PHYSICS

XI

UNIT 5

WORK, ENERGY AND POWER

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WORK

WORKDONE BY A CONSTANT FORCE

WORK

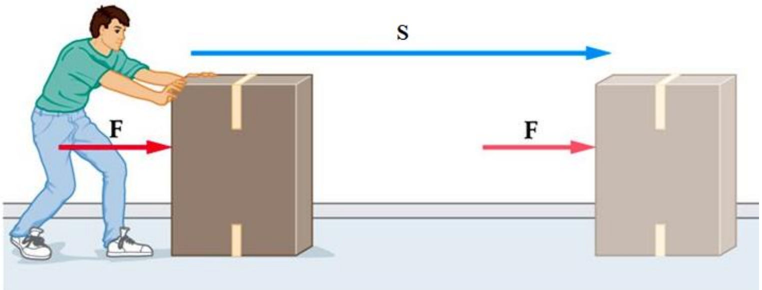
Work in physics is defined as the product of a force and the displacement in the direction of the force.

In equation form,

$$Work = (Force)\,(displacement\ in\ the\ direction\ of\ force)$$

$$W = F\,s$$

Work is a scalar quantity.

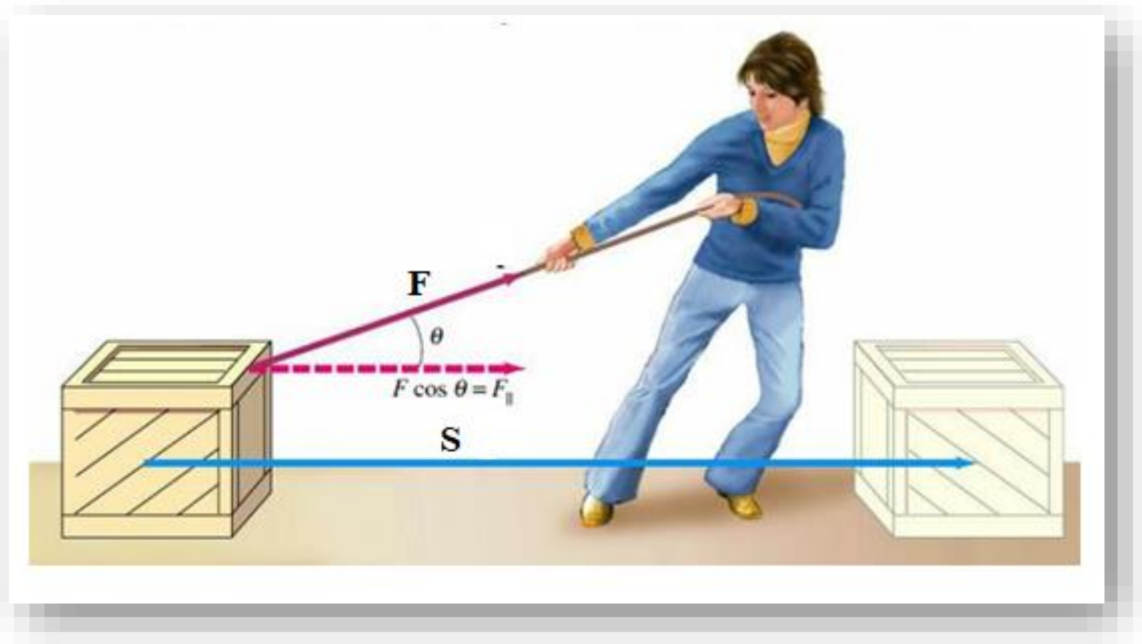


FORCE F IS MAKING AN ANGLE Θ WITH THE DIRECTION OF DISPLACEMENT OF THE BODY

DEFINITION-2:

The work done on an object by a constant force is defined as the product of the magnitude of the displacement and the component of the force parallel to the displacement .

Formula:



In equation form, we can write

$$W = (F_{11})\,S$$

When F_{11} is the component of the constant force $\vec{F}$ parallel to the displacement 'S'. We can also write

$$W = (F\cos\theta)\,S$$

$$W = F\,S\cos\theta \quad \text{\_\_\_\_\_\_\_\_\_\_(i)}$$

When F is the magnitude of the constant force, S is the magnitude of the displacement of the object, and θ is the angle between the direction of force and the displacement. The Cosθ factor appears in equation (1) because $F\cos\theta = F_{11}$ is the component of $\vec{F}$ parallel to S

WORK AS SCALAR PRODCUT:

From above equation, work is defined as the scalar product of 'force' and 'displacement'

$$W = \vec{F}.\vec{S}$$

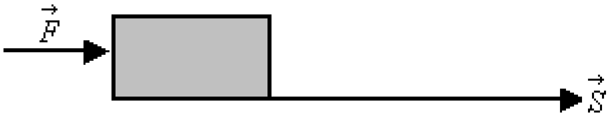
SPECIAL CASES

Work can be either positive or negative, depending on the value of angle between F and S. Following are some important cases.

CASE I: Positive work (if $\theta = 0^0$)

If the force acts on a body and displacement $\vec{S}$ is produced in the direction of the force, then the work done is positive

The angle between the force and the displacement is $\theta = 0^o$



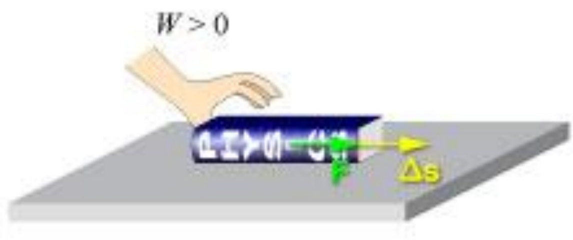
$W = F\ S \cos\theta$

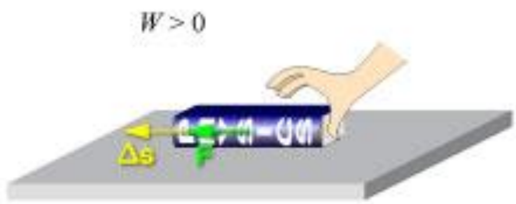
$W = FS \cos 0^o$

$W = FS(1)$ $[\cos 0^o = 1]$

$W = FS$(work is positive)

Example





Positive work is done when pushing a textbook to the right across a level tabletop at a constant velocity.

Positive work is also done when pushing a textbook to the left across a level tabletop at a constant velocity

CASE II: Zero work (If $\theta = 90^o$)

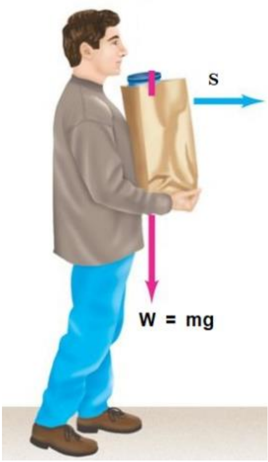
If the force acts at the right angles to the direction of the motion of a body, then the angle θ between the direction of the motion and the direction of the force is **90º.** Now, $\cos 90^o = 0$

$W = F\cos\theta$

$= F\cos 90^o$

$= FS\ (0)$

$=$ Zero work.



Example

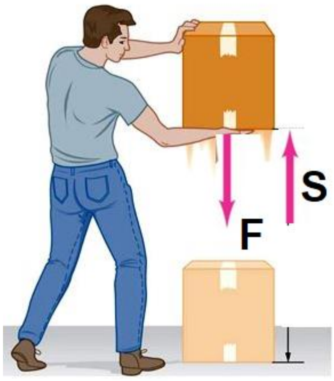
1. Work done by centripetal force is always zero.
2. The work done of the gravity on box hold by walking a person is zero

CASE III: **Negative work (If θ = 180º)**

When the force is opposite to the direction of displacement, the work done is negative.

