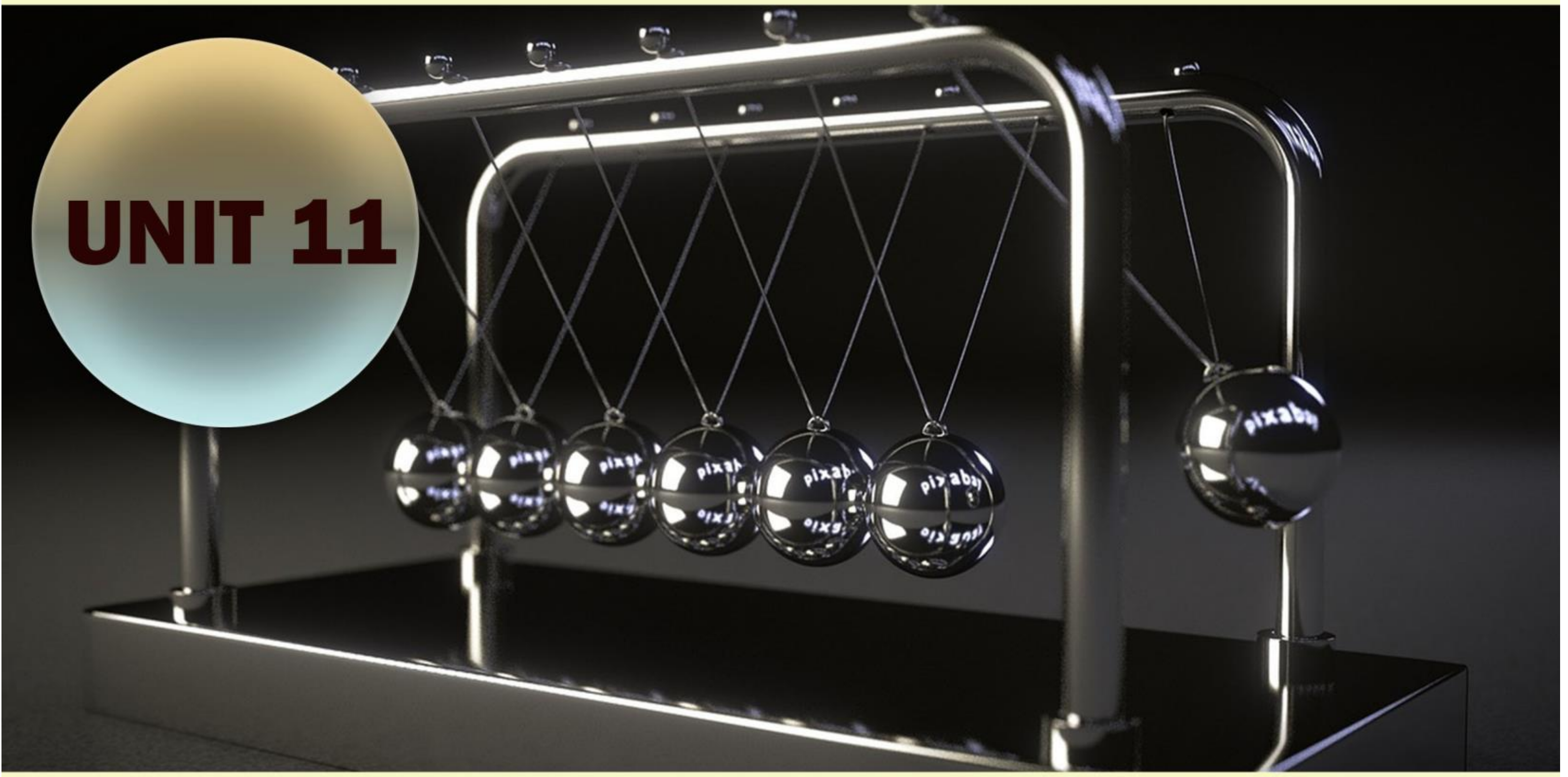
PHYSICS XI



OSCILLATIONS

PROF:IMRAN HASHMI

VIBRATORY MOTION

When a body moves to and fro about its mean position, it is said to possess vibratory or oscillatory motion.

Examples

Some examples are:

1. The motion of the sitar's string
2. Prongs of a tuning fork
3. The motion of atoms in a solid etc.

PERIODIC MOTION

A motion, which repeats itself in equal intervals of time, is called periodic motion.

Example

Some examples are:

1. The motion of a simple pendulum
2. The motion mass-spring system. etc.

SIMPLE HARMONIC MOTION

DEFINITION:

An object moves with simple harmonic motion whenever its acceleration is directly proportional to its displacement from some equilibrium position and is oppositely directed or directed towards an equilibrium position is known as simple harmonic motion.

MATHEMATICAL EXPRESSION:

$$a \propto -x$$

Where , **a** is an acceleration

x is a displacement from the **equilibrium** position,

A negative sign shows the acceleration is directed toward the **equilibrium** position

Example

Some examples of SHM are

1) The motion of simple pendulum,
2) The motion of projection of a body moving in a circle etc.

CHARACTERISTICS AND CONDITIONS OF SIMPLE HARMONIC MOTION

1) **There must be a restoring force, directly proportional to displacement.**
2) **The acceleration should be directly proportional to displacement.**
3) **The acceleration should be directed toward mean position.**
4) **The system should be frictionless.**
5) **The body must have inertia.**
6) **Total energy of the vibrating body should be constant.**

MOTION UNDER ELASTIC RESTORING FORCE

A mass attached to an ideal massless spring and free to move over a frictionless horizontal surface performs simple harmonic motion.

Proof:

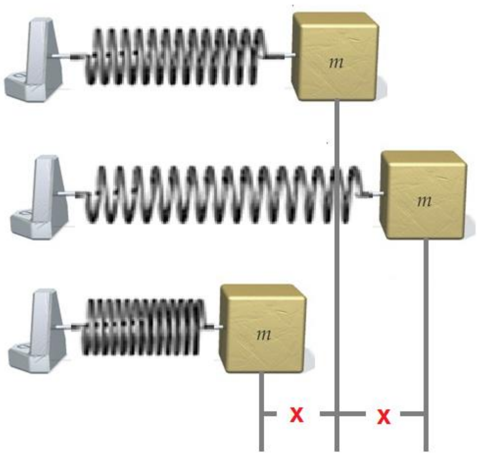
Consider an object of mass 'm' attached to an ideal spring placed on a frictionless horizontal surface. If the spring is stretched or compressed a small distance x from its upstretched position and then released it exerted force on the object obey the equation.

