CHAPTER = 1 PHYSICS AND MEASUREMENTS

BOOK NUMERICAL

1 What is the percent uncertainty in the measurement 3.67 ± 0.25 m?

DATA	SOLUTION
$x = 3.67\ m$ $\Delta x = 0.25\ m$ $percentage\ uncertainty = \frac{\Delta x}{x} \times 100$	$percentage\ uncertainty = \frac{\Delta x}{x} \times 100$ $percentage\ uncertainty = \frac{0.25}{3.67} \times 100$ $percentage\ uncertainty = 6.8\ \%$

2 What is the area, and its approximate uncertainty, of a circle with radius ($3.7 \text{x} 10^4$ ± 500) cm

DATA $r = 3.7 \times 10^4\ cm$ $\Delta r = 500\ cm$ $A = ?$ **SOLUTION** $A = \pi r^2$ $A = \pi (3.7 \times 10^4)^2$ $A = 4.3 \times 10^9\ cm^2$	$fraction\ uncertainty\ in\ area$ $\frac{\Delta A}{A} = 2 \left(\frac{\Delta r}{r}\right)$ $\Delta A = 2 \left(\frac{\Delta r}{r}\right) \times A$ $\Delta A = 2 \left(\frac{500}{3.7 \times 10^4}\right) \times 4.3 \times 10^9$ $\Delta A = 0.11 \times 10^9\ cm^2$ **RESULT** $(A \pm \Delta A) = (4.3 \times 10^9 \pm 0.11 \times 10^9)\ cm^2$ $(A \pm \Delta A) = (4.3 \pm 0.11)\ 10^9\ cm^2$

3 An aero plane travels at 850 km/h. How long does it take to travel 1.00 km?

DATA	SOLUTION
$v = 850\ km/h$ $S = 1.00\ km$	$S = v\,t$ $t = \frac{S}{v} = \frac{1.00}{850} = 0.001176\ h$ $t = 0.001176\ h = 0.001176 \times 3600$ $t = 4.23\ sec$

4 **A rectangular holding tank 25.0 m in length and 15.0 m in width is used to store water for short period of time in an industrial plant. If 2980 m^3 water is pumped into the tank. What is the depth of the water?**

DATA	SOLUTION
$L = 25.0\ m$ $W = 15.0\ m$ $V = 2980\ m^3$ $depth\ of\ water, \quad d = ?$	$V = L \times W \times h \qquad (h = d)$ $V = L \times W \times d$ $d = \frac{V}{(L \times W)}$ $d = \frac{2980}{(25.0 \times 15.0)} = \frac{2980}{375}$ $d = 7.946\ m$

5 **Find the volume of rectangular underground water tank has storage facility of 1.9 m by 1.2 m by 0.8 m.**

DATA	SOLUTION
$L = 1.9\ m$ $W = 1.2\ m$ $h = 0.8\ m$ $V = ?$	$V = L \times W \times h$ $V = 1.9 \times 1.2 \times 0.8$ $V = 1.9 \times 1.2 \times 0.8$ $V = 1.824\ m^3$

6 **Two students derive following equations in which x refers to distance traveled, v the speed, the acceleration, and t the time and the subscript (o) means a quantity at time t=0:**

(a) $x = v\,t^2 + 2\,a\,t$

(b) $x = v_0\,t + 2\,a\,t^2$ **, which of these could possibly be correct to dimensional check**

(ii) $x = v\,t^2 + 2\,a\,t$ **SOLUTION** $L.H.S = [x]$ $L.H.S = [\mathrm{L}]$ $R.H.S = [v][t^2] + 2\,[a]\,[t]$ $R.H.S = [L\,T^{-1}][T^2] + 2\,[L\,T^{-2}]\,[T]$ $R.H.S = [L\,T^{-1}\,T^2] + 2\,[L\,T^{-2}\,T]$ $\{2 \text{ has no dimension}\}$ $R.H.S = [L\,T^{-1}\,T^2] + [L\,T^{-2}\,T]$ $R.H.S = [L\,T^{1}] + [L\,T^{-1}]$ we have [L.H.S] ≠ [R.H.S] Hence, the given equation is dimensionally incorrect.	**(ii)** $x = v_0\,t + 2\,a\,t^2$ **SOLUTION** $L.H.S = [x]$ $L.H.S = [\mathrm{L}]$ $R.H.S = [v][t] + 2\,[a]\,[t^2]$ $R.H.S = [L\,T^{-1}][T] + 2\,[L\,T^{-2}]\,[T^2]$ $R.H.S = [L\,T^{-1}\,T] + 2\,[L\,T^{-2}\,T^2]$ $\{2 \text{ has no dimension}\}$ $R.H.S = \left[L\,\left(\frac{1}{T}\right)T\right] + \left[L\,\left(\frac{1}{T^2}\right)T^2\right]$ $R.H.S = [L] + [L]$ $R.H.S = [L]$ we have [L.H.S] = [R.H.S] Hence, the given equation is dimensionally correct

