

CHAPTER = 4**ROTATIONAL AND CIRCULAR MOTION****BOOK NUMERICAL**

- 1 A car mechanic applies a force of 800 N to a wrench for the purpose of loosening a bolt. He applies the force which is perpendicular to the arm of the wrench. The distance from the bolt to the mechanic's hand is 0.40 m. Find out the magnitude of the torque applied?

DATA

$$F = 800 \text{ N}$$

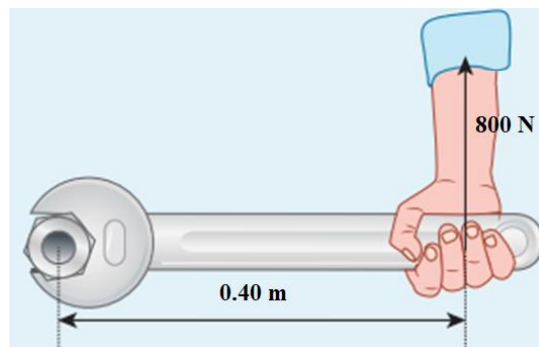
$$r = 0.40 \text{ m}$$

SOLUTION

$$\tau = r F \sin \theta$$

$$\tau = 0.40 \times 800 \sin 90$$

$$\tau = 320 \text{ Nm}$$



- 2 A car accelerates uniformly from rest and reaches a speed of 22 m/s in 9 s. If the diameter of a tire is 58 cm, find
- the number of revolutions the tire makes during this motion, assuming no slipping, and
 - the final rotational speed of the tire in revolutions per second

DATA

$$V_i = 0$$

$$V_f = 22 \text{ m/s}$$

$$t = 9 \text{ s}$$

$$D = 58 \text{ cm}$$

$$r = \frac{D}{2} = \frac{58}{2}$$

$$r = 29 \text{ cm} = \frac{29 \text{ cm}}{100} = 0.29 \text{ m}$$

SOLUTION:

Acceleration of the car

$$a = \frac{V_f - V_i}{t}$$

$$a = \frac{20 - 0}{9}$$

$$a = 2.469 \text{ m/s}^2$$

Angular acceleration of the car

$$a = r \alpha$$

$$\alpha = \frac{a}{r} = \frac{2.469}{0.29}$$

$$\alpha = 8.51 \text{ rad/s}^2$$

Angular speed of the car

$$\omega_f = \frac{V_f}{r} \quad \text{and} \quad \omega_f = 0$$

$$\omega_f = \frac{22}{0.29} = 75.86 \text{ rad/s}$$

Angular displacement

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\theta = (0)(9) + \frac{1}{2}(8.51)(9)^2$$

$$\theta = 0 + 344.655$$

$$\theta = 344.665 \text{ rad}$$

Angular displacement in revolution

$$\theta = \frac{344.665}{2\pi}$$

$$\theta = 54.85 \text{ rev}$$

Angular speed of the car in rev/s

$$\omega_f = \frac{75.86}{2\pi} = 12.07 \text{ rev/s}$$

- 3 An ordinary workshop grindstone has a radius of 7.5 cm and rotates 6500 rev/min.
- (a) Calculate the magnitude of centripetal acceleration at its edge in m/s^2 and convert it into multiples of g .
- (b) What is the linear speed of a point on its edge?

DATA

$$r = 7.5 \text{ cm} = \frac{7.5}{100} = 0.075 \text{ m}$$

$$\omega = 6500 \text{ rev/min}$$

$$\omega = \frac{6500 \times 2\pi}{1 \times 60} = 680.678 \text{ rad/s}$$

DATA

$$r = 7.5 \text{ cm} = \frac{7.5}{100} = 0.075 \text{ m}$$

$$\omega = 6500 \text{ rev/min}$$

$$\omega = \frac{6500 \times 2\pi}{1 \times 60} = 680.678 \text{ rad/s}$$

SOLUTION:

(a) Centripetal acceleration

$$a = r \omega^2$$

$$a = (0.075) (680.678)^2$$

$$a = (0.075) (4.633 \times 10^5)$$

$$a = 3.47 \times 10^4 \text{ m/s}^2$$

Centripetal acceleration in term of g

$$a = 3.47 \times 10^4 \times \frac{g}{g}$$

$$a = \frac{3.47 \times 10^4}{9.8} \times g$$

$$a = 3.54 \times 10^3 g$$

(a) Linear speed

$$V = r \omega$$

$$V = (0.075) (680.678)$$

$$V = 51.01 \text{ m/s}$$

- 4 A satellite is orbiting the Earth with an orbital velocity of 3200 m/s. What is the orbital radius?