(Applied force) = k*x*.

Where x is the displacement of the object from its equilibrium position and k is the positive constant called the **spring constant.**

Restoring force = - (Applied force)

F = - k (Hooke's law)



The negative sign in the above equation significance that the force exerted by the spring is always directed opposite the displacement of the object. We can use Newton's second law of motion to calculate the acceleration of the object.

$$m\,a = -k\,x \qquad [\,F = ma\,]$$

$$a = -\frac{k}{m}x$$

$\frac{k}{m}$ = is constant during the motion of the object.

Then, $a = -(\text{constant})\,x$

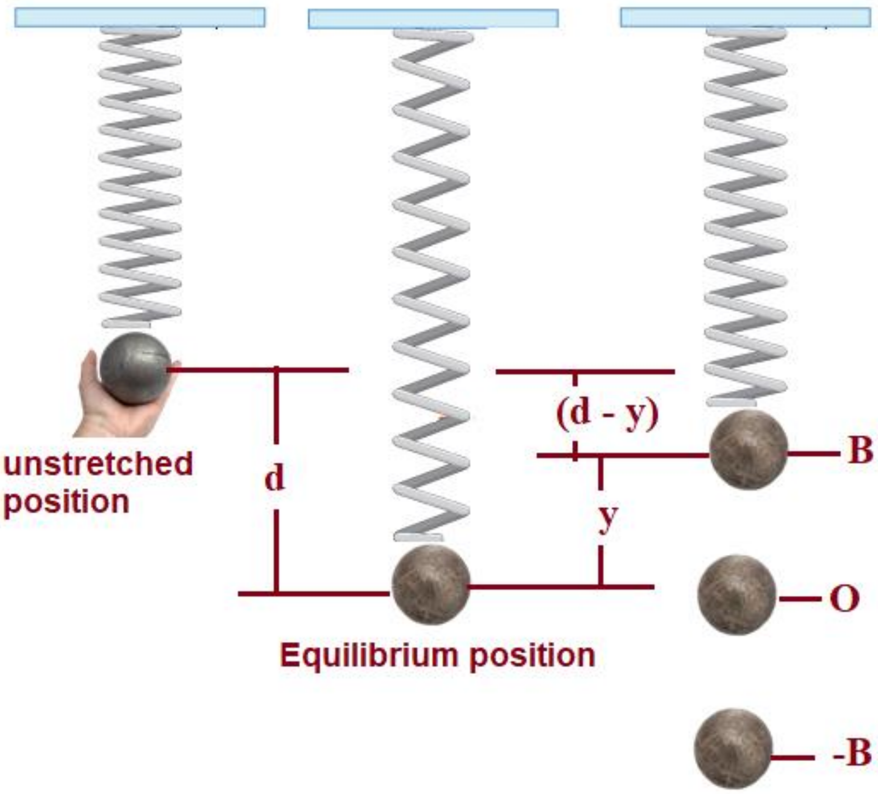
Or $\mathbf{a \propto -x}$

This is an equation representing simple harmonic motion **(SHM)**, which shows that a mass attached to a spring performs **S H M**,

MOTION OF A MASS ATTACHED TO A SPRING:

An oscillating mass on a vertical spring also exhibits SHM.

Suppose an object of mass **m** and weight **mg** is hung from the spring with spring constant **k**. The spring is stretched downward under gravity to a distance **d** from its relaxed point **A** and settled at **0** the equilibrium position.



Taking the y-axis in an upward direction the net force acting on mass at equilibrium is

$$F_{net} = \text{spring force} - \text{weight}$$
$$F_{net} = kd - mg$$
$$0 = kd - mg$$
$$mg = kd \quad \ldots\ldots\ldots (i)$$

if the upward direction of y is taken as positive then the net force acting on the mass at B is

$$F_{net} = \text{spring force} - \text{weight}$$
$$F_{net} = k(d - y) - mg$$
$$F_{net} = k\,d - ky - mg \quad \ldots\ldots. (ii)$$

Substituting the expression k d = mg in equation (ii)

$$F_{net} = mg - ky - mg$$
$$ma = -ky$$
$$a = -\frac{k}{m} y$$

$\frac{k}{m}$ *is constant*

$$a = -\,constant\ y$$
$$a \propto -y$$

INTRODUCTION TO SHM GRAPHS

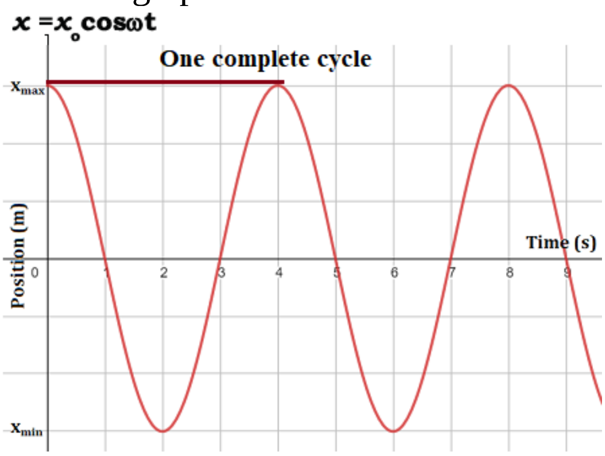
SHM is characterized by its sinusoidal nature, which is best understood through its graphical representations

Displacement in SHM

Displacement in SHM is the distance from the mean position in a specific direction.

Graphical Representation of Displacement

1. Displacement-Time Graph: A sinusoidal curve representing how displacement varies with time.
2. Amplitude (x_0): The peak value of displacement; the highest point on the displacement graph.
3. Period (T): The duration for a complete oscillation, visible as the distance between two consecutive peaks on the graph.