7 **One hectare is defined as 10^4 m^2 . one acre is 4 x 10^4 ft^2. How many acres are in one hectare?**

DATA $1\ hectare = 10^4\ m^2$ $1\ acre = 4 \times 10^4\ ft^2$ $1\ m = 3.28\ ft$ $1\ ft = \left(\frac{1}{3.28}\right) m$ $how\ many\ acres\ in\ 1\ hectare = ?$	**SOLUTION** $1\ acre = 4 \times 10^4\ ft^2$ **We know that** $1\ ft = \left(\frac{1}{3.28}\right) m$ $1\ acre = 4 \times 10^4 \left(\frac{1}{3.28}\right)^2 m^2$ $1\ acre = 4 \left(\frac{1}{3.28}\right)^2 hectare$ $1\ hectare = \frac{(3.28)^2}{4}\ acre$ $1\ hectare = 2.69\ acre$

8 **A watch factory claims that its watches gain or lose not more than 10 seconds in a year. How accurate is this watch, express as percentage?**

DATA	SOLUTION
$\Delta t = \pm 10\ s$ $t = 1\ year = 3.15 \times 10^7\ s$ $\frac{\Delta t}{t}\ \% = ?$	$\frac{\Delta t}{t} = \frac{10}{3.15 \times 10^7} \times 100$ $\frac{\Delta t}{t} = 3.1746 \times 10^{-5}\ \%$

9 **The diameter of Moonis 3480 km. What is the volume of the Moon? How many Moons would be needed to create a volume equal to that of Earth?**
$(r_e = 6380\ km)$

DATA	Number of Moons (n)
$D_m = 3480\ km$ $r_m = \frac{D_m}{2} = \frac{3480}{2} = 1740\ km$ $r_m = 1740 \times 10^3\ m$ $r_e = 6380\ km = 6380 \times 10^3\ m$ **SOLUTION** **Volume of Moon** $V_m = \frac{4}{3}\pi r_m^3$ $V_m = \frac{4}{3}\pi (1740 \times 10^3)^3$ $V_m = 2.22 \times 10^{10}\ m^3$	$V_e = n\ V_m$ $n = \frac{V_e}{V_m}$ $n = \frac{\frac{4}{3}\pi r_e^3}{\frac{4}{3}\pi r_m^3}$ $n = \frac{r_e^3}{r_m^3} = \left(\frac{6380 \times 10^3}{1740 \times 10^3}\right)^3$ $n = (3.666)^3$ $n = 49.29$

WORKED EXAMPLES

Worked example 1.1

Show that the equation for impulse $Ft = mV_f - mV_i$ is dimensionally correct.

Solution:

$$Ft = mV_f - mV_i$$

By using dimensional formula, we can write.

$$[MLT^{-2}]\,[T] = [M]\,[LT^{-1}] - [M]\,[LT^{-1}]$$

$$[MLT^{-2+1}] = [MLT^{-1}] - [MLT^{-1}]$$

$$[MLT^{-1}] = [MLT^{-1}]$$

Worked Example 1.2

If the radius of sphere is measured 9 cm with an error of 0.02 cm. Find the approximate error in calculating its volume.

$$R = 9\text{cm}$$

$$\Delta R = 0.02\text{cm}$$

The volume of sphere is

$$V = \frac{4}{3}\pi R^3$$

$$V = \frac{4}{3}(3.14)(9)^3$$

$$\boxed{V = 3052.08\ \text{cm}^3}$$

Error in volume is

$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$$= 3\left(\frac{0.02}{9}\right)$$

$$\frac{\Delta V}{V} = 0.00666$$

$$\frac{\Delta V}{V} = 0.00666 \times 3052.08$$

$$\Delta V = 20.34\ \text{cm}^3$$

Worked Example: 1.3

If the radius of a sphere is measured as 7.5 cm with error 0.03 cm, find the approximate error in calculating its volume.