DATA

$$v = 3200 \text{ m/s}$$

$$r = ?$$

SOLUTION:

(b) Centripetal acceleration

$$v = \sqrt{\frac{G M}{R}}$$

Squaring both the side

$$(v)^2 = \left(\sqrt{\frac{G M}{R}} \right)^2$$

$$v^2 = \frac{G M}{R}$$

$$R = \frac{G M}{v^2}$$

$$R = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(3200)^2}$$

$$R = \frac{3.988 \times 10^{14}}{1.024 \times 10^7}$$

$$R = 3.894 \times 10^7 \text{ m}$$

- 5 A satellite wishes to orbit the earth at a height of 100 km (approximately 60 miles) above the surface of the earth. Determine the speed, acceleration and orbital period of the satellite. (Given: $M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}$, $R_{\text{Earth}} = 6.37 \times 10^6 \text{ m}$)

DATA

$$h = 100 \text{ km} = 100\,000 \text{ m} = 10^5$$

$$R = R_e + h$$

$$R = (6.37 \times 10^6 + 10^5) = 6.47 \times 10^6 \text{ m}$$

$$M_e = 5.98 \times 10^{24} \text{ kg}$$

$$v = ?$$

SOLUTION:

Orbital speed

$$v = \sqrt{\frac{GM}{R}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.47 \times 10^6}}$$

$$v = \sqrt{\frac{3.988 \times 10^{14}}{6.47 \times 10^6}}$$

$$v = \sqrt{6.1638 \times 10^7}$$

$$v = 7.85 \times 10^3 \text{ m/s}$$

$$a = \frac{v^2}{R}$$

$$a = \frac{(7.85 \times 10^3)^2}{6.47 \times 10^6}$$

$$a = \frac{6.1638 \times 10^7}{6.47 \times 10^6} = 9.524 \text{ m/s}^2$$

orbital period

$$T = \sqrt{\frac{4\pi R^3}{GM_E}}$$

$$T = \sqrt{\frac{4(3.142)(6.47 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}}$$

$$T = 1.44 \text{ hours}$$

- 6 A thin disk with a 0.3m diameter and a total moment of inertia of $0.45 \text{ kg} \cdot \text{m}^2$ is rotating about its center of mass. There are three rocks with masses of 0.2kg on the outer part of the disk. Find the total moment of inertia of the system.?

DATA

$$I_d = 0.45 \text{ kg m}^2$$

$$D = 0.3 \text{ m}$$

$$r = \frac{0.3}{2} = 0.15 \text{ m}$$

$$m_1 = 0.2 \text{ kg}$$

$$m_2 = 0.2 \text{ kg}$$

$$m_3 = 0.2 \text{ kg}$$

SOLUTION:

$$I = I_d + I_{m_1} + I_{m_2} + I_{m_3}$$

$$I = I_d + m_1 r^2 + m_2 r^2 + m_3 r^2$$

$$m_1 = m_2 = m_3 = m$$

$$I = I_d + m r^2 + m r^2 + m r^2$$

$$I = I_d + 3 m r^2$$

$$I = 0.45 + 3(0.2)(0.15)^2$$

$$I = 0.45 + 0.0135 = 0.4635 \text{ kg m}^2$$

- 7 What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

DATA

$$r = 1.20 \text{ km}$$

$$r = 1200 \text{ m}$$

$$v = 105 \text{ km/h}$$

$$v = \frac{105 \times 1000}{3600} = 29.16 \text{ m/s}$$

$$\theta = ?$$

SOLUTION:

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

$$\theta = \tan^{-1} \left\{ \frac{(29.16)^2}{(1200) \times (9.8)} \right\}$$

$$\theta = \tan^{-1} \left\{ \frac{850.305}{11760} \right\}$$

$$\theta = \tan^{-1} (0.0723)$$

$$\theta = 4.13^\circ$$

- 8 A 1500 kg car moving on a flat horizontal road negotiates a curve as shown. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully

DATA

$$m = 1500 \text{ kg}$$

$$r = 35.0 \text{ m}$$

$$\mu = 0.523$$

$$V = ?$$

SOLUTION:

$$f = \mu R$$

$$f = \mu mg \quad \{ R = W = mg \}$$

$$f = (0.523) (1500) (9.8)$$

$$f = 7688 \text{ N}$$

$$\text{centripetal force} = \text{friction force}$$

$$\frac{mv^2}{r} = f$$

$$\frac{1500 v^2}{35.0} = 7688$$

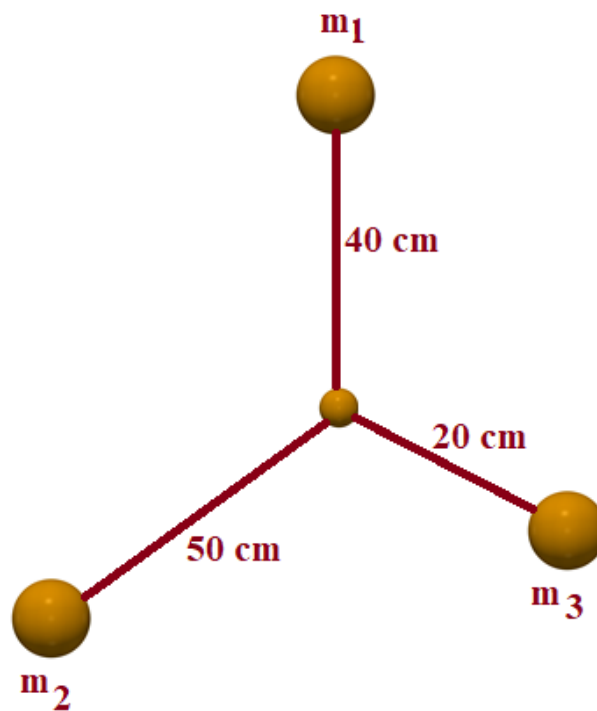
$$v^2 = \frac{7688 \times 35.0}{1500}$$

$$v^2 = 179.38$$

$$v = \sqrt{179.38}$$

$$v = 13.39 \text{ m/s}$$

- 9 A system of points is shown in the figure. Each particle has the same mass of 0.3 kg and all lie in the same plane. What is the moment of inertia of the system about a given axis?



DATA

$$m = 0.3 \text{ kg}$$

$$r_1 = 40 \text{ cm} = \frac{40}{100} = 0.4 \text{ m}$$

$$r_2 = 50 \text{ cm} = \frac{50}{100} = 0.5 \text{ m}$$

$$r_3 = 20 \text{ cm} = \frac{20}{100} = 0.2 \text{ m}$$

$$I = ?$$

SOLUTION:

$$I = m r_1^2 + m r_2^2 + m r_3^2$$

$$I = (0.3) (0.4)^2 + (0.3) (0.5)^2 + (0.3) (0.2)^2$$

$$I = 0.048 + 0.075 + 0.012$$

$$I = 0.135 \text{ kg m}^2$$

- 10 (a) What is the angular momentum of a 2.9 kg uniform cylindrical grinding wheel of radius 20 cm when rotating 1550 rpm? (b) How much torque is required to stop it in 6 s?



DATA

$$m = 2.9 \text{ kg}$$

$$r = 20 \text{ cm} = \frac{20}{100} = 0.2 \text{ m}$$

$$\omega = 1550 \text{ rpm (rev per min)}$$

$$\omega = \frac{1550 \times 2\pi}{60} = \frac{9738.937}{60}$$

$$\omega = 162.315 \text{ rad/s}$$

$$L = ?$$

$$t = 6 \text{ s}$$

$$\tau = ?$$

SOLUTION:

$$L = I \omega$$

$$\text{moment of inertia of solid cylinder} = \frac{1}{2} m r^2$$

$$L = \frac{1}{2} m r^2 \omega$$

$$L = \frac{1}{2} (2.9) (0.2)^2 (162.315)$$

$$L = (0.5)(2.9) (0.04) (162.315)$$

$$L = 9.414 \text{ Js}$$

For torque required to stop

$$\tau = \frac{\Delta L}{\Delta t}$$

$$\tau = \frac{0 - 9.414}{6}$$

$$\tau = - \frac{9.414}{6}$$

$$\tau = - \frac{9.414}{6}$$

$$\tau = - 1.569 \text{ Nm}$$

- 11 Determine the angular momentum of the Earth (a) about its rotation axis (Assume the Earth as uniform sphere), and (b) in its orbit around the Sun (Take Earth as a particle orbiting the Sun). The Earth has mass 6×10^{24} kg and radius 6.4×10^6 m, and is 1.5×10^8 km from the Sun

