D.C CIRCUITS

CHAPTER = 10 **BOOK NUMERICAL**

1 The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.5 Ω, what is the maximum current that can be drawn from the battery?

Data:	SOLUTION:
$V = 12\ \text{V}$ $r = 0.5\ \Omega$ $I = ?$	$I = \frac{V}{r}$ $I = \frac{12}{0.5}$ $I = 24\ A$

2 A negligibly small current is passed through a wire of length 12m and uniform cross- section $4.0 \times 10^{-8}\ m^2$, and its resistance is measured to be 6.0 Ω. What is the resistivity of the material at the temperature of the experiment?

Data:	SOLUTION:
$L = 12\ \text{m}$ $A = 4.0 \times 10^{-8} \text{m}^2$ $R = 6.0\ \Omega$ $\rho = ?$	$R = \frac{\rho\ L}{A}$ $\rho = \frac{R\ A}{L}$ $\rho = \frac{(6.0)\ (4.0 \times 10^{-8})}{12}$ $\rho = 2 \times 10^{-8}\ \Omega\, m$

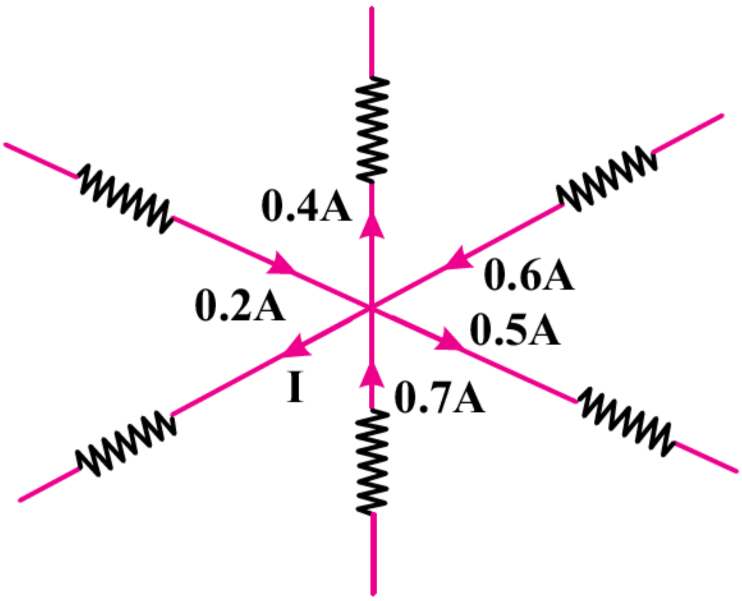
3 In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 40.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 74.0 cm, what is the emf of the second cell?

Data:	SOLUTION:
$(emf)_1 = 1.20\ V$ $L_1 = 40.0\ \text{cm}$ $L_2 = 74.0\ \text{cm}$ $(emf)_2 = ?$	$\frac{(emf)_1}{(emf)_2} = \frac{L_1}{L_2}$ $\frac{1.20}{(emf)_2} = \frac{40.0}{74.0}$ $(emf)_2 = 2.22\ V$

4 (a) Three resistors 1 Ω, 2 Ω, and 3 Ω are combined in series. What is the total resistance of the combination?
(b) If the combination is connected to a battery of emf 24 V and negligible internal resistance, obtain the potential drop across each resistor.

Data: $R_1 = 1\ \Omega$ $R_2 = 2\ \Omega$ $R_3 = 3\ \Omega$ $R = ?$ $V = 24\ \text{V}$ **SOLUTION:** $R = R_1 + R_2 + R_3$ $R = 1 + 2 + 3 = 6\ \Omega$ Total current $I = \frac{V}{R}$	$I = \frac{24}{6} = 4\ A$ Potential across R_1 $V_1 = I\,R_1 = (4)(1) = 4\ V$ Potential across R_2 $V_2 = I\,R_2 = (4)(2) = 8\ V$ Potential across R_3 $V_3 = I\,R_3 = (4)(3) = 12\ V$

5 From the given circuit find the value of I.



$$I + 0.5 + 0.4 = 0.2 + 0.6 + 0.7$$
$$I + 0.9 = 0.15$$
$$I = 0.15 - 0.9$$
$$I = 0.6\ A$$

6 In a meter bridge with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 5:3. Find the value of the other resistance.