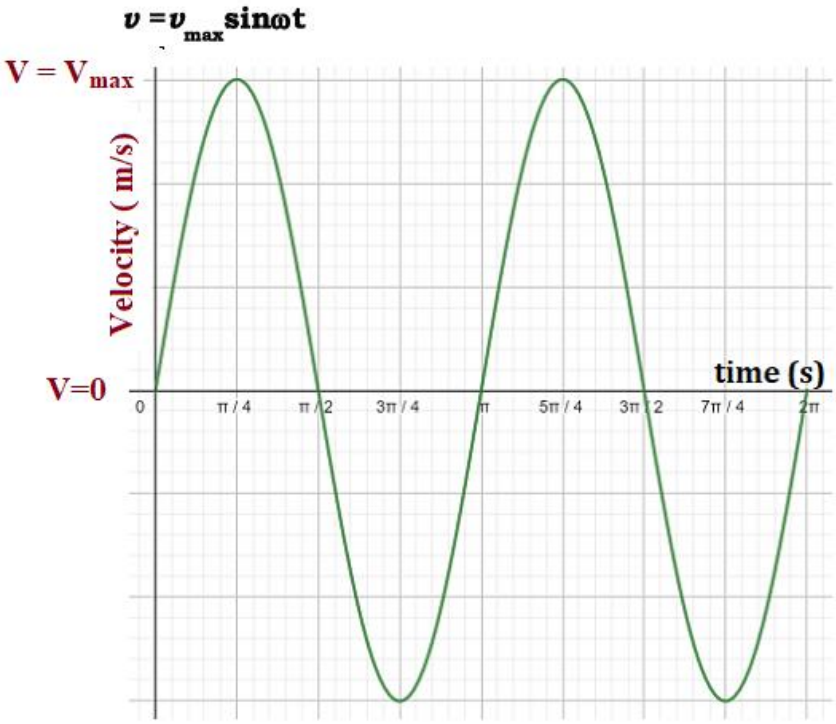


VELOCITY IN SHM

Velocity in SHM is the rate of change of displacement. It gives us an idea about how fast the object is moving and in which direction.

Velocity-Time Graph in SHM

1. sine Curve Representation: Velocity in SHM follows a cosine curve, which is a phase-shifted version of the displacement graph.
2. Maximum Velocity (v_0): The peak value of the velocity graph, occurs when the object crosses the equilibrium position.
3. Zero Velocity: Points where the curve touches the time axis, indicating the object is momentarily at rest (at maximum displacement points).

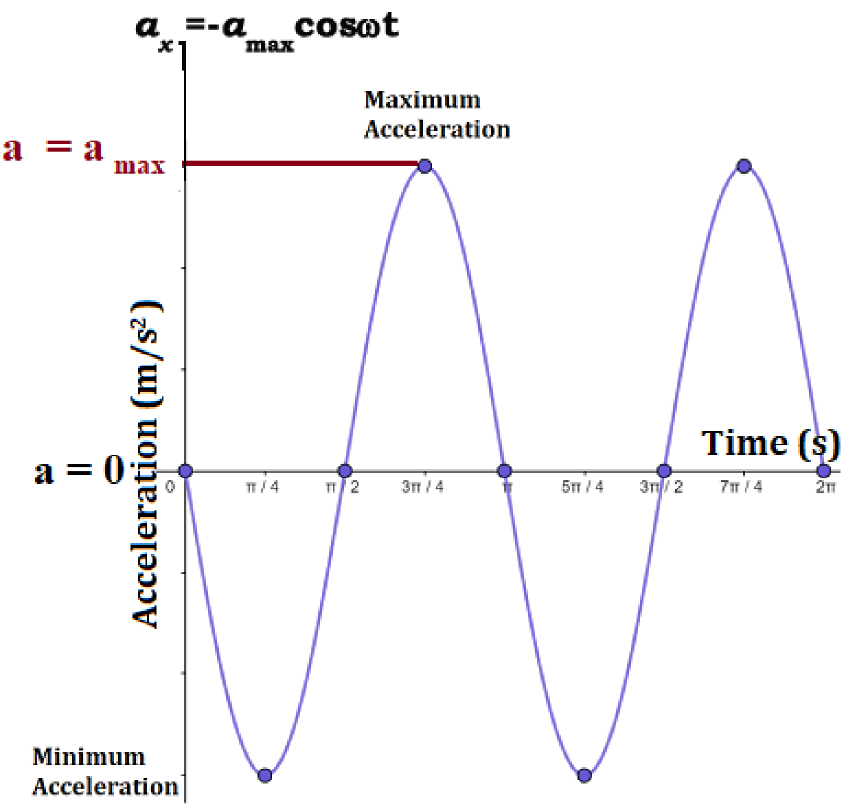


ACCELERATION IN SHM

Acceleration in SHM, defined as the rate of change of velocity, is always directed towards the mean position and varies with displacement.

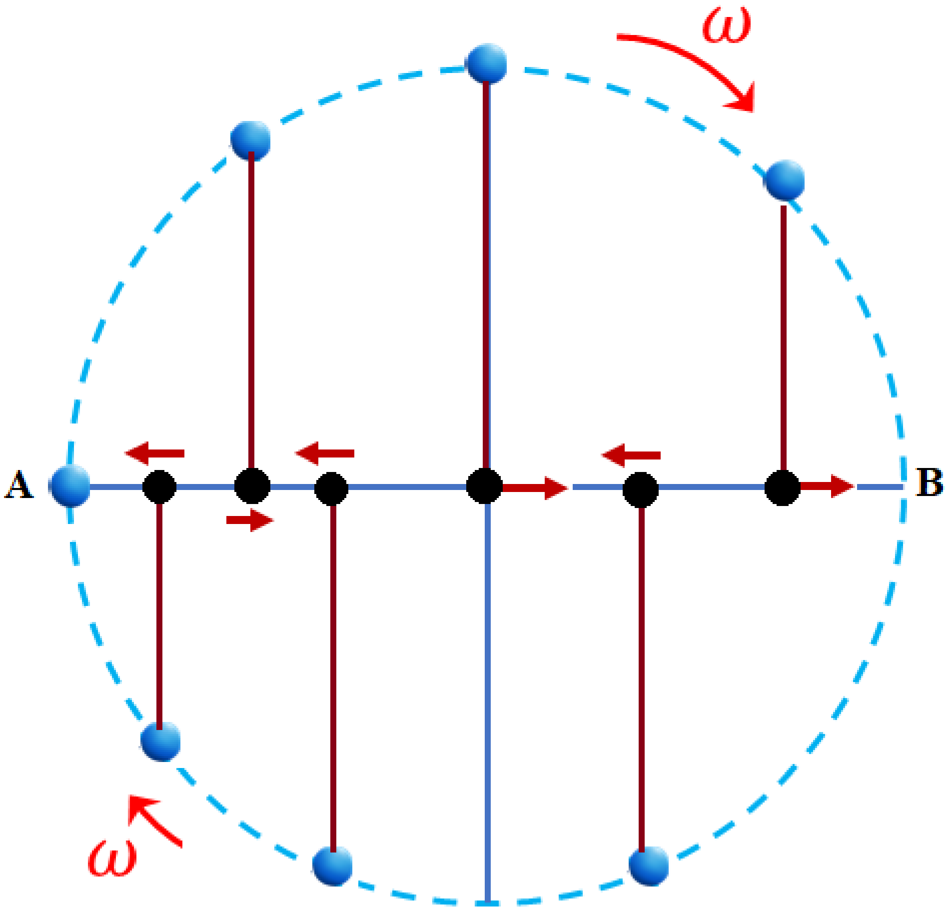
Acceleration-Time Graph

1. Inverted cos Curve: The acceleration graph in SHM typically looks like an inverted sine wave, showing how it changes over time.
2. Maximum Acceleration: Corresponds to the points of maximum displacement.
3. Zero Acceleration: When the object passes through the equilibrium position, acceleration becomes zero