DATA

$$M_e = 5.98 \times 10^{24} \text{ kg}$$

$$R_e = 6.38 \times 10^6 \text{ m}$$

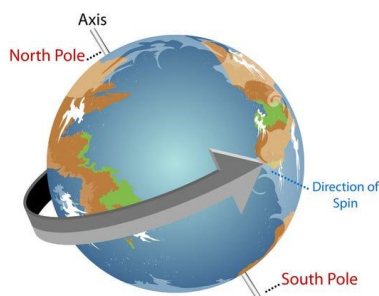
$$R = 1.5 \times 10^8 \text{ km} = 1.5 \times 10^{11} \text{ m}$$

$$T = 24 \text{ h} = 24 \times 3600 = 86400 \text{ sec}$$

$$L =$$

SOLUTION:

THE EARTH ROTATES ON ITS AXIS



Angular momentum is given by

$$L = I \omega$$

$$\text{moment of inertia of Earth} = \frac{2}{5} M_e R_e^2$$

$$I_e = \frac{2}{5} (5.98 \times 10^{24}) (6.38 \times 10^6)^2$$

$$I_e = 9.736 \times 10^{37} \text{ kg m}^2$$

$$\text{angular speed of Earth} = \frac{2\pi}{T}$$

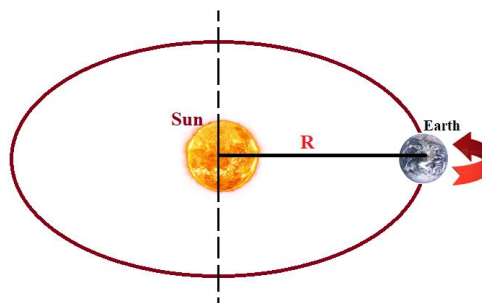
$$\omega = \frac{2 \times 3.14}{86400}$$

$$\omega = 7.272 \times 10^{-5} \text{ rad/s}$$

$$L = (9.736 \times 10^{37}) (7.272 \times 10^{-5})$$

$$L = 7.08 \times 10^{33} \text{ kg m}^2$$

EARTH ORBITS AROUND THE SUN



Angular momentum is given by

$$L = I \omega$$

$$\text{moment of inertia} = M_e R_e^2$$

$$I_e = (5.98 \times 10^{24}) (1.5 \times 10^{11})^2$$

$$I_e = 1.3455 \times 10^{47} \text{ kg m}^2$$

$$\text{angular speed of Earth} = \frac{2\pi}{T}$$

$$\omega = \frac{2 \times 3.14}{365.3 \times 24 \times 3600}$$

$$\omega = 1.99 \times 10^{-7} \text{ rad/s}$$

$$L = (1.3455 \times 10^{47}) (1.99 \times 10^{-7})$$

$$L = 2.677 \times 10^{40} \text{ kg m}^2$$

CHAPTER = 4**ROTATIONAL AND CIRCULAR MOTION****WORKED EXAMPLES**

- 1 The platter of the hard drive of a computer rotates at 7300 rpm (a) What is the angular velocity of the platter? (b) if the reading head of the drive is located 3.1 cm from the rotation axis, what is the linear speed of the point on the platter just below it? (c) If a single bit requires 0.55 μm of length along the direction of motion, how many bits per second can the writing head write when it is 3.1 cm from the axis?

Data:

$$\omega = 7300 \text{ rpm}$$

$$(a) \omega = ?$$

$$(b) r = 3.1 \text{ cm} = 0.031 \text{ m}$$

$$V = ?$$

$$(c) \text{ Singal bit length} = 0.55 \mu\text{ m}$$

$$\text{Singal bit length} = 0.55 \times 10^{-6} \text{ m}$$

$$r = 3.1 \text{ cm}$$

$$r = 0.031 \text{ m}$$

$$\text{bits per second} = ?$$

SOLUTION:

$$\omega = 7300 \text{ rpm}$$

$$\omega = \frac{7300 \text{ rev}}{\text{min}}$$

$$\omega = \frac{7300 \times 2\pi}{60}$$

$$\omega = 764.45 \text{ rad/s}$$

$$V = r \omega$$

$$V = (0.031) (764.45)$$

$$V = 23.69 \text{ m/s}$$

The number of bits passing the head per second is

$$\text{bits per second} = \frac{\text{linear speed}}{\text{single bit length}}$$

$$\text{bits per second} = \frac{23.69 \frac{\text{m}}{\text{s}}}{0.55 \times 10^{-6} \frac{\text{m}}{\text{bit}}}$$

$$\text{bits per second} = \frac{23.69}{0.55 \times 10^{-6}} \times \left(\frac{\text{m}}{\text{s}}\right) \frac{\text{bit}}{\text{m}}$$

$$\text{bits per second} = \frac{23.69}{0.55 \times 10^{-6}} \times \left(\frac{1}{\text{s}}\right) \frac{\text{bit}}{1}$$

$$\text{bits per second} = 43 \times 10^6 \left(\frac{\text{bits}}{\text{s}}\right)$$

- 2 A motor cycle wheel turns 3620 times while being ridden for 6.50 minutes. What is the angular speed in rev/min (rpm)?

DATA

$$\theta = 3620 \text{ rev}$$

$$t = 6.50 \text{ min}$$

$$\omega = \text{rev/min}$$

SOLUTION

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{3620}{6.5}$$

$$\omega = 557 \text{ rev/min}$$

3 What is the magnitude of force and the centripetal acceleration of a car having mass of 300 kg following a curve of radius 500 m at a speed of 100 km/h? also compare the acceleration with that due to gravity for this fairly gentle curve taken at highway speed.

DATA

$$m = 300 \text{ kg}$$

$$r = 500 \text{ m}$$

$$v = 100 \text{ km/h}$$

$$v = \frac{100 \times 1000}{3600}$$

$$v = 27.77 \text{ m/s}$$

SOLUTION

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{(27.77)^2}{500} = \frac{771.17}{500}$$

$$a_c = 1.54 \text{ m/s}^2$$

$$F_c = \frac{m v^2}{r}$$

$$F_c = \frac{(300)(27.77)^2}{500} = \frac{(300)(771.17)}{500}$$

$$F_c = \frac{231351}{500}$$

$$F_c = 462.7 \text{ N}$$

To compare the a_c with g by taking ratio, we get

$$\frac{a_c}{g} = \frac{1.54}{9.8}$$

$$\frac{a_c}{g} = 0.157$$

$$a_c = 0.157 g$$

4 Curve on some test tracks and race courses, such as M-1 Islamabad -Lahore Motorway, are very steeply banked. This banking, with the support of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100 m radius curve banked at 30° should be driven if the road were frictionless.

DATA

$$r = 100 \text{ m}$$

$$\theta = 30^\circ$$

$$v = ?$$

SOLUTION

$$\tan \theta = \frac{v^2}{r g}$$

$$v^2 = (\tan \theta) (r) (g)$$

$$v = \sqrt{(r) (g)(\tan \theta)}$$

$$v = \sqrt{(100) (9.8)(\tan 30)}$$

$$v = \sqrt{(100) (9.8)(0.5773)}$$

$$v = \sqrt{565.754}$$

$$v = 23.78 \text{ m/s}$$

- 5** The International Space Station orbits at an altitude of 400 km above the surface of the Earth. What is the space station's orbital velocity?

DATA

$$h = 400 \text{ km} = 400000 \text{ m}$$

$$R = (R_e + h)$$

$$R = (6.38 \times 10^6 + 400000) \text{ m}$$

$$R = 6.78 \times 10^6 \text{ m}$$

$$v = ?$$

SOLUTION

$$v = \sqrt{\frac{G M_e}{R}}$$

$$v = \sqrt{\frac{6.673 \times 10^{-11} \times 5.98 \times 10^{24}}{6.78 \times 10^6}}$$

$$v = \sqrt{\frac{3.99 \times 10^{14}}{6.78 \times 10^6}}$$

$$v = \sqrt{5.8849 \times 10^7}$$

$$v = 7.671 \times 10^3 \text{ m/s}$$

- 6** A basketball spinning on the finger of an athlete has angular velocity $\omega = 120.0 \text{ rad/s}$. The moment of inertia of a sphere that is hollow, where M is the mass and R is the radius. If the basketball has a weight of 0.6000 kg and has a radius of 0.1200 m, what is the angular momentum of this basketball?