Data:

$$R_1 = 15\,\Omega$$

$$\frac{L_1}{L_2} = 5:3$$

$$\frac{L_2}{L_1} = \frac{5}{3}$$

$$R_2 = ?$$

SOLUTION:

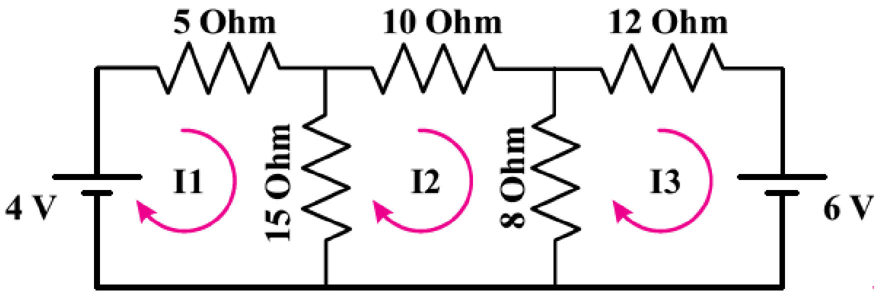
$$\frac{R_2}{R_1} = \frac{L_2}{L_1}$$

$$\frac{R_2}{15} = \frac{5}{3}$$

$$R_2 = 15 \times \frac{5}{3}$$

$$R_2 = 25\,\Omega$$

7 By using KVL, find Current flowing through 10 Ω resistance



SOLUTION:

For loop 1

$$4 - 5\,I_1 - 15\,(I_1 - I_2) = 0$$

$$4 - 5\,I_1 - 15\,I_1 + 15\,I_2 = 0$$

$$4 - 20\,I_1 + 15\,I_2 = 0$$

$$20\,I_1 - 15\,I_2 = 4$$

$$20\,I_1 = 15\,I_2 + 4 \;\ldots\ldots\ldots.(i)$$

$$I_1 = \frac{15\,I_2 + 4}{20} \;\ldots\ldots\ldots.(i)$$

For loop 2

$$-15\,(I_2 - I_1) - 10\,I_2 - 8(I_2 - I_3) = 0$$

$$-15\,I_2 + 15\,I_1 - 10\,I_2 - 8I_2 + 8\,I_3 = 0$$

$$15\,I_1 - 33\,I_2 + 8\,I_3 = 0 \;\ldots\ldots(ii)$$

For loop 3

$$-8(I_3 - I_2) - 12\,I_3 - 6 = 0$$

$$-8\,I_2 - 20\,I_3 = 6$$

$$20\,I_3 = 8\,I_2 - 6$$

$$I_3 = \frac{8\,I_2 - 6}{20} \;\ldots\ldots.(iii)$$

Substituting the expression I_1 and I_3 in equation(ii)

$$15\left(\frac{15\,I_2 + 4}{20}\right) - 33\,I_2 + 8\left(\frac{8\,I_2 - 6}{20}\right) = 0$$

$$\frac{225\,I_2 + 60 - 660\,I_2 + 64\,I_2 - 48}{20} = 0$$

$$225\,I_2 + 60 - 660\,I_2 + 64\,I_2 - 48 = 0$$

$$-371\,I_2 + 12 = 0$$

$$I_2 = \frac{12}{371}$$

$$I_2 = 0,0323\,A$$

WORKED EXAMPLE

1 Calculate the resistance of 100 meter rolls of 2.5 mm^2 copper wire if the resistivity of copper at 20°C is 1.72×10^{-8} Ω m.