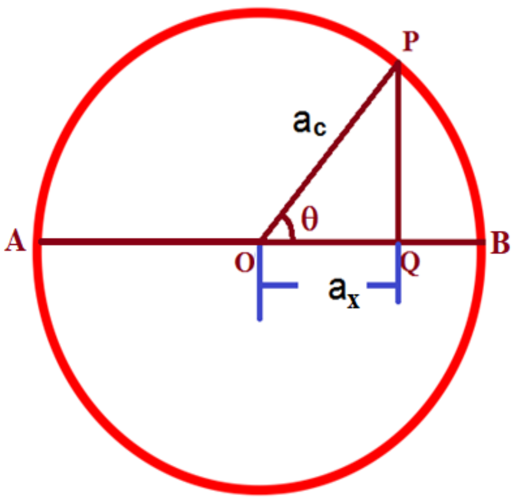
SIMPLE HARMONIC MOTION RELATED TO UNIFORM CIRCULAR MOTION

simple harmonic motion has an interesting relationship to particles rotating in a circle with uniform speed. Consider a mass m rotating is a circle of radius r with a speed v as shown in the figure. The easiest way to view this motion is to shine a light that casts a shadow of the object on a screen, as shown in the figure, while the object itself moves on a circular path, it shadow moves back and forth in a straight line on the screen.



PROJECTION OF UNIFORM CIRCULAR MOTION ALONG A DIAMETER OF THE CIRCLE.

Consider a particle of mass **m** rotating in a circle of radius **r** with a speed of **v,** then projection '*Q*' of the particle moves back and forth along the diameter of the circle.



The centripetal acceleration of particle 'P' a_c into its rectangular components

$$a_x = a_c \cos\theta$$

$$a_x = x_0\,\omega^2 \cos\theta$$

And

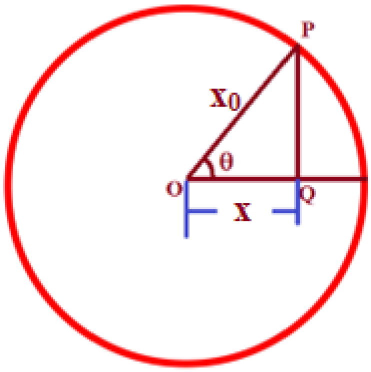
$$a_y = a_c \sin\theta$$

$$a_y = x_0\,\omega^2\,\sin\theta$$

Since a_x is the component along the diameter AOB and always directed towards the equilibrium position '0'. At any instant t the direction of the acceleration vector is opposite to the direction of the displacement vector. Therefore;

$$a_x = -\,x_0\,\omega^2\,\cos\theta \quad \ldots\ldots\ldots\ldots..(i)$$

Consider the right-angle triangle OQP



$$Cos\theta = \frac{x}{x_0}.................(ii)$$

Substituting cosθ expression in equation (i)

$$a_x = -\,x_0\,\omega^2\left(\frac{x}{x_0}\right)$$

$$a_x = -\,\omega^2\,x$$

$$a_x = -\,(constant)\,x$$

$$a_x \propto -\,x$$

This is an equation representing simple harmonic motion, which indicates that the acceleration of 'Q' is directly proportional to its displacement and is directed towards mean position. Hence the motion of projection 'Q' along the diameter of the circle is simple harmonic motion.

TIME PERIOD

DEFINITION

Time taken by a vibrating body to complete one cycle is called its time period.

Formula for projection 'Q'

For projection '*Q*' it is equal to the time taken by particle '*P*' to complete one cycle with angular velocity 'ω'.

or $$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\omega} \qquad \text{———————} \qquad (1)$$

Formula for Spring-Mass system

The acceleration of the object attached to a spring is

$$a = -\ a = -\frac{k}{m}x$$

But $a = -\,\omega^2\,x$

Comparing these two above equation , we get

$$-\omega^2 x = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{1}{\omega} = \sqrt{\frac{m}{k}}$$

Substitute the value of 1/ω in equation (1), we get,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Unit

Time period is measured in 'seconds'.

Dimension

The dimension of time period are [T]

FREQUENCY

DEFINITION

Number of cycles performed by a vibrating body in one second is called its frequency.

Formula for Projection '*Q*'

$$f = \frac{\omega}{2\pi}$$

Formula for Spring-Mass system

We know that the frequency of the system is

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi}\omega$$

Substitute the value of ω in above equation, we get,

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Unit

Frequency is measured in (cycle/sec) or Hz

1 Hz = 1 cycle/second

INSTANTANEOUS DISPLACEMENT

Displacement is the distance from equilibrium position at any instant.

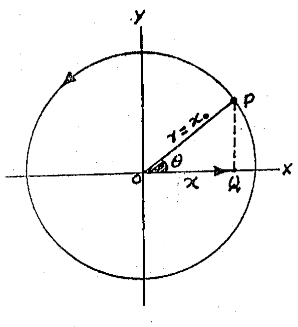
When a particle '*P*' moves in a circle of radius '*r*' its projection '*Q*' performs SHM along the diameter.

Let,

ω= uniform angular speed of '*P*'

t = time internal

x = instantaneous displacement of '*Q*'



In the right angle triangle, ΔOPQ

$$\text{Cos}\,\theta = \frac{OQ}{OP}$$

$$\text{Cos}\,\theta = \frac{x}{r}$$

$$x = r\,\text{Cos}\,\theta$$

$$x = x_0\,\text{Cos}(\omega t + \Phi) \qquad r = x_0 \qquad \text{and} \qquad \theta = \omega t + \Phi$$

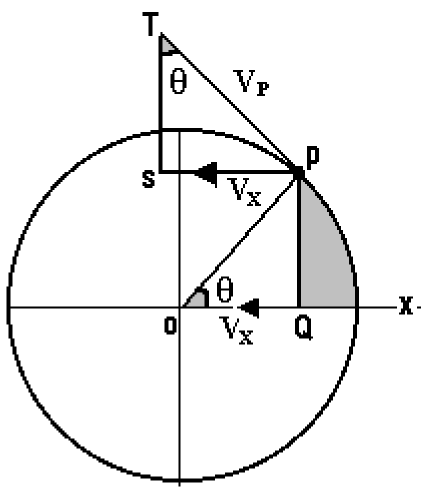
Where,

x_o = maximum displacement from mean position known as **Amplitude**.