When θ = 180º & Cos 180º = -1

$$W = FS\cos\theta$$
$$= FS\cos 180^o$$
$$= FS(-1)$$
$$= -FS \text{ (negative work)}$$



Example

When a bucket is lifting up, the work of the gravitational force is negative and work done by the muscular force of the hand lowering down the bucket is negative.

UNITS:

1. In SI unit s, work is measured in **Newton – meters**, A special name is given to this unit, the joule (J)

1 J = 1 N.m

2. In *CGS* system the unit of work is called 'erg' and is defined as, when the force of 1 dyne displace a body by 1 cm in the direction of the force, , the work done is 1 erg.

1 erg = 1 dyne.cm

3. In British system the unit of work is measured in 'foot-pound'

Relation

it is easy to show that

1 joule = 10^7 ergs

1 joule = 0.7376ft-lb

Practical unit

In atomic and nuclear physic a much smaller unit is used, called electron –volt

$$1eV = 1.602 \times 10^{-19} J$$

DEFINATION OF ELECTRON VOLT

1 eV is the kinetic energy acquired by an electron in falling through a potential difference of 1 volt

DIMENSIONS:

$\therefore$ $W = FS\cos\theta$

$= (MLT^{-2}).(L)$

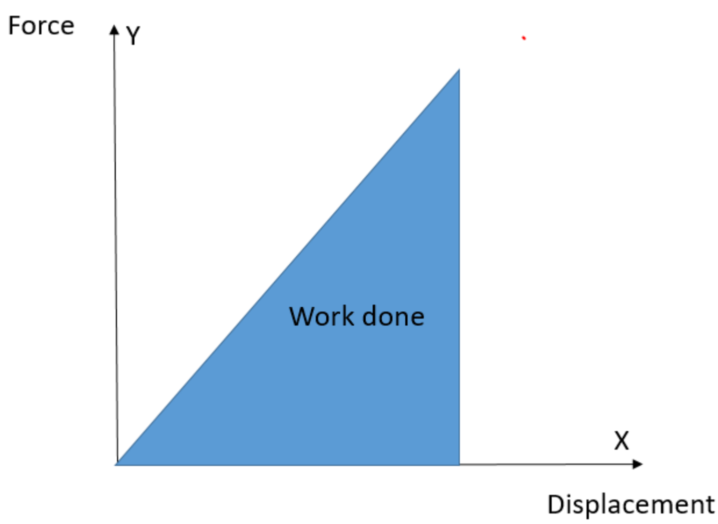
$= ML^2T^{-2}$ **are the dimension of work**.

WORK FORM FORCE AND DISPLACEMENT GRAPH:

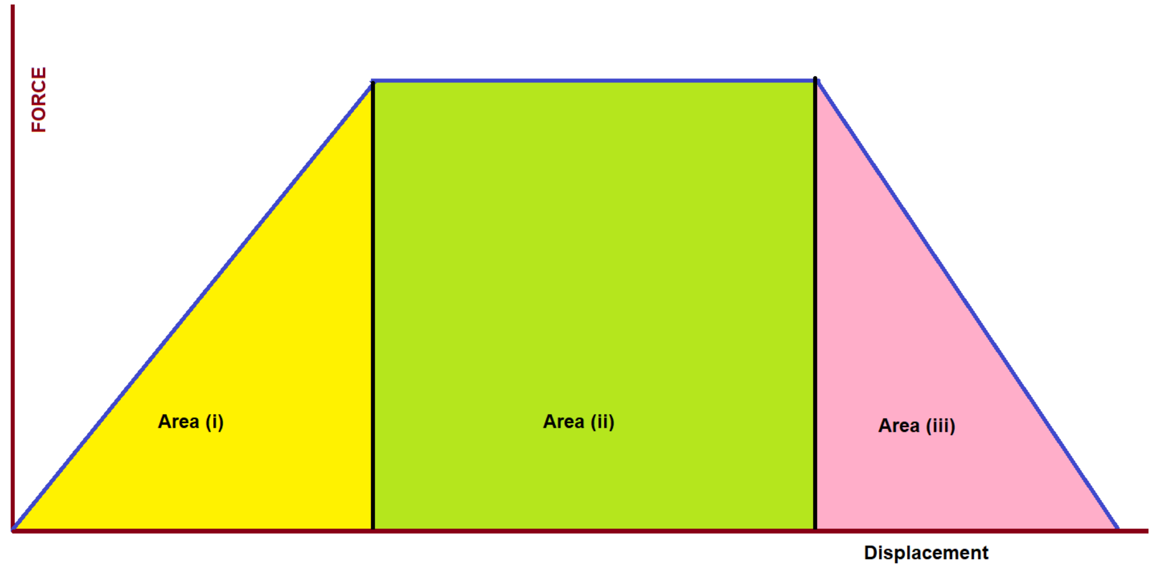
To calculate the work done by a force-displacement graph, you need to find the area under the graph. The area represents the work done.

In this case, the area under the graph is triangular

$$work = area\ of\ triangle$$
$$work = \frac{1}{2} \times base \times height$$



If the graph is not a straight line as shown in figure, you will need to break the area under the graph into smaller shapes (e.g., rectangles, triangles) and calculate the area of each shape separately. Then. you sum up all the shapes to find the total work done.



$$\text{Total work done} = \text{area(i)} + \text{area(ii)} + \text{area(iii)}$$

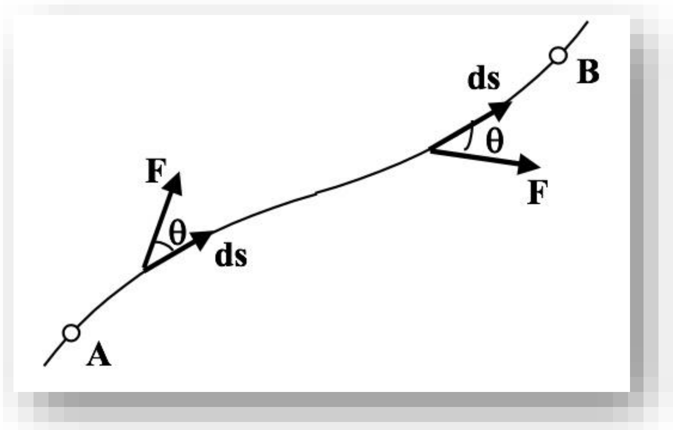
$$\text{Total work done} = \text{area of traingle} + \text{arearectangle} + \text{areatraingle}$$

$$\text{Total work done} = \frac{1}{2}\,(\text{base})(\text{height}) + (\text{length} \times \text{breadth}) + \frac{1}{2}\,(\text{base})(\text{height})$$

WORK DONE BY VARIABLE FORCE

A variable force is a force that changes in magnitude or direction as a function of time, position, or any other relevant variable. Unlike a constant force, which remains unchanged, a variable force can have different values at different. points or moments.

