 u_2 = m_1 v_1 - m_1 u_1$$

$$\therefore m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\left(\begin{array}{c} \text{Total momentum} \\ \text{before collision} \end{array} \right) = \left(\begin{array}{c} \text{Total momentum} \\ \text{after collision} \end{array} \right)$$

This result is known as the **conservation of momentum**.