Φ = initial phase or phase constant.

INSTANTANEOUS VELOCITY

Let a particle '*P*' rotate in a circle of radius '*r*' with uniform linear speed V_p. The speed of projection ***Q*** is the component of V_p along *x*-axis i.e. V_x.



In ΔPST

$$\text{Sin}\ \theta = \text{Sin}\theta = \frac{\text{Perp}}{\text{Hyp}}$$

$$\therefore \text{Sin}\theta = \frac{V_X}{V_P}$$

or $V_x = V_p\,\text{Sin}\theta$ ————— (i)

We kow that

$V_p = r\,\omega$

$$Sin\theta = \text{Sin}\theta = \sqrt{1 - \text{Cos}^2\theta}$$

Subtituting these values in equation (i), we get

$$V_x = r\omega\sqrt{1-\cos^2\theta}$$

$$V_x = r\omega\sqrt{1-\frac{x^2}{r^2}} \qquad \because \cos\theta = \frac{x}{r}$$

$$V_x = r\omega\sqrt{\frac{r^2-x^2}{r^2}}$$

$$V_x = r\omega\frac{\sqrt{r^2-x^2}}{\sqrt{r^2}}$$

$$V_x = r\omega\frac{\sqrt{r^2-x^2}}{r}$$

$$V_x = \omega\sqrt{r^2-x^2}$$

$$V_x = \omega \quad V_x = \omega\sqrt{x_0^2-x^2} \qquad r = x_0$$

This is an expression for instantaneous speed of projection 'Q'.

FOR SPRING-MASS SYSTEM

We know for spring-mass system that $\omega = \sqrt{\frac{k}{m}}$

$$V_x = \sqrt{\frac{k}{m}}\sqrt{x_0^2-x^2}$$

This is an expression for instantaneous speed of spring-mass system

MAXIMUM VELOCITY

The projection '*Q*' has maximum velocity when it passes through the mean position. When the displacement is zero(x = 0)

Put x=0 in equation for instantaneous velocity.

$$V_{max} = \omega\sqrt{x_0^2-0^2}$$

$$V_{max} = \omega\sqrt{x_0^2}$$

$$V_{max} = \omega\, x_0$$

Similarly for spring-mass system

$$V_{max} = \sqrt{\frac{k}{m}}\ x_0$$

$$V_{max} = \sqrt{\frac{k}{m}}\ x_0$$

MINIMUM VELOCITY

The projection '*Q*' has minimum velocity when its displacement from the equilibrium is a maximum

$$x = x_0$$

Put x = x_0 in equation for instantaneous velocity

$$V_{minimum} = \omega\sqrt{x_0^{\ 2} - x_0^{\ 2}}$$

$$V_{minimum} = 0$$

INSTANTANEOUS KINETIC ENERGY

In an ideal system with no friction or other non-conservative forces, the total energy is conserved. For example, the total energy **E** of a mass on a spring is the sum of its kinetic energy (K.E) and potential energy (P.E). therefore,

E = K.E + P.E ———————— (i)

Since E remain the same throughout the motion, it follow that there is a continual tradeoff between kinetic and potential energy.

The kinetic energy of the mass is

$$K.E = \frac{1}{2}mv^2$$

For a body executing simple harmonic motion, the instantaneous velocity of the mass is

$$V_x = \sqrt{\frac{k}{m}}\ \sqrt{x_0^{\ 2} - x^2}$$

Substituting the value of velocity from the above equation in kinetic energy equation.

$$K.E = \frac{1}{2} m \left[\sqrt{\frac{k}{m}} . \sqrt{x_0^2 - x^2} \right]^2$$

$$K.E = \frac{1}{2} m \left[\left(\sqrt{\frac{k}{m}} . \right)^2 \left(\sqrt{x_0^2 - x^2} \right)^2 \right]$$

$$K.E = \frac{1}{2} m \frac{k}{m} \left(x_0^2 - x^2 \right)$$

$$K.E = \frac{1}{2} k \left(x_0^2 - x^2 \right)$$

MAXIMUM K.E

When a body passes through mean position its K.E is maximum i.e. when $x = 0$ K.E = $K.E_{(max)}$

$$K.E_{max} = \frac{1}{2} k \left(x_0^2 - 0 \right)$$

$$K.E_{max} = \frac{1}{2} k x_0^2$$

MINIMUM K.E

When a body at extreme position, its K.E is minimum i.e. when $x = x_0$

$$K.E_{min} = \frac{1}{2} k \left({x_0}^2 - {x_0}^2 \right)$$

$$K.E_{min} = 0$$

INSTANTANEOUS POTENTIAL ENERGY

We know that P.E = work done

= (Force) (displacement)

In a system executing SHM, force is proportional to displacement i.e.

F = O, when displacement = x = 0

And F = Kx when displacement = x

Now,

$$F_{av} = \frac{O + Kx}{2}$$

$$F_{av} = \frac{1}{2}Kx$$

$$P.E = (F_{av})(\text{displacement})$$

$$P.E = \left(\frac{1}{2}kx\right)(x)$$

$$P.E = \frac{1}{2}kx^2$$

MAXIMUM P.E

Potential energy is maximum when displacement is maximum i.e., $P.E = P.E_{max}$ when $x = x_o$ (at extreme position)

$$P.E_{max} = \frac{1}{2}k\,x_o^2$$

TOTAL ENERGY

At any instant the total energy '*E*' of a body performing SHM is equal to the sum of *K.E* & *P.E*.

$$E = K.E + P.E$$

$$E = \frac{1}{2}k\left(x_o^2 - x^2\right) + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kx_o^2 - \frac{1}{2}kx^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}k\,x_o^2$$

or $E = \text{constant}$

Thus, Total energy of a body executing SHM is always constant.

SIMPLE PENDULUM

DEFINITION

A point mass suspended from a frictionless, rigid support by a light, inextensible string is known as simple pendulum.

EXPLAINATION

Consider a pendulum of length '*L*' and mass of bob 'm'.

Let x be the displacement of bob from mean position. The forces acting on the bob are

T = tension in the string

W = weight of bob vertically downward.

Weight ‘w’ can be resolved in to two components.