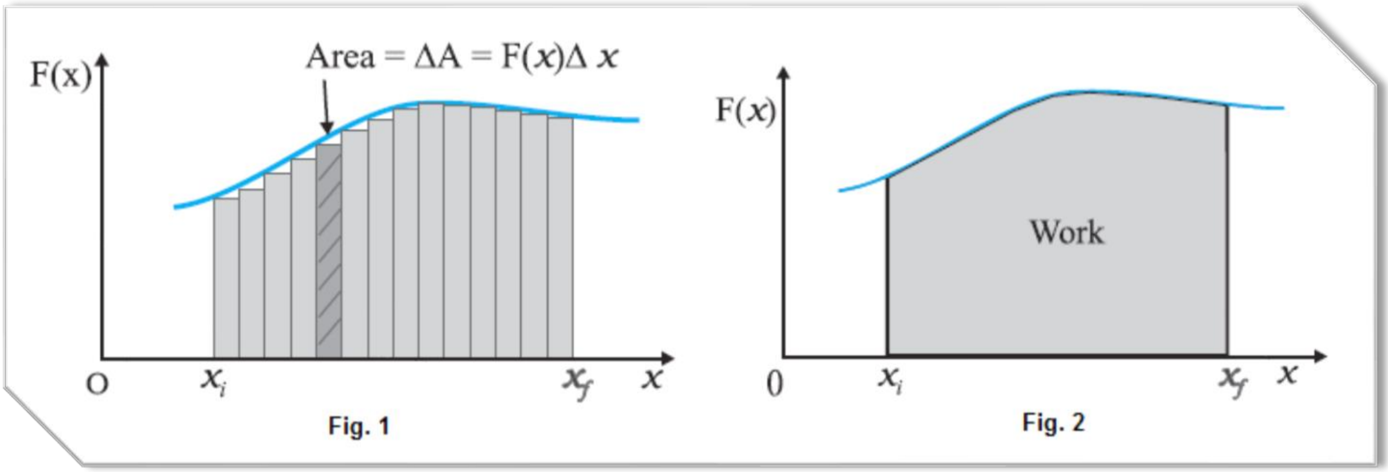
In this condition, we consider the variable force to be variable for any elementary displacement ds as shown in figure, work done in that elementary displacement is evaluated. Total work is obtained by integrating the elementary work from initial to final limits.



$$\Delta W = \vec{F} \cdot \vec{ds}$$
$$\Delta W = \sum \left(\vec{F} \cdot \vec{ds}\right)$$

Work done by variable force and its graphical calculation:

Consider a body covers displacement from x_i to x_f, when a variable force acts on it. To clarify the situation, we plot the graph between force and displacement covered by the body as shown in figure(1 and (2)



The area under the covered line is representing the work done by the body. To calculate the work done by the body we divide the covered displacement in to small segments $\Delta x_1, \ \Delta x_2, \ \Delta x_3, \ \dots\dots\dots.\Delta x_n$ and the corresponding forces for each segment are $\vec{F}_{1x}, \ \vec{F}_{2x}, \ \vec{F}_{3x}, \ \dots\dots\dots.\vec{F}_{nx}$. The total work done is given by

$$W_T = W_1 + W_2 + W_3 + \dots\dots\dots.+W_n$$

$$W_T = \vec{F}_{1x} \cdot \Delta\vec{x}_1 + \vec{F}_{2x} \cdot \Delta\vec{x}_2 + \vec{F}_{3x} \cdot \Delta\vec{x}_3 + \dots\dots\dots.+\vec{F}_{nx} \cdot \Delta\vec{x}_n$$

$$W_T = \sum_{i=1}^{n} (\vec{F}_{ix} \cdot \Delta\vec{x}_i)$$

Above equation represents the total work done by body when variable force acts on it

KINETIC ENERGY

DEFINITION:

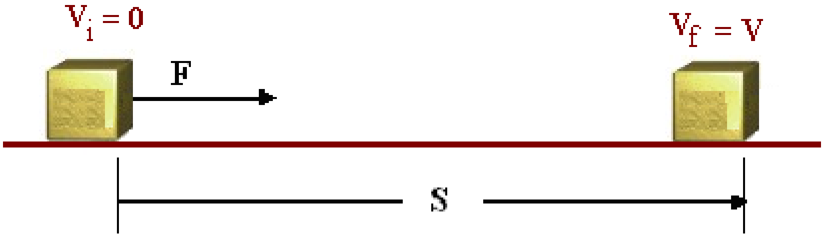
Energy possessed by a body by virtue of its motion is called its kinetic energy. It is a scalar quantity

FORMULA

$$K.E = \frac{1}{2}mv^2$$

DERIVATION

Consider a body of mass m is placed on a horizontal surface. Suppose a force F is applied on an object and it produces displacement S in the direction of force Along x-axis as shown in figure



The work done is given by

$$W = F\,S\,Cos\theta$$

Force and displacement are in the same direction, angle between them is zero

$W = F\,S\,Cos0^0$ $[\cos 0^0 = 1]$

W = F S ______________ (i)

If F is the only force acting, then Newton's second law of motion tells us that

F = ma ______________ (ii)

Since the acceleration is constant, we can use kinematics equation for constant acceleration.

$$2as = V_f^2 - V_i^2$$

$$2as = v^2 - 0$$

$$S = \frac{v^2}{2\,a} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (iii)}$$

Substituting the expression of S from equation (iii), and F = ma in equation (i)

$$W = (ma)\left(\frac{v^2}{2\,a}\right)$$

$$W = \frac{1}{2}mv^2$$

The work done is equal to the change in quantity $\frac{1}{2}mv^2$. This quantity is called kinetic energy of the object.

$$KE = \frac{1}{2}mv^2$$

This is an expression for Kinetic energy.

POTENTIAL ENERGY

DEFINITION:

The Energy a body possesses by its position in a field of force is called potential energy. It is a scalar quantity.

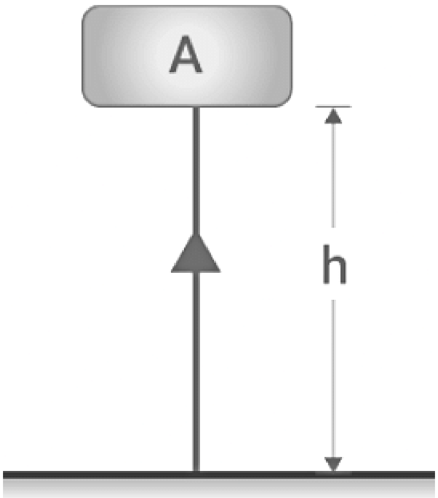
TYPES:

Potential energy of a system may be one of the following types.

1. **Gravitational potential energy.**
2. **Elastic potential energy.**
3. **Electric potential energy etc.**

FORMULA FOR POTENTIAL ENERGY

The potential energy of a body is due to its higher position above the Earth and it is equal to the work done on the body, against gravity, in moving the body to that position.



Suppose a body of mass "m" is raised to a height "h" above the earth's surface. The force acting on the body is the gravitational pull of the Earth (mg) which acts in the downward direction. To lift the body above the surface of the Earth, we have to work against this force of gravity.

Work done = force x distance

Work done = m g h

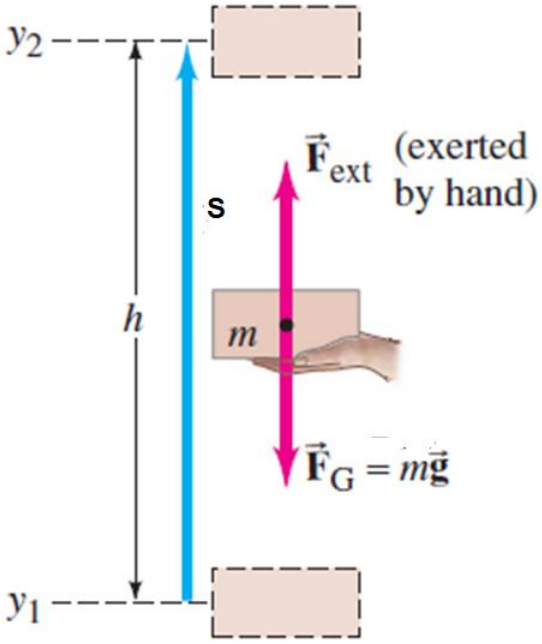
This work gets stored up in the body as potential energy.