Data:

$L = 100 \text{ m}$

$A = 2.5 \text{ x mm}^2$

$A = 2.5 \text{ x } 10^{-6} \text{m}^2$

$\rho = 1.72 \text{ x } 10^{-8} \, \Omega \text{ m}$

$R = ?$

SOLUTION:

$$R = \frac{\rho \;\; L}{A}$$

$$R = \frac{(1.72 \text{ x } 10^{-8}) \;\; (2.5 \text{ x } 10^{-6})}{12}$$

$$R = 0.688 \, \Omega$$

2 A 20-meter length of cable has a cross-sectional area of 1mm^2 and a resistance of 5 Ω . Calculate the conductivity of the cable.

Data:

$L = 200 \text{ m}$

$A = 1 \text{ mm}^2 = 1 \times 10^{-6} \; m$

$R = 5 \, \Omega$

$\rho = ?$

SOLUTION:

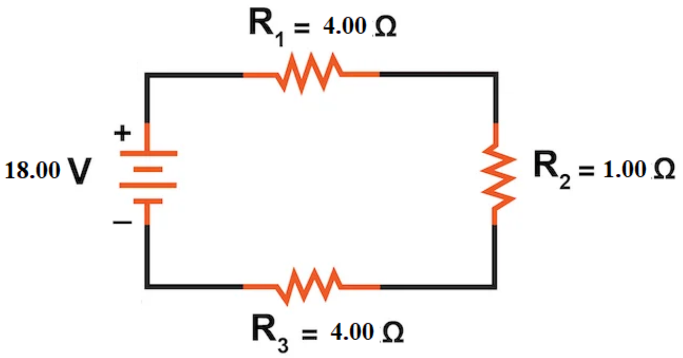
Conductivity is given by

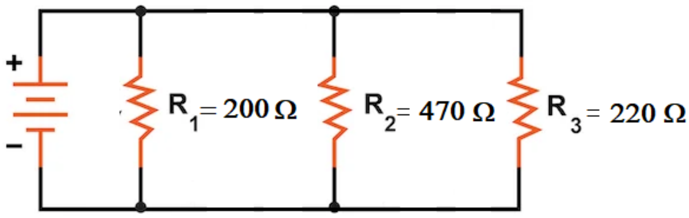
$$\sigma = \frac{L}{R \;\; A}$$

$$\sigma = \frac{200}{(5) \;\; (1 \text{ x } 10^{-6})}$$

$$\sigma = 4 \times 10^{7} \; S/m$$

3 Determine the equivalent resistance for each of the two circuits shown below





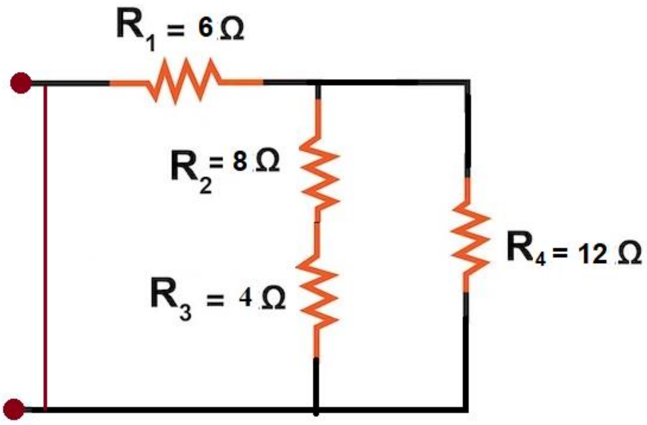
Three resistors are in series

$$R = R_1 + R_2 + R_3$$
$$R = 4 + 1 + 4$$
$$R = 9\,\Omega$$

Three resistors are in series

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R} = \frac{1}{200} + \frac{1}{470} + \frac{1}{220}$$
$$\frac{1}{R} = 0.005 + 0.00212 + 0.00454$$
$$\frac{1}{R} = 0.1166$$
$$R = 85.76\,\Omega$$