$W_x = W \cos\theta$

$W_x = mg \cos\theta$ (along the string)

$W_Y = W \sin\theta$

$W_Y = mg \sin\theta$ (perpendicular to string)

Since the bob does not move along the string.

Therefore, **T = W = W Cos θθ**

Hence the net force W_Y, responsible for the motion of bob, W_Y also called restoring force.

Restoring force = F = - mg Sinθ

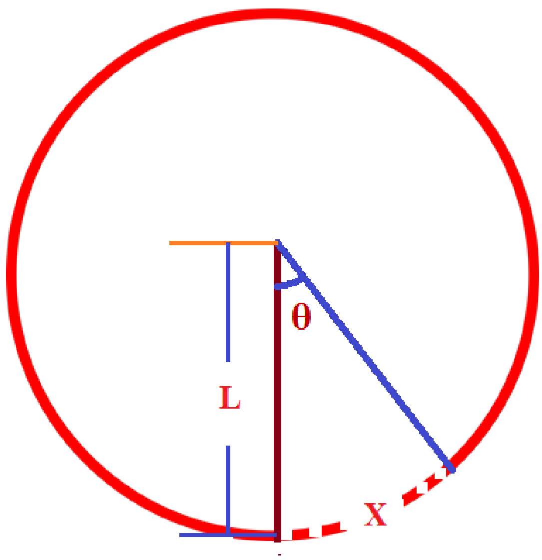
The minus sign indicates that F is directed towards mean position.

For small angle θ (measured in radian), the sinθ is approximately equal to the angle itself, that is,

$$\sin\theta \approx \theta$$

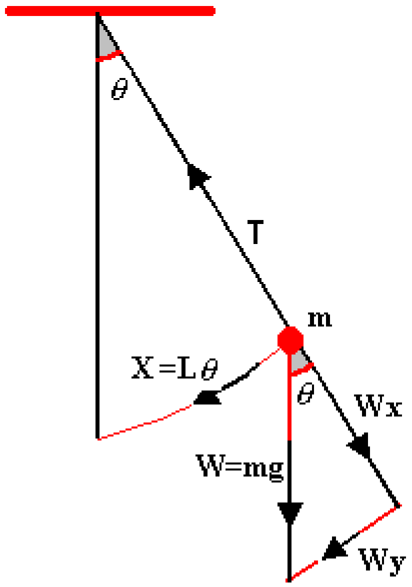
$$F = -mg\,\theta$$

The arc length displacement of the mass from mean position is



$$\because x = L\,\theta$$

equivalently,



$$\theta = \frac{x}{L}$$

$$F = -mg\frac{x}{L}$$

Using Newton's 2nd law of motion

$$ma = -mg\frac{x}{L}$$

$$a = -g\frac{x}{L}$$

For a particular pendulum $\frac{g}{L}$ = constant

$$a = -(\text{constant})\,x$$

$$a \;\alpha\; -x$$

This is an equation representing SHM, which proves that pendulum execute SHM

TIME PERIOD OF PENDULUM

DEFINITION

Time required by a simple pendulum to complete one cycle is called its time period.

Formula

We know that: - $a = -\omega^2 x$

For pendulum: - $a = -\frac{g}{l}x$

Comparing these two equations, we get

$$-\omega^2 x = -\frac{g}{l}x$$

$$\omega^2 = \frac{gl}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$\frac{1}{\omega} = \sqrt{\frac{l}{g}}$$

But time period is given by

$$T = 2\pi.\frac{1}{\omega}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

unit and dimension

Unit of time period is second. It had dimension of time i.e. T.

FREQUENCY OF PENDULUM

DEFINITION

Number of cycles performed by a pendulum in one second is called its frequency.

FORMULA

Mathematically frequency is defined as the reciprocal of time period.

$$f = \frac{1}{2\pi}.\omega$$

Substituting the value of ω in the above equation

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

UNIT And DIMENSION

It is measured in hertz (Hz)

It has dimension of inverse time i.e. T^{-1}

SECOND'S PENDULUM

DEFINITION

A simple pendulum whose time period is two second is called second's pendulum.

Length of Second's Pendulum

Consider $T = 2\pi\sqrt{\frac{l}{g}}$ Put π = 3.142

g = 9.8 m/s^2

$2 = 2\,(3.142)\sqrt{\frac{l}{9.8}}$ T = 2 sec

$\frac{1}{3.142} = \sqrt{\frac{l}{9.8}}$ Squaring both sides, we get

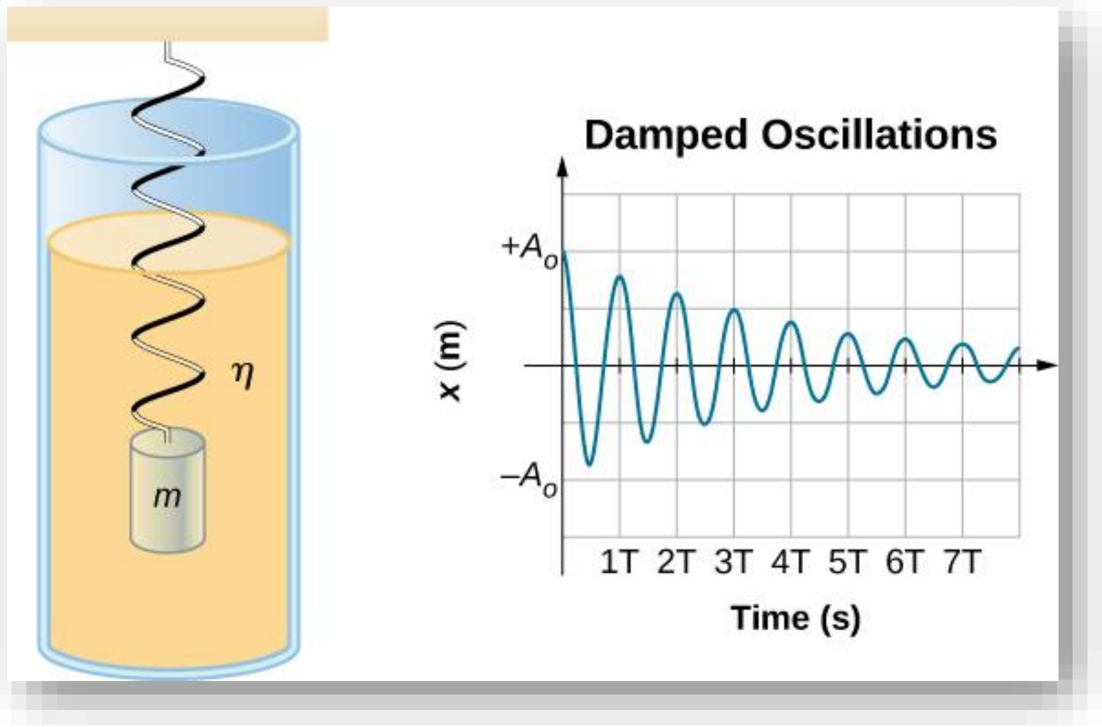
$\frac{1}{(3.142)^2} = \frac{l}{9.8}$ Or $l = \frac{9.8}{(3.142)^2} = -0.99m$