Thus,

Potential energy = m g h

GRAVITATIONAL POTENTIAL ENERGY:

Consider an object of mass m to be lifted vertically, an upward force at least equal to its weight, mg, must be exerted on it, say by a person's hand. To lift the object without acceleration, the person exerts an "external force" $F_{ext} = mg$ If it is raised a vertical height h, from position y_1 to y_2 in Fig (upward direction chosen positive), a person does work equal to the product of the "external" force she exerts, $F_{ext} = mg$ upward, multiplied by the vertical displacement h. That is



$$W_{ext} = FS\cos 0^0$$
$$W_{ext} = Fh\,(1)$$
$$W_{ext} = (mgy_2 - mgy_1)$$
$$W_{ext} = mg\,(y_2 - y_1) \ldots\ldots (1)$$

Gravity is also acting on the object as it moves from y_1 to y_2 and does work on the object equal to

$$W_{Grav} = FS\cos 180^0$$
$$W_{Grav} = Fh\,(-1)$$
$$W_{Grav} = -(mgy_2 - mgy_1)$$
$$W_{Grav} = -mg\,(y_2 - y_1) \ldots\ldots (2)$$

Thus, to raise an object of mass *m* to a height *h requires* an amount of work equal to *mgh* (Eq. 1). And once at height *h*, the object has the *ability* to do an amount of work equal to mgh. We can say that the work done in lifting the object has been stored as gravitational potential energy.

$$(PE)_{Grav} = mgh \ldots\ldots\ldots (3)$$

The higher an object is above the ground, the more gravitational potential energy it has. We combine Eq. (1) with Eq. (3):

$$W_{ext} = (mgy_2 - mgy_1)$$
$$W_{ext} = (PE_2 - PE_1) = \Delta PE_{Grav}$$

That is, the change in potential energy when an object moves from a height y_1 to a height y_2 is equal to the work done by a net external force to move the object from position 1 to position 2 without acceleration. Equivalently, we can define the change in gravitational potential energy, in terms of the work done by gravity itself. Starting from Eq. (2), we obtain

$$W_{Grav} = -(mgy_2 - mgy_1)$$
$$W_{Grav} = -mg\,(y_2 - y_1)$$
$$W_{Grav} = -(PE_2 - PE_1) = -\Delta PE_{Grav}$$
$$\Delta PE_{Grav} = -W_{ext}$$

That is, the change in gravitational potential energy as the object moves from position 1 to position 2 is equal to the negative of the work done by gravity itself. Gravitational potential energy depends on the v*ertical height* of the object *above some reference level.*

CONSERVATIVE FIELD

DEFINITION-1:

A field of force, in which the work done by the force on an object moving from one point to another depends only on the initial and final position and is independent of the particular path taken is called conservative field.

DEFINITION-2:

A field of force, in which the net work done by a force on an object moving around any closed path is zero, is called conservative field.

EXAMPLES:

(i) Gravitational field of earth (ii) Electrostatic field.

GRAVITATIONAL FIELD IS A CONSERVATIVE FIELD

PROOF – 1:

The work done in moving a body in gravitational field depends upon the initial and final positions i.e., it is independent of the particular path taken . To prove this statement, we consider a triangular path ABC as shown, and carry a body of mass **m** from A to C along two different paths.

PATH I

Work done along AC ($W_{A\to C}$)

$$\text{Work} = F\, S_3\, \cos\theta$$

$$W_{A\to C} = W\, S_3\, \cos\beta \qquad W = mg$$

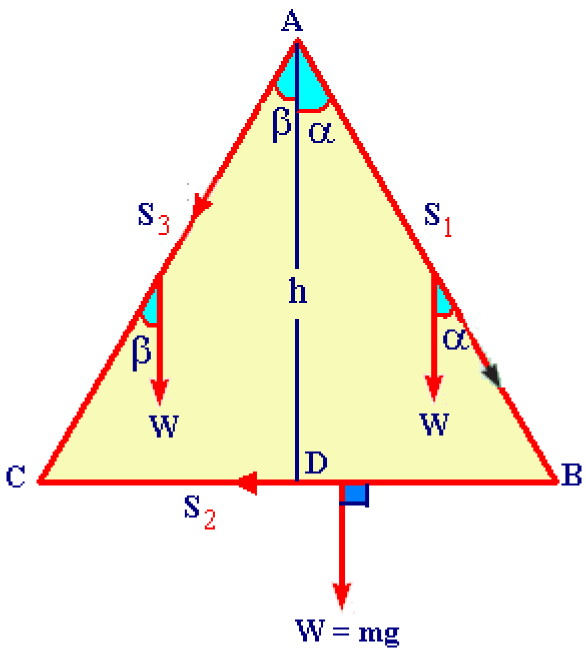
$$W_{A\to C} = mg\, S_3\, \cos\beta \quad \text{..............(i)}$$

From the triangle ACD

$$\cos\beta = \frac{AD}{AC}$$

$$\cos\beta = \frac{h}{S_3}$$

$$h = S_3 \cos\beta \text{ .... (ii)}$$



Substituting the value of h from equation (ii) in equation (i), we get

$$W_{A\to C} = mgh \text{ .... (iii)}$$

PATH II

Work done along ABC = $W = W_{A\to B} + W_{B\to C}$

First we find the work done from A to B

$$W_{A\to B} = \vec{F} \cdot \vec{S_1}$$

$$W_{A\to B} = F\, S_1\, \cos\alpha$$

$$W_{A\to B} = W\, S_1\, \cos\alpha \qquad W = mg \quad h = S_1 \cos\alpha$$

$$W_{A\to B} = mg\, h \quad \text{..............(iv)}$$

work done from B to C

$$W_{B\to C} = \vec{F} \cdot \vec{S_2}$$

$$W_{B\to C} = F\, S_2\, \cos 90^0 \qquad \alpha = 90^0$$

$$W_{B\to C} = W\, S_2\, \cos 90^0 \qquad W = mg \quad \cos 90^0 = 0$$

$$W_{A\to B} = mg\, S_2\,(0) \quad \text{..............(v)}$$

Now, from equation (iii), (iv) and (v),we get

$$W_{A\to B} + W_{B\to C} = W_{A\to C}$$

$$mgh + 0 = mgh$$

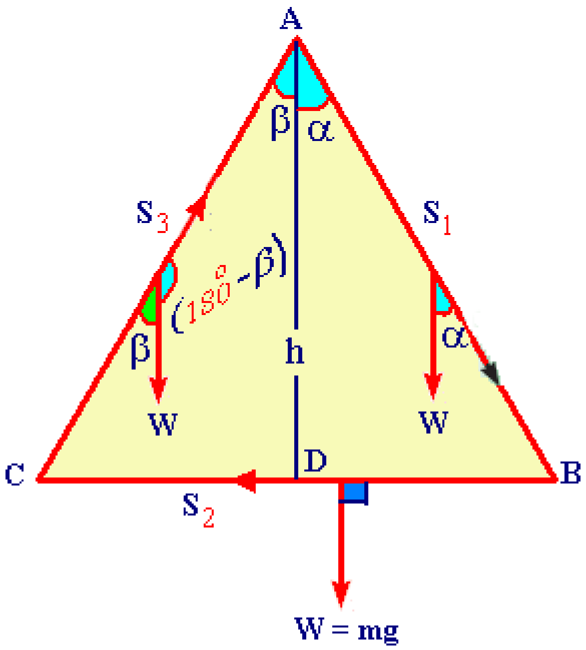
$$mgh = mgh$$

thus, gravitational field is conservative field.

<u>PROOF –2</u>

In gravitational field the net work done by the force on an object moving around a closed path is zero , therefore gravitational field is conservative field.