DATA

$$\omega = 120.0 \text{ rad/s}$$

$$m = 0.6000 \text{ kg}$$

$$r = 0.1200 \text{ m}$$

$$L = ?$$

SOLUTION

Moment of inertia of hollow sphere is given by

$$I = \frac{2}{3} m r^2$$

$$I = \frac{2}{3} (0.6000) (0.1200)^2$$

$$I = \frac{2}{3} (0.6000) (0.01440)$$

$$I = 0.005760 \text{ kg m}^2$$

$$I = 5.760 \times 10^{-3} \text{ kg m}^2$$

The angular momentum is:

$$L = I \omega$$

$$L = (5.760 \times 10^{-3}) (120.0)$$

$$L = 0.6912 \text{ kg m}^2/\text{s}$$

CHAPTER = 4**ROTATIONAL AND CIRCULAR MOTION****ADDITIONAL NUMERICAL**

- 1 A string 1 meter long would break when its tension is 69.6N. Find the greatest speed at which a ball of mass 2kg can be whirled with the string in a vertical circle.

Data:

Length of the string

= radius of the circle $r = 1 \text{ m}$

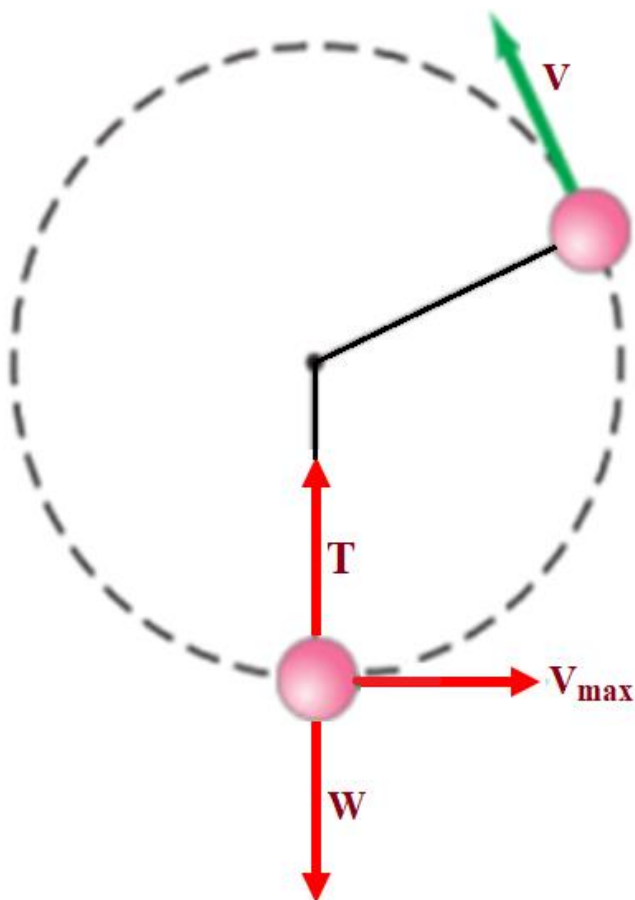
Tension in the string $T = 69.6 \text{ N}$

Mass of the ball $m = 2 \text{ kg}$

Maximum speed of the ball $V = ?$

Solution:

When a ball is whirled in a vertical circle tension in the string will be maximum when the ball is at the lowest level from the ground, at this position the unbalanced force which provides the necessary centripetal force will be $T - W$.



$T - W = \text{centripetal force}$

$$T - mg = \frac{mV^2}{r}$$

$$69.6 - (2 \times 9.8) = \frac{2V^2}{1.0}$$

$$69.6 - 19.6 = 2V^2$$

$$50 = 2V^2$$

$$V^2 = 25$$

$$V = \sqrt{25}$$

$$V = 5 \text{ ms}^{-1}$$

The greatest speed with which the ball can be whirled is 5 m/s.

2. A ball of mass 0.2 kg is tied to the end of a string and whirled in a horizontal circle of radius 0.4m. If the ball makes 10 complete revolutions in 4 second, determine the linear speed, centripetal acceleration and centripetal force.