DAMPING

Damping is the process where energy is dissipated from an oscillating system

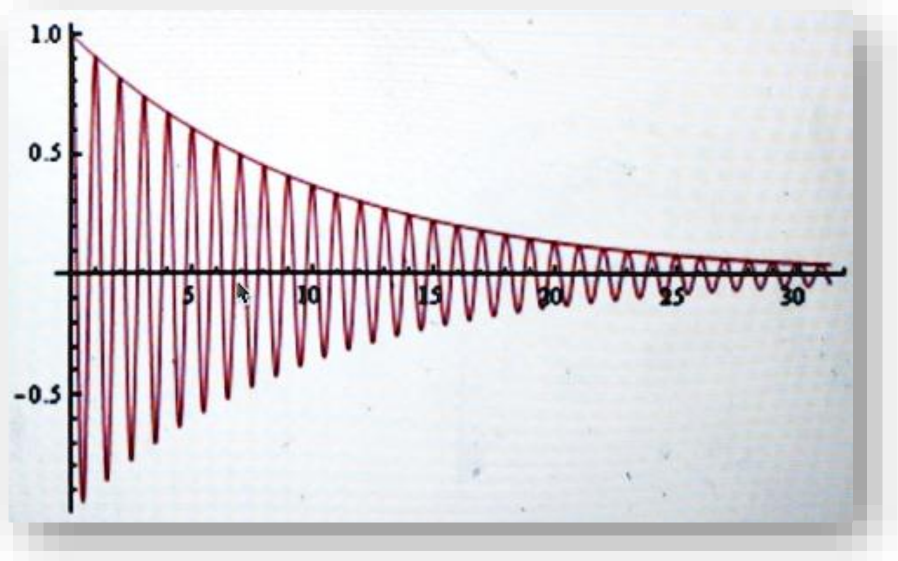
DAMPED HARMONIC MOTION

Damped harmonic oscillation is a type of motion where energy is lost at each cycle. The amplitude of the oscillations decreases over time and eventually, the motion will stop. This happens when the damping force is greater than the restoring force.



DAMPED OSCILLATION

Damped oscillation refers to an oscillatory motion in which the amplitude of the oscillation gradually decreases over time. This decrease in amplitude is due to the dissipation of energy from the system, often due to friction or other resistive forces.



DAMPED HARMONIC OSCILLATOR CASES

In a damped harmonic oscillator, three cases are distinguished based on the damping level:

- **Large Damping:** In systems with very large damping, oscillations do not occur; instead, the system slowly moves towards equilibrium. The displacement of the oscillator moves more slowly towards equilibrium than critically damped systems.
- **Critical Damping:** Critical damping occurs when the damping constant equals the square root of 4 times the mass multiplied by the spring constant. Systems under critical damping return to equilibrium as quickly as possible, like shock absorbers in cars, without overshooting.
- **Small Damping:** In underdamped systems, oscillations occur while the amplitude decreases exponentially until the system comes to rest. These systems oscillate through the equilibrium position and eventually approach zero amplitude.

Damped Oscillation Example

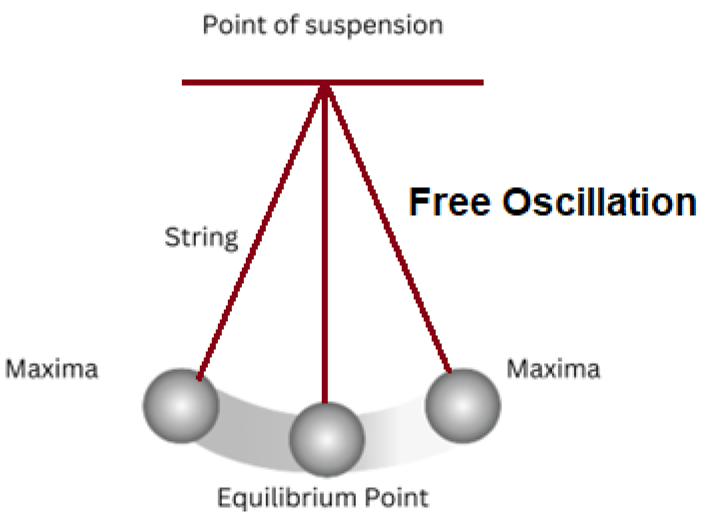
Examples of damped oscillation include:

- **Mass on a Spring:** When a mass is attached to a spring and immersed in a fluid, the system undergoes damped oscillation. The amplitude of the oscillation decreases over time due to the damping force exerted by the fluid.
- **Pendulum in a Viscous Medium:** A pendulum swinging in a medium with significant viscosity experiences damped oscillation. The damping force from the medium causes the pendulum's amplitude to decrease over time, eventually leading to the pendulum coming to rest.

FREE OSCILLATION

If an oscillator is displaced and then released it will begin to vibrate. If no more external forces are applied to the system, it is a *free oscillator*

The oscillation of a simple pendulum is a good example of free oscillation. When a simple pendulum is set into oscillation, it vibrates with its natural frequency. If it is not disturbed by some external force, it will continue to do so with its natural frequency.



FORCED OSCILLATIONS

The phenomenon of setting a body into vibrations with the external periodic force having a frequency different from natural frequency of body is called forced vibrations and the resulting oscillatory system is called a forced oscillator.

EXAMPLE OF FORCED OSCILLATION

when a child is on a swing, they will be pushed at one end after each cycle to keep swinging and prevent air resistance from damping the oscillations. These extra pushes are the forced oscillations, without them, the child will eventually come to a stop.