To prove this statement, consider a closed triangular path ABCA as shown. Let us calculate the work done around closed path of mass 'm' in gravitational field of earth.



work done along AB = $W_{A\to B}$

$$W_{A\to B} = \vec{F}\cdot\vec{S_1}$$

$$W_{A\to B} = F\,S_1\,\text{Cos}\,\alpha$$

$$W_{A\to B} = W\,S_1\,\text{Cos}\,\alpha \;\;..........(i)$$

From the triangle ACD

$$\text{Cos}\,\alpha = \frac{AD}{AC}$$

$$\text{Cos}\,\alpha = \frac{h}{S_1}$$

$$h = S_3\,\text{Cos}\,\alpha \;............ (ii)$$

Substituting the value of h from equation (ii) in equation (i), we get

$$W_{A\to B} = mgh \;\;.............(iii)$$

<u>work done from B to C</u>

$$W_{B\to C} = \vec{F}\cdot\vec{S_2}$$

$$W_{B\to C} = F\,S_2\,\text{Cos}90^0 \qquad \alpha = 90^0$$

$$W_{B\to C} = W\,S_2\,\text{Cos}90^0 \qquad W = mg \quad \text{Cos}90^0 = 0$$

$$W_{A\to B} = mg\,S_2\,(0) \;\;.............(iv)$$

work done along CA = $W_{C\to A}$

$$W_{C\to A} = \vec{F}\cdot\vec{S_3}$$

$$W_{C\to A} = F\,S_3\,\text{Cos}\,\theta$$

$$W_{C\to A} = W\,S_3\,\text{Cos}(180-\beta) \qquad \theta = 180-\beta$$

$$W_{C\to A} = W\,S_3\,(-\text{Cos}\,\beta) \qquad \text{Cos}(180-\beta) = -\text{Cos}\,\beta$$

$$W_{C\to A} = -\,mg\,S_3\,\text{Cos}\,\beta$$

$$W_{C\to A} = -\,mg\,h \qquad (h = S_3\,\text{Cos}\,\beta)$$

$$\text{Total work} = W_{A\to B} + W_{B\to C} + W_{C\to A}$$

$$\text{Total work} = mgh + 0 + (-mgh)$$

$$\text{Total work} = 0$$

Thus, the gravitational field is a conservative field.

ABSOLUTE POTENTIAL ENERGY

DEFINITION:

The amount of work needed to move a body form a point in the gravitational field of earth to zero point (infinity) is called its absolute P.E at that point. Absolute PE is a *scalar quantity*.

DERIVATION:

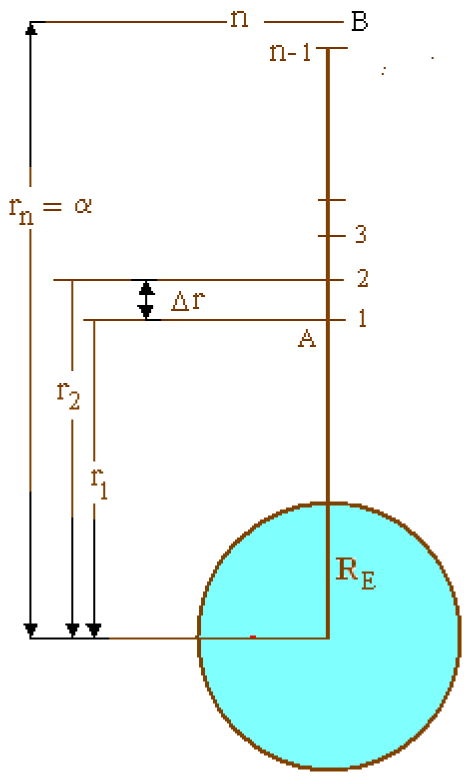
Consider a body of mass "m" which is lifted from point "A" to a very far-off point "B", in the gravitational field. We know that Work done = P. E = $\vec{F}\cdot\vec{S}$, But this formula for work cannot be used, because gravitational force does not remain constant between A and B. To remove this difficulty, we divide the distance between A & B into a large number of small intervals, each of length Δr. This interval Δr. is so small that the gravitational force remains constant during the interval. The constant force may be taken as the average force during Δr.

(i) **The magnitude of gravitational force at point** 1

$$F_1 = G\frac{m\,Me}{r_1^2}$$

(ii) **The magnitude of gravitational force at point** 2

$$F_2 = G\frac{m\,Me}{r_2^2}$$



$$\therefore F_{aV} = \frac{F_1 + F_2}{2}$$

$$F_{aV} = \frac{G\frac{mMe}{r_1^2} + G\frac{mMe}{r_2^2}}{2}$$

$$F_{aV} = G\frac{mMe}{2}\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right)$$

$$F_{aV} = G\frac{mMe}{2}\left(\frac{r_2^2 + r_1^2}{r_1^2\ r_2^2}\right)$$

$$F_{aV} = G\frac{m\,Me}{2}\left(\frac{(r_1 + \Delta r)^2 + r_1^2}{r_1^2\ r_2^2}\right)$$

$$\Delta r = r_2 - r_1$$

$$F_{aV} = G\frac{m\ Me}{2}\left(\frac{r_1^2 + 2\,r_1\,\Delta r + (\Delta r)^2 + r_1^2}{r_1^2\ r_2^2}\right)$$

$\therefore r_2 = r_1 + \Delta r$ and Δr. is very small value its square value [$(\Delta r.)^2$] is negligible

$$F_{aV} = G\frac{m\,Me}{2}\left(\frac{r_1^2 + 2r_1\,\Delta r + r_1^2}{r_1^2\ r_2^2}\right)$$

$$F_{AV} = G\frac{mMe}{2}\left(\frac{2r_1^2 + 2r_1\,\Delta r}{r_1^2\ r_2^2}\right)$$

$$F_{AV} = G\frac{mMe}{\cancel{2}}\left(\frac{\cancel{2}r_1\left(r_1+\Delta r\right)}{r_1^2\ r_2^2}\right) \qquad \therefore r_2 = r_1 + \Delta r$$

$$F_{AV} = G\,m\,Me\left(\frac{r_1\ r_2}{r_1^2\ r_2^2}\right)$$

$$F_{AV} = G\,m\,Me\left(\frac{r_1\ r_2}{r_1 r_2 \times r_1 r_2}\right)$$

$$F_{AV} = G\frac{m\,Me}{r_1\ r_2}$$

Now, work done in moving the body from point 1 to point 2.

$$W_{1\to2} = \vec{F}_{av}.\Delta\vec{r}$$

$$W_{1\to2} = F_{av}\ \Delta r\ \ Cos\theta$$

$$W_{1\to2} = F_{av}\,\Delta r\ Cos0^o$$

$$W_{1\to2} = G\frac{mMe}{r_1\ r_2}\left(r_2 - r_1\right)$$

$$W_{1\to2} = G\ mMe\left(\frac{r_2 - r_1}{r_1\ r_2}\right)$$

$$W_{1\to2} = G\ mMe\left(\frac{r_2}{r_1\ r_2} - \frac{r_1}{r_1\ r_2}\right)$$

$$W_{1\to2} = G\ mMe\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly,

$$W_{2\to3} = G\ mMe\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$W_{(N-1)\to N} = G\ mMe\left(\frac{1}{r_{(N-1)}} - \frac{1}{r_N}\right)$$

Hence, Total work done

$$W_{1\to N} = W_{1\to2} + W_{2\to3} + \ldots\ldots\ldots\ldots\ldots + W_{(N-1)\to N}$$

Substituting all work done expression in the above equation, we get

$$W_{1\to N} = GmMe\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + GmMe\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \ldots\ldots\ldots + GmMe\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

$$W_{1\to N} = GmMe\left[\frac{1}{r_1} - \frac{1}{\cancel{r_2}} + \frac{1}{\cancel{r_2}} - \frac{1}{\cancel{r_3}} + \ldots\ldots\ldots + \frac{1}{\cancel{r_{n-1}}} - \frac{1}{r_n}\right]$$

$$W_{1\to N} = GmMe\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

This is the potential energy of the body at point "B" with respect to the point "A". Hence, the P.E of the body at point "B" is given by

$$\Delta U = -W$$

$$U_A - U_B = -GmMe\left(\frac{1}{r_1} - \frac{1}{r_N}\right) \qquad \Delta U = U_A - U_B$$

For gravitational potential energy at point "B", we have $r_n = \alpha$

$$\frac{1}{r_N} = \frac{1}{\alpha}$$

$$\frac{1}{r_N} = 0$$

and $U_B = 0$

$$U_A = -G\frac{mMe}{r_1}$$

In general when point A lies at any distance r from center of earth then, $r_1 = r$ and $U_A = U$

$$U = -G\frac{mMe}{r}$$

Therefore the absolute Potential energy of a body of mass 'm' lying a the surface of the earth is given by

$$U_{abs} = -G\frac{mMe}{r}$$

Where R_E is the radius of earth.