<p><u>Data:</u></p> <p>m = 200gm = 0.2 kg r = 0.6 m Number of revolutions = 10 t = 2 s V = ? ; a = ?; F_c = ?</p> <p><u>SOLUTION:</u></p> <p>Time period,</p> $T = \frac{\text{Total Time}}{\text{No. of revolutions}}$ $T = \frac{4}{10} = 0.4 \text{ sec}$ <p>We have</p> $S = Vt$ $V = \frac{S}{t}$	$V = \frac{2 \pi r}{T}$ $V = \frac{2(3.142)(0.4)}{0.4}$ $V = 6.28 \text{ m/sec}$ <p>Centripetal acceleration</p> $a_c = \frac{V^2}{r}$ $a_c = \frac{(6.28)^2}{0.4}$ $a_c = 98.596 \text{ m/sec}^2$ <p>Then</p> $F = m a$ $F = 0.2 \times 98.596$ $F = 19.7192 \text{ N}$
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3. Calculate the centripetal acceleration and centripetal force acting on a man whose mass is 64 kg when he is resting on the ground at the equator. (radius of earth = 6.4 x 10⁶ m)

<p><u>Data:</u></p> <p>Mass of the man m = 64 kg Radius of the earth R = 6.4 x 10⁶ m Centripetal acceleration a_o = ? Centripetal force F_o = ? Period of revolution T = 1 day T = 24 hrs. T = 24 x 60 x 60 sec. T = 86400 s</p> <p><u>Solution:</u></p> $a_c = \frac{4\pi^2 R}{T^2}$	$a_c = \frac{4 \times (\pi)^2 \times 6.4 \times 10^6}{(24 \times 60 \times 60)^2}$ $a_c = \frac{2.5266 \times 10^8}{7.46496 \times 10^9}$ $a_c = 0.03385 \text{ m/s}^2$ $F = m a_c$ $F = 64 \times 0.03385$ $F = 2.1664 \text{ N}$
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- 4 Tarzan swings on a vine of length 5m in a vertical circle under the influence of gravity. When the vine makes an angle of 30 degree with the vertical, Tarzan has a speed of 4m/sec. Find (a) centripetal acceleration at this instant (b) the tangential acceleration.

Data:

$$L = r = 5 \text{ m}$$

$$\theta = 30^\circ$$

$$V = 4 \text{ m/s}$$

$$a_c = ?$$

$$a_t = ?$$

$$a = ?$$

Solution:

(a) The only component of weight responsible to bring the Tarzan back to his initial position is and it is along tangent to the arc i.e.

$$F_t = W_{\perp}$$

$$ma_t = W \sin \theta$$

$$ma_t = mg \sin \theta$$

$$a_t = 9.8 \sin 30^\circ$$

$$a_t = 9.8 \times 0.5$$

$$a_t = 4.9 \text{ m/s}^2$$

$$(b) a_c = \frac{V^2}{R}$$

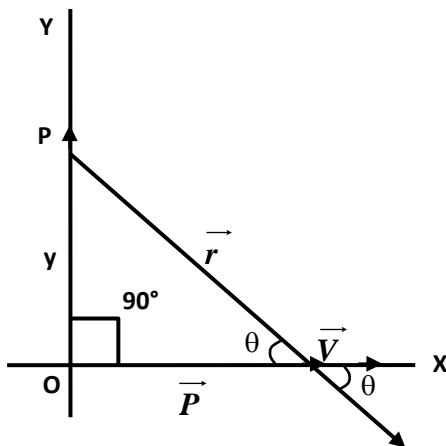
$$a_c = \frac{(4)^2}{5}$$

$$a_c = 3.2 \text{ m/s}^2$$

- 5 A particle with mass 4 kg move along the x-axis with a velocity $v = 15t$ m/sec, where $t = 0$ is the instant that the particle is at the origin.

(a) At $t = 2$ sec, what is the angular momentum of the particle about a point P located on + y – axis, 6m from the origin?

(b) What torque about P acts on the particle?