RESONANCE

When the driving frequency applied to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations increases significantly

If the frequency of external driving force **f** continues to increase and if it becomes an equal or integral multiple of natural frequency $\mathbf{f_o}$ of the system such that

$$f_{external} = f_1$$

$$or \quad f_{external} = 2f_1$$

$$or \quad f_{external} = 3f_1$$

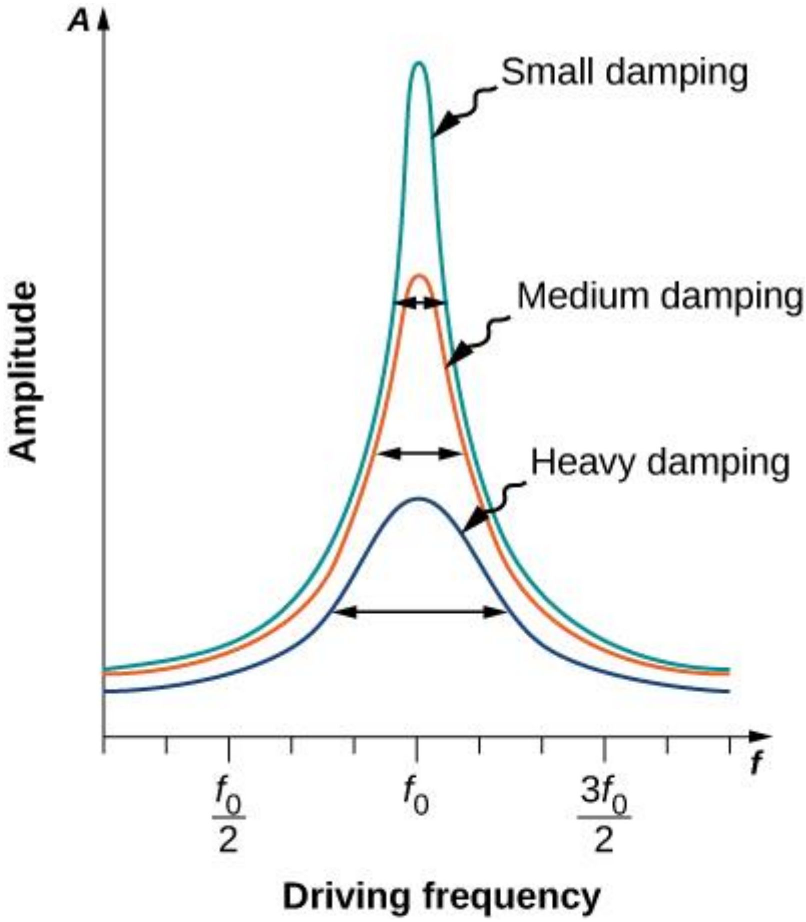
$$or \quad f_{external} = n\,f_1$$

IMPORTANT THINGS FOR RESONANCE

There are three things needed for the incident of Resonance, and they are:

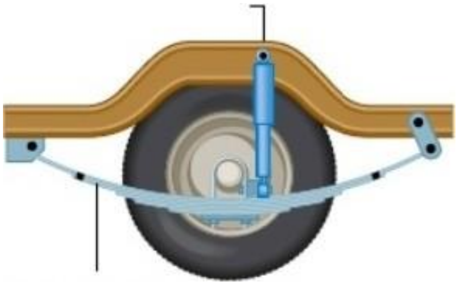
1. An Object or a System that has a natural frequency.
2. Driving Force whose frequency is the same as the natural frequency of a system.
3. The elements which can destroy the energy of the system must be least.

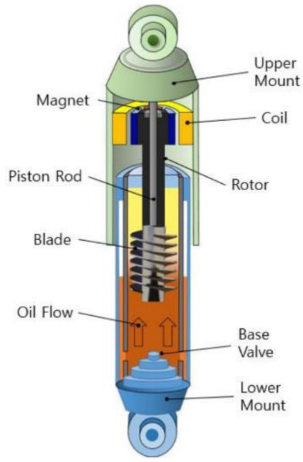
At resonance the driving force is always in the same direction as the object's velocity. Since the driving force is always doing positive work, the energy of the oscillator builds up until the dissipation of energy balances the energy added by the driving force. For an oscillator with little damping, the amplitude becomes large. When the driving force is not at resonance, some negative work is stored in the system. Hence the net work done by the driving force decreases as the driving frequency moves away from the resonance. Therefore, the oscillator's energy and amplitude is smaller than at resonance



PRACTICAL EXAMPLE OF DAMPED OSCILLATIONS

Damping is not always disadvantageous. An example of damped oscillation can be seen in a shock absorber used in vehicles as shown in figure. A shock absorber is a device that is integrated into the suspension system of a car or motorcycle. Its primary purpose is to dampen the oscillations caused by irregularities in the road surface or when the vehicle encounters bumps. In order to compress or expand the shock absorber viscous oil must flow through the holes in the piston. The viscous force dissipates energy regardless of which direction the piston moves. The shock absorber enables the spring to smoothly return to its equilibrium length without oscillating up and down





Frequency response and Sharpness of Resonance (Q-Factor)

In most physics and engineering problems the oscillators are analyzed in the limit of small amplitudes. In mechanics problems, an oscillating spring or other structural element has some nonlinearity in its stress-strain curve as the driving force increases and reaches close to the elastic limit.

An oscillating system does not like that an external force resonates with its natural frequency. If you do this the system responds and sometimes its response is catastrophic, the collapse of Tacoma Narrows Bridge is a textbook example of this fact.

On the contrary, when the damping forces are sufficiently strong to restrict the oscillation's amplitude at resonance, the oscillator behaves linearly. This behavior arises because, at resonance, the energy supplied to the oscillator from an external source precisely matches the energy loss due to work done against the damping forces. Increasing damping diminishes the sharpness of resonance (Fig. thereby reducing its strength.

The sharpness of resonance depends mainly on two factors: amplitude and damping. The Q-factor quantifies the sharpness of resonance. It signifies the reduction of the oscillation's amplitude over time, which corresponds to the decay of energy in an oscillating system. It is approximately defined as the number of free oscillations the oscillator undergoes before its amplitude decays to zero. In the case of light damping, the Q-factor will be large, whereas it will be small for significant damping. Mathematically, the Q-factor is the ratio of energy stored to energy lost per oscillation, and it is a dimensionless quantity.

$$Q = E_{stored}/E_{lost}$$

DO YOU KNOW?

In 1940 **Tacoma Narrows Bridge in Washington USA** was collapsed due to increase in amplitude as heavy wind blowing across the bridge resonated with the natural frequency of oscillation of the bridge. This decreases the damping and with the increasing amplitude enormous amount of energy is stored in it which causes the bridge to collapse.

The collapsed, Tacoma Narrows Bridge.

The newly built Tacoma Narrows bridges opened in 1950 (right) and 2007 (left). These bridges are built with much higher resonant frequencies.