The minus sign indicates that the potential energy is negative at any finite distance, that is the Potential energy is zero at infinity and decreases as the separation distance decreases. This is because the gravitational force acting on the Particle by the earth is attractive. As the particle moves from infinity, the work W_∞ is positive which means U is negative.

Escape velocity

Escape velocity on Earth or any other planet is defined as the minimum velocity with which the body has to be projected vertically upwards from the surface of the Earth or any other planet so that it just crosses the gravitational field of earth or of that planet and never return on its own.

Work is done at the cost of kinetic energy given to the body at the surface of the earth. If V_{es} is the escape velocity of the body projected from the surface of the earth. Then kinetic energy of the body will escape out of the gravitational field.

$$\frac{1}{2} m v_{es}^2 = \frac{G\, m\, M_e}{R}$$

$$v_{es}^2 = \frac{2\, G\, M_e}{R}$$

$$v_{es} = \sqrt{\frac{2\, G\, M_e}{R}} \;\ldots\ldots\ldots (i)$$

The gravitational acceleration at the surface of the earth is given by

$$g = \frac{G\ M_e}{R^2}$$

$$G\ M_e = g\ R^2 \quad \ldots\ldots (ii)$$

Substituting the expression G M_e in equation (i)

$$v_{es} = \sqrt{\frac{2\ g\ R^2}{R}}$$

$$v_{es} = \sqrt{2\ g\ R}$$

ESCAPE VELOCITY OF EARTH

$$v_{es} = \sqrt{2\ g\ R}$$

$$v_{es} = \sqrt{2 \times 9.8 \times 6.38 \times 10^6}$$

$$v_{es} = \sqrt{1.25 \times 10^8}$$

$$v_{es} = 11.2 \times 10^3\ m/s$$

$$v_{es} = 11.2 \times K\ m\ s^{-1}$$

POWER

DEFINITION:

Power is defined as the rate at which the work is done. It is scalar quantity.

FORMULA:

The average power "P_{av}" when an amount of work is done in a time interval Δ**t** is

$$P_{av} = \frac{\text{Total work}}{\text{Time interval}}$$

$$P_{av} = \frac{\Delta W}{\Delta t}$$

Instantaneous power is the limit of this ratio when $\Delta t \rightarrow 0$.

$$P = \underset{\Delta t \rightarrow 0}{\text{Limit}} \frac{\Delta W}{\Delta t}$$

The work done in the process is equal to the energy transformed from one form to another form. We can say that power is the rate at which energy is transformed.

$$P = \frac{\Delta E}{\Delta t}$$

POWER AS SCALAR PRODUCT:

Power is defined as

$$Power = \frac{Total work}{Time\ interval}$$

$$P = \frac{W}{t}$$

$$P = \frac{\vec{F} \cdot \vec{S}}{t}$$

$$P = \vec{F} \cdot \frac{\vec{S}}{t}$$

$$P = \vec{F} . \vec{V}$$

UNITS:

$$\because P_{av} = \frac{\Delta W}{\Delta t} = \frac{J}{Sec}$$

1. The SI unit of power is (joule per second), called **watt** (W).
2. The CGS unit of power is **erg/s**
3. The British (F.P.S) unit of power is **ft- lb/Sec**. Another bigger unit in this system is **horse power** (hp).

1 hp = 746 watt
1 hp = 550 ft-lb/Sec

DIMENSIONS:

The dimension of power are ML^2T^{-3}

WORK ENERGY THEOREM:

It states that the total work done on the body is equal to the change in kinetic energy

Consider a body of mass m moving with initial velocity v_1 after travelling through displacement S its final velocity becomes v_2 under the effect of force F.

As we know that

$$2\,a\,S = v_f^2 - v_i^2$$

$$a = \frac{v_f^2 - v_i^2}{2\,S}$$

hence external force acting on the body is

$$F = m\,a$$

$$F = m\left(\frac{v_f^2 - v_i^2}{2\,S}\right) \ldots\ldots(i)$$

Therefore, work done on the body by an external force is

$$W = F\,S \ldots\ldots\ldots\ldots..(ii)$$

Substituting the expression for force from equation (i) in equation (ii)

$$W = m\left(\frac{v_f^2 - v_i^2}{2\,S}\right)S$$

$$W = m\left(\frac{v_f^2 - v_i^2}{2}\right)$$

$$W = \frac{1}{2}\,m\left(v_f^2 - v_i^2\right)$$

$$W = \frac{1}{2}\,m\,v_f^2 - \frac{1}{2}\,m\,v_i^2$$

$$W = (K.E)_f - (K.E)_i$$

$$W = \Delta(K.E)$$

LAW OF CONSERVATION OF ENERGY

Energy can neither be created nor destroyed. It can only be transformed from one form to another. An equal increase in the other forms of energy accompanies a loss in one form of energy. The total energy remains constant

PROOF

Consider a body of mass m is paced at point "p" at a high h from the ground's surface.

The potential energy of a body at point P

P.E = m g h

The potential energy of a body at point P

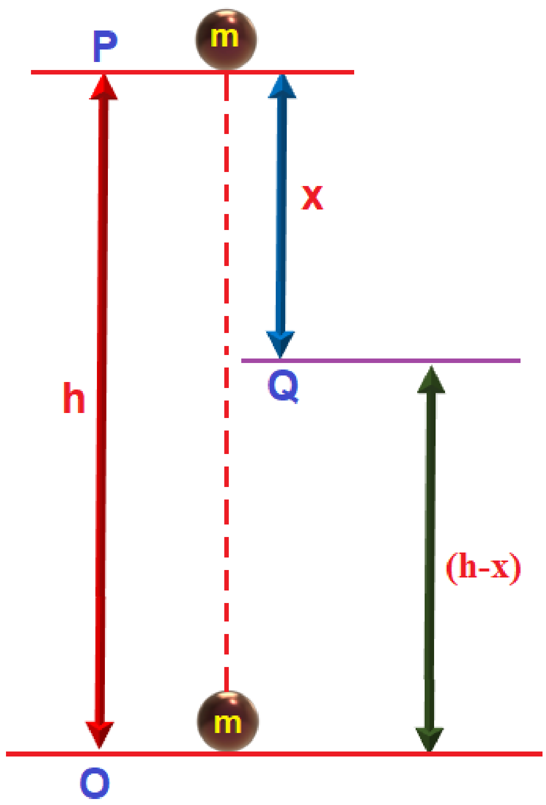
K.E = 0

The Total energy of a body at point P

E = K.E + P.E

E = 0 + mgh

E = mgh …………(i)



If the body is allowed to fall freely under the action of gravity then its P.E will go on decreasing whole its K.E will go on increasing just before hitting the ground the P.E of the body will be minimum or zero while K.E of the body will be maximum. If 'v' is the velocity of the body just before hitting the ground. The formula can find the velocity of the body

$$2\,a\,S = v_f^2 - v_i^2$$

$$2\,g\,h = v^2 - 0$$

$$2\,gh = v^2$$

The potential energy of a body at point O

P.E = 0

The potential energy of a body at point O

$$K.E = \frac{1}{2}\,m\,v^2$$

Substituting the expression for velocity in the above equation

$$K.E = \frac{1}{2}\,m \times 2\,g\,h$$

$$K.E = m\;g\,h$$

The Total energy of a body at point O

E = K.E + P.E

E = mgh + 0

E = mgh …………(ii)

Consider a point "Q" at a high x from the point P and (h-x) from the ground's surface
The formula can find the velocity of the body

$$2\,a\,S = v_f^2 - v_i^2$$

$$2\,g\,x = v^2 - 0$$

$$2\,gx = v^2$$

The potential energy of a body at point Q

$$P.E = mg\,(h - x) = mgh - mgx$$

The potential energy of a body at point Q

$$K.E = \frac{1}{2} m v^2$$

Substituting the expression for velocity in the above equation

$$K.E = \frac{1}{2} m \times 2 g x$$

$$K.E = m g x$$

The Total energy of a body at point O

$$E = m g x + mgh - mgx$$

$$E = mgh \ldots\ldots (iii)$$

From equations (i), (ii), and (iii) it can be seen that the total energy of the body remains constant everywhere provided there is no force of friction involved during the motion of the body

SHORT REASONING QUESTIONS

QUESTION 1: How does work relate to the transfer of energy?