DATA:

$$m = 4\text{kg},$$

$$v = 15t \text{ m/sec}$$

$$t = 2 \text{ sec},$$

$$y = 6\text{m}$$

$$L = ?$$

$$\tau = ?$$

SOLUTION:

$$(i) \vec{L} = \vec{r} \times \vec{p}$$

$$L = r p \sin \theta$$

$$L = r (m v) \sin \theta$$

$$L = (m v) (r \sin \theta) \dots \dots \dots (i)$$

$$\sin \theta = \frac{\text{perp}}{\text{hyp}}$$

$$\sin \theta = \frac{y}{r}$$

$$y = r \sin \theta$$

$$6 = r \sin \theta$$

$$v = 15t$$

$$v = 15 \times 2 = 30 \text{ m/s}$$

Putting $V = 30 \text{ m/s}$ and $r \sin \theta = 6$ in eq(i)

$$L = (4 \times 30) (6)$$

$$L = 720 \text{ Js}$$

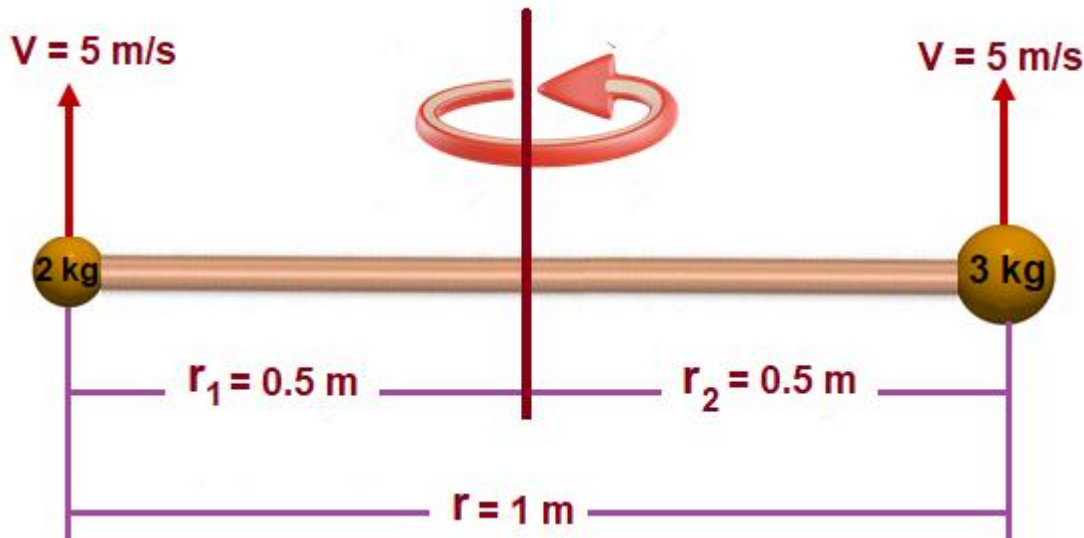
$$(ii) \tau = \frac{\Delta L}{\Delta t} \because \text{at } t = 0$$

$$\tau = \frac{L_f - L_i}{t_f - t_i} \quad (L_i = 0)$$

$$\tau = \frac{720 - 0}{2 - 0}$$

$$\tau = 360 \text{ Nm}$$

- 6 A light rigid rod 1 m in length rotates in the xy-plane about a pivot through the rod's centre. Two particles of mass 2 kg and 3 kg are connected to its ends. Determine the angular momentum of the system about the origin at the instant the speed of each particle is 5 m/sec.



Data:

$$m_1 = 2 \text{ kg}$$

$$m_2 = 3 \text{ kg}$$

$$L = ?$$

$$V_1 = 5 \text{ m/s}$$

$$V_2 = 5 \text{ m/s}$$

SOLUTIONS

The angular momentum of 1st particle about centre of rod is

$$\vec{L}_1 = \vec{r}_1 \times \vec{p}_1$$

$$L_1 = (r_1)(P_1) \sin \theta$$

$$L_1 = (r_1)(m v_1) \sin \theta$$

$$L_1 = (0.5)(2 \times 5) \sin 90$$

$$L_1 = 5 \text{ Js}$$

The angular momentum of 2nd particle about centre of rod is

$$\vec{L}_2 = \vec{r}_2 \times \vec{p}_2$$

$$L_2 = (r_2)(P_2) \sin \theta$$

$$L_2 = (r_2)(m v_2) \sin \theta$$

$$L_2 = (0.5)(3 \times 5) \sin 90$$

$$L_2 = 7.5 \text{ Js}$$

Total angular momentum about centre of rod is

$$L = L_1 + L_2$$

$$L = 5 + 7.5$$

$$L = 12.5 \text{ Js}$$