$$R = 7.5\text{cm}$$

$$\Delta R = ?$$

Volume of sphere is

$$V = \frac{4}{3}\pi R^3$$

$$V = \frac{4}{3}(3.14)(7.5)^3$$

$$V = 1766.25\text{cm}^3$$

Error in volume is

$$\frac{\Delta V}{V} = 3\frac{\Delta R}{R}$$

$$\frac{\Delta V}{V} = 3\times\frac{0.03}{7.5}$$

$$\frac{\Delta V}{V} = 0.012\times V$$

$$\frac{\Delta V}{V} = 0.012\times 17666.25$$

$$\Delta V = 21.2\text{cm}^3$$

Worked Example: 1.4

Consider the length of cube is 5.75 ± 0.3 cm and you want to find absolute uncertainty in volume.

Solution

Volume of the cube is

$$V=L^3$$

$$V=(5.75)^3$$

$$V=190cm^3$$

Uncertainty in volume is

$$\frac{\Delta V}{V}=3\frac{\Delta L}{L}$$

$$\frac{\Delta V}{V}=3\times\frac{0.3}{5.75}$$

$$\frac{\Delta V}{V}=0.1565$$

Or

$$\frac{\Delta V}{V}=15.65\%$$

Volume of the cube with uncertainty is $190 \pm 15.65\%$ cm^3

Self Assessment Question:

A girl needs to calculate the volume of her pool, so that she knows how much water she will need to fill it. She measures the length, width, and height as under.

Length $= 5.56 \pm 0.14$ m

Width $= 3.12 \pm 0.08$ m

Height $= 2.94 \pm 0.11$ m

What will the pool's volume with uncertainty?

Solution:

Volume of the pool is

$$V = l \times w \times h$$

$$V = 5.56 \times 3.12 \times 2.94$$

$$\boxed{V = 51.0\ m^3}$$

Fractional uncertainty in volume is

$$\frac{\Delta V}{V} = \frac{\Delta \ell}{\ell} + \frac{\Delta \omega}{\omega} + \frac{\Delta h}{h}$$

$$\frac{\Delta V}{V} = \frac{0.14}{5.56} + \frac{0.08}{3.12} + \frac{0.11}{2.94}$$

$$\frac{\Delta V}{V} = 0.088$$

$$\frac{\Delta V}{V} = 0.088 \times 100$$

$$= 8.8\%$$

The volume of the pool is $51 \pm 8.8\%\ m^3$

SHORT REASONING QUESTIONS

Q1. Give an example of:

(i) a physical quantity that has a unit but no dimensions.

(ii) a physical quantity that has neither unit nor dimensions.

(iii) a constant that has a unit.

(iv) a constant that has no unit.

(i) Angular displacement (θ) has a unit (radian) but it has no dimension.

(ii) The strain is a physical quantity that has neither unit nor dimension.

(iii) The gravitational constant (G) has a unit (Nm^2/kg^2)

(iv) Avogadro's number is a constant and has no unit.

Q2. When rounding the product or quotient of two measurements, is it necessary to consider significant digits?

Yes, when rounding the product or quotient of two measurements. The least number of significant figures in any number of the problem determines the number of significant figures in the answer.

For example, if you multiply 2.34 m by 1.23 m, the product is 2.8882 m. However, since 2.34 m has only 3 significant digits, the product should also be rounded to 3 significant digits, giving a final answer of 2.89 m.

Q3. Drive the equation for the period of oscillations of a mass suspended on a vertical spring by dimensional analysis. i.e., $T = 2\pi\sqrt{\frac{m}{k}}$

The period of the mass-spring system depends upon the mass of the body connected by spring and spring constant. $T \propto m^a k^b$

$$T = c\, m^a k^b \rightarrow \text{(i)}$$

$$[T] = [M]^a[MT^{-2}]^b$$

$$M^0[T] = [M]^{a+b}[T]^{-2b}$$

Equating the co-efficient, we get

$$a + b = 0 \text{ and } -2b = 1$$

$$b = -\frac{1}{2}, \quad a = -b \Rightarrow a = \frac{1}{2}$$

Now equation (i) becomes,

$$T = c\, m^{\frac{1}{2}} k^{-\frac{1}{2}}$$

$$T = c\,\frac{m^{\frac{1}{2}}}{k^{\frac{1}{2}}}$$

$$T = c\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}} (c = 2\pi)$$

Q4. Find the dimensions of the following.