ANSWER: WORK-ENERGY THOREM

"Work done on a body is equal to the change in its kinetic energy provided the body moves horizontally without any resistance."

Proof:

Consider a body of mass 'm' is moving with some velocity '$\vec{V_1}$'. A force '$\vec{F}$' produces some acceleration '$\vec{a}$' in it and it moves along a straight line with a velocity '$\vec{V_2}$' and covers a displacement '$\vec{S}$' in the direction of the force.

According to 3rd equation of motion:

$$2aS = V_2^2 - V_1^2$$

$$a = \frac{V_2^2 - V_1^2}{2S} \quad \ldots \quad (i)$$

Since,

$$W = \vec{F}.\vec{S}$$

$$\Rightarrow W = FS\cos 0^o$$

$$\because \ F = ma$$

and $\quad \cos 0^o = 1$

$$\therefore \ W = ma\,S \ \ldots \ (ii)$$

Using eq. (i) in eq. (ii)

$$W = m\left(\frac{V_2^2 - V_1^2}{2S}\right)S$$

$$\Rightarrow W = \frac{1}{2}m(V_2^2 - V_1^2)$$

$$\Rightarrow W = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$$

$$\because \ K.E = \frac{1}{2}mV^2$$

$$\therefore \ W = K.E_2 - K.E_1$$

$$\Rightarrow W = \Delta K.E$$

⇒ Work done = Change in K.E

QUESTION 2: How is power related to work and time?

ANSWER: Power is closely related to work and time, and it can be defined as the rate at which work is done or the rate at which energy is transferred or transformed. The relationship between power (P), work (w), and time (t) can be expressed by the following formula:

$$P = \frac{W}{t}$$

QUESTION 3: What is the difference between average power and instantaneous power?

ANSWER:

Average Power:
Average power is the power averaged over a certain period of time, It provides the average rate at which work is done or energy is transferred during that time interval.

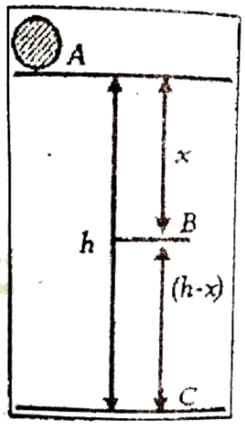
$$P_{avg} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power:
Instantaneous power is the power at a specific moment in time. It represents the rate at which work is being done or energy is being transferred at that instant.

$$P_{ins} = \frac{dW}{dt}$$

QUESTION 4: What is the law of conservation of energy?

ANSWER: PROOF OF LAW OF CONSERVATION OF ENERGY
With reference to the problem of a freely falling body, such as a body of mass m' placed at a point 'A' which is at a height 'h' from the surface of earth. The body possesses the P.E. equal to 'mgh' with respect to point 'C' lying at the surface of the earth. But the K.E of the body at point 'A' is zero. i.e.



AT POINTA

Total Energy T.E = K.E + P.E

P.E = 0 + mgh

Total Energy at A = mgh

AT POINT C
Now we calculate the kinetic energy at point 'C' for this we make the following data.

Initial velocity = V_i = 0 (at point 'P')
Final velocity = V_f = V (at point 'O')
Acceleration = g
Distance = S = h

Now by using third equation of motion.

$$2aS = V_2^2 - V_1^2$$

$$2gh = V^2 - (O)^2$$

$$V^2 = 2gh$$

Hence the kinetic energy of the body is:

$$\mathbf{K.E = \frac{1}{2} mV^2}$$

Put the value of n^2 in above equation

$$\mathbf{K.E = \frac{1}{2} m.2gh}$$

$$\mathbf{K.E = mgh}$$

QUESTION 5: How does energy efficiency play a role in various energy transformations?

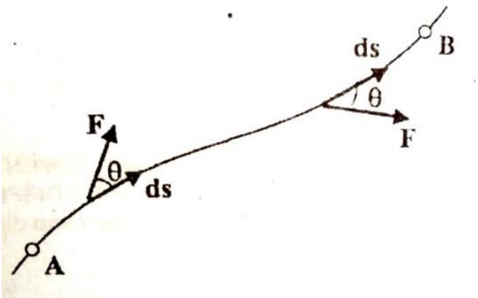
ANSWER: Energy efficiency is vital in energy transformations because it minimizes waste and optimizes the useful output of energy. It plays a significant role in reducing costs, conserving resources, and lowering environmental impact in processes such as power generation, transportation, lighting, and manufacturing. High energy efficiency results in more effective and sustainable energy use.

QUESTION 6: How is the work done by a variable force calculated?

ANSWER: WORK DONE BY VARIABLE FORCE

The work done by a variable force is the amount of energy transferred or expended when a force, which can vary in magnitude and direction with respect to position or displacement, acts on an object and causes it to move over a certain distance.

Consider a particle which moves along a curved path subject to a variable force.



The work done can be written as:

$$dW = \vec{F}.d\vec{s}$$

The Total Work done from point A to B will be:

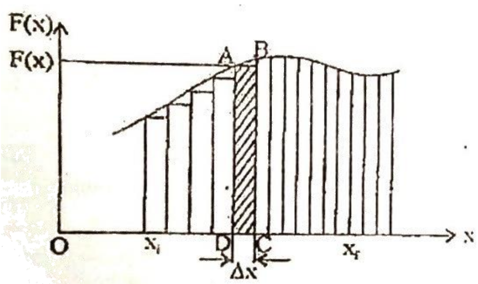
OR Generally $W_{A \text{ to } b} = \sum F.\Delta s$

$$W_{A\to B} =: \int_A^B \vec{F}.d\vec{s} = \int_A^B (F\cos\theta)ds$$

WORK DONE BY VARIBALE FORCE (GRAPHICAL REPRESENTATION):

Consider a body covers displacement from X_i to X_f, when a variable force acts on it.

The Area under the line is the total work done by the body. To calculate the work done we divide total area into 'n' number of small segments as shown in the graph.



Where:

$\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4, \dots, \Delta x_n$ are covered displacements and
$Fx_1, Fx_2, Fx_3, Fx_4, \dots, Fx_n$ are their corresponding forces.

For Total Work Done we can say that $W = W_1 + W_2 + W_3 + W_4 \dots + W_n$

For Total Work Done we can say that $W = \sum_{i=1}^{n} W_i$

$$W = \vec{F}_1.\overrightarrow{\Delta x_1} + \vec{F}_2.\overrightarrow{\Delta x_2} + \vec{F}_3.\overrightarrow{\Delta x_3} + \vec{F}_4.\overrightarrow{\Delta x_4} + \dots + \vec{F}_n.\overrightarrow{\Delta x_n} = \sum_{i=1}^{n} \vec{F}_{xi}.\overrightarrow{\Delta x_i}$$

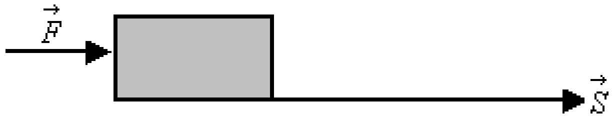
The above the equation of total work done when a variable force acts on it.