(a) Work: $[F][S] = [MLT^{-2}][L] = [ML^2T^{-2}]$

(b) Energy: $[m][c^2] = [M][L^{2T^{-2}}] = [ML^2T^{-2}]$

(c) Power: $\frac{[W]}{[T]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$

(d) Momentum: $[m][v] = [M][LT^{-1}] = [ML^2T^{-1}]$

Q5. You measure the radius of a wheel to be 4.16 cm. If you multiply by 2 to get the diameter, should the result as 8 cm or 8.32 cm? Justify your answer.

Diameter $= 2 \times 4.16$

Diameter $= 8.32\text{cm} = 8\text{cm}$

The correct answer is 8 cm because it contains only one significant figure.

Q6. If $y = a + bt + ct^2$ where y is in meters and t in seconds, what is the unit of c?

$$y = a + bt + ct^2$$

$$\text{Unit of y} = \text{Unit of c} \times \text{Unit of } t^2$$

$$\text{Unit of c} = \frac{\text{Unit of y}}{\text{Unit of } t^2}$$

$$\text{Unit of c} = \frac{m}{s^2} = ms^{-2}$$

Q7. Differentiate between accuracy and precision.

Accuracy refers to how close a measurement is to the true or accepted value. Precision refers to how close measurements of the same item are to each other.

Precision is independent of accuracy. A measurement can be accurate but not necessarily precise.

Accuracy can be determined by a single measurement but precision needs several measurements to be determined.

Q8. Define dependent and independent variables.

In statistics and mathematical modeling, dependent and independent variables are terms used to describe the relationship between two variables.

An independent variable is a variable that is manipulated in an experiment or study to observe the effect it has on a dependent variable.

A dependent variable is a variable that is being measured or observed in an experiment or study and is expected to change as a result of the manipulation of the independent variable.

Q9. Differentiate systematic error and random error.

Systematic errors:

- These errors arise due to faulty instruments, the surrounding environment, and inaccurate observation.
- These errors occur only in one direction.
- These errors can be corrected.

Random errors:

- These errors occur randomly due to unknown sources.
- These errors occur in any direction.
- These errors can be minimized by taking several observations.

Q10. Enlist the limitations of dimensional analysis.

Some limitations of dimensionality are:

- It doesn't give information about the dimensional constant.
- The formula containing trigonometric function, exponential functions, logarithmic function, etc. cannot be derived
- It gives no information about whether a physical quantity is a scalar or vector.

Q11. Describe the least count of the vernier caliper and micrometer screw gauge.

Vernier Calipers:

- Least Count: The least count of a Vernier caliper is the smallest measurement that can be accurately read on the caliper's scale.
- Formula: It is typically calculated as the difference between one main scale division and one Vernier scale division. It's expressed as:
  Least Count (LC)= Value of one main scale division - Value of one Vernier scale division

Micrometer Screw Gauge:

- Least Count: The least count of a micrometer screw gauge is the smallest measurement that can be accurately read on the gauge.
- Formula: It is typically calculated as the pitch of the screw divided by the number of divisions on the circular scale. It's expressed as:
  Least Count (LC) = Pitch of the screw / Number of divisions on the circular scale

  In both cases, a smaller least count indicates a higher precision in measurements, allowing for more accurate readings. These instruments are invaluable in fields where precision measurements are crucial, such as engineering, manufacturing, and quality control.

Q12. Describe extrapolation methods.

Extrapolation is a technique used to predict values beyond the range of the known data it is often used in data science to make predictions about future trends.

There are two common types of extrapolations:

- Linear extrapolation: This method uses a straight line to predict future values. It is best suited for data sets that exhibit a linear trend.
- Polynomial extrapolation: This method uses a polynomial equation to predict future values. It is more complex than linear extrapolation, but it can be more accurate for data sets that exhibit a non-linear trend.