SELF ASSESSMENT QUESTIONS

Q-1 **When does work have a positive value, and when does it have a negative value? Provide examples of each case.**

Ans: POSITIVE WORK:

The work done on an object is said to be positive work when force and displacement are in same direction.



The angle between the force and the displacement is θ = 0º

$W = F\ S \cos\theta$

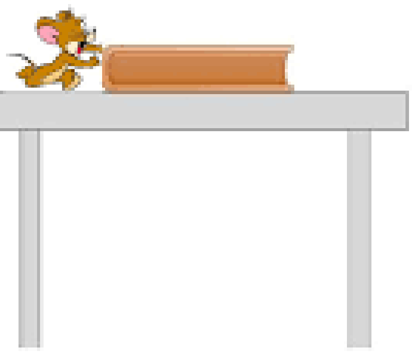
$W = FS \cos 0^o$

$W = FS(1)$ $[\cos 0^o = 1]$

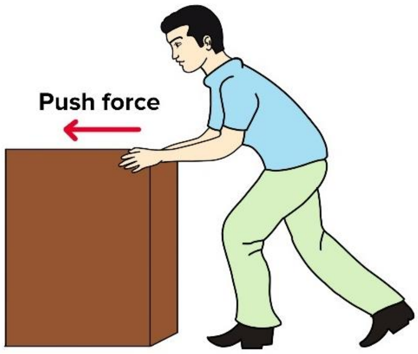
W = FS(work is positive)

Example

1 Positive work is done when pushing a textbook to the right across a level tabletop at a constant velocity



2 Pushing a box through a distance.



NEGATIVE WORK:

The work done is said to be negative work when force and displacement are in opposite direction.

When the force is opposite to the direction of displacement, the work done is negative.

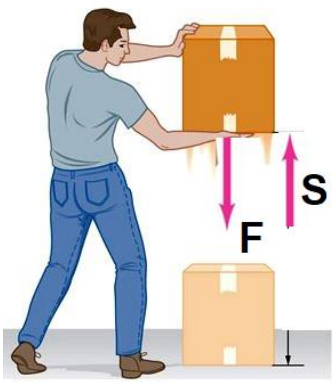
When θ = 180º & Cos 180º = -1

W = FS cosθ

= FS cos 180º

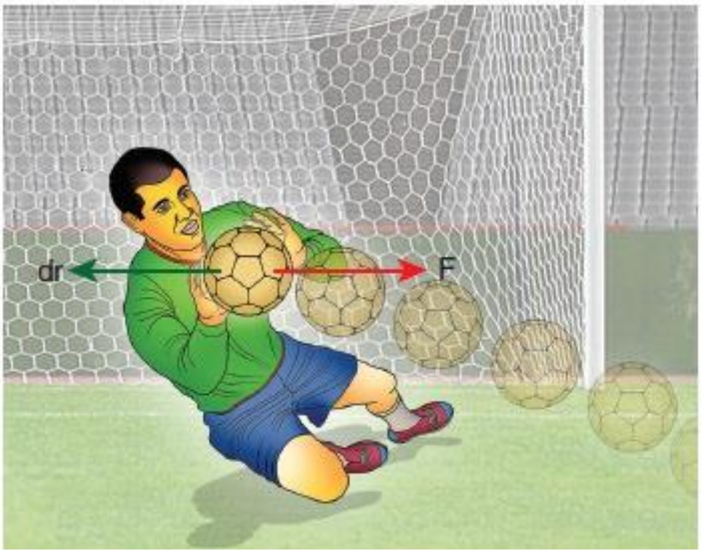
= FS (-1)

= - FS (negative work)



Example

1 The goal keeper catches the ball coming towards him by applying a force such that the force is applied in a direction opposite to that of the motion of the ball till it comes to rest in his hands. During the time of applying the force, he does a negative work on the ball.



2 When a bucket is lifting up, the work of the gravitational force is negative

Q-2 In what units is work typically measured. and how are the units related to force and displacement?

Ans **UNITS:**

1. In SI unit s, work is measured in **Newton – meters**, A special name is given to this unit, the joule (J)
   One joule is defined as the amount of work done when a force of 1 Newton moves an object through a distance of 1 metre.

   1 J = 1 N.m

2. In *CGS* system the unit of work is called 'erg' and is defined as, **when the force of 1 dyne displace a body by 1 cm in the direction of the force, , the work done is 1 erg.**

   1 erg = 1 dyne.cm

3. In British system the unit of work is measured in **'foot-pound'**

Relation

it is easy to show that

1 joule = 10^7 ergs

1 joule = 0.7376 ft-lb

Practical unit

In atomic and nuclear physics, a much smaller unit is used, called electron –volt

1eV = 1.602 x 10^{-19} J

Q-3 How does the concept of work. done by a variable force differ from work done by a constant force?

Ans: The work done by a constant force of magnitude F, as we know, that displaces an object by Δx can be given as

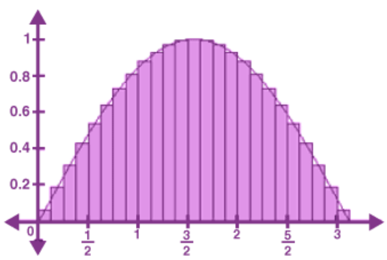
W = F Δx

The variable force's work is a little more complicated. As the job progresses, the force might alter both magnitude and direction. Variable force work is the most common in our daily lives.

Because force is proportional to the object's displacement from its equilibrium position, the force acting at each instant during the spring's compression and the extension will be different. Thus, the infinitesimally small amounts of work performed during each instant must be counted in order to determine the total amount of work performed.

Adding successive rectangles, the total work done can be written as

$$W = \sum_{x_i}^{x_f} F(x)\Delta x$$



Q-4 How can you represent a variable force graphically to analyze the work done?

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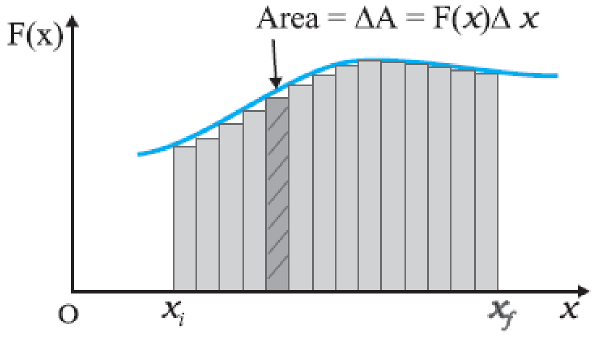

Fig. 1

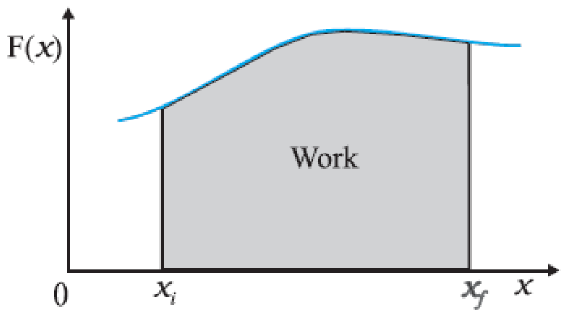

Fig. 2

Where:

$\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4, \ldots, \Delta x_n$ are covered displacements and
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For Total Work Done we can say that $W = W_1 + W_2 + W_3 + W_4 \ldots + W_n$
For Total Work Done we can say that $W = \sum_{i=1}^{n} W_i$

$$W = \vec{F}_1.\overrightarrow{\Delta x_1} + \vec{F}_2.\overrightarrow{\Delta x_2} + \vec{F}_3.\overrightarrow{\Delta x_3} + \vec{F}_4.\overrightarrow{\Delta x_4} + \cdots + \vec{F}_n.\overrightarrow{\Delta x_n} = \sum_{i=1}^{n} \vec{F}_{xi}.\overrightarrow{\Delta x_i}$$

The above the equation of total work done when a variable force acts on it.