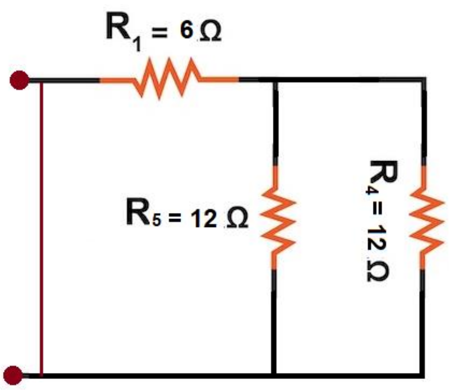
4 Find the equivalent resistance of the given circuit.



Two resistors R_2 *and* R_3 are in series

$$R_5 = R_2 + R_3$$
$$R_5 = 8 + 4 = 12\,\Omega$$

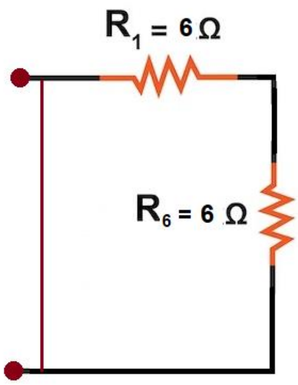
New circuit is



Two resistors R_4 *and* R_5 are in parallel

$$\frac{1}{R_6} = \frac{1}{R_4} + \frac{1}{R_5}$$
$$\frac{1}{R_6} = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$
$$R_6 = 6\,\Omega$$

New circuit is



Two resistors R_1 *and* R_6 are in series

$$R_e = R_1 + R_6$$
$$R_5 = 6 + 6$$
$$R_5 = 12\,\Omega$$

5 **A silver wire 2-m long is to have a resistance of 0.5 Ω .What should its diameter be ? (ρ = 1.52 × 10^{-8} Ω . m)**

Data:	
$L = 2\text{ m}$ $R = 0.5\ \Omega$ $D = ?$ $\rho = 1.52 \text{ x } 10^{-8}\ \Omega\text{ m}$ **SOLUTION:** $R = \frac{\rho\ L}{A}$ $R = \frac{\rho\ L}{\pi\ r^2}$ $r^2 = \frac{\rho\ L}{\pi\ R}$	$r^2 = \frac{\rho\ L}{\pi\ R}$ $r^2 = \frac{(1.52 \text{ x } 10^{-8})\ (2)}{3.142\ \times 0.5}$ $r^2 = 1.935 \times 10^{-8}$ $r = \sqrt{1.935 \times 10^{-8}}$ $r = 1.391 \times 10^{-4}\ m$ $D = 2 \times 1.391 \times 10^{-4}\ m$ $D = 2.782 \times 10^{-4}\ m$

6 **An electric drill rated at 400 W is connected to a 240 V power line. How much current does it draw ?**

Data:	SOLUTION:
$P = 400\text{ W}$ $V = 240\text{ V}$ $I = ?$	$P = V\,I$ $\text{I} = \frac{\text{P}}{\text{V}} = \frac{400}{240}$ $\text{I} = 1.66\text{ A}$

7 **A source of what potential difference is needed to charge a battery of 24V e.m.f. and internal resistance of 0.1 Ω at a rate of 70 A?**

Data:	SOLUTION:
$E = 24\text{ V}$ $r = 0.1\ \Omega$ $I = 70\ A$	$V = E + I\,R$ $V = 24 + (80 \times 0.1)$ $V = 24 + 8$ $V = 32\ V$

8 **A battery of 20 V is connected to a 10 Ω load and a current of 1.8 A flows. Find the internal resistance of the battery and its terminal voltage.**

Data:	**SOLUTION:**
$E = 20\text{ V}$ $R = 10\,\Omega$ $I = 1.8\,A$ $r = ?$	$E = V - I\,R$ $E = (I\ R) - I\,r$ $20 = (1.8 \times 10) - (1.8 \times r)$ $20 = 18 - 1.8\,r$ $20 - 18 = 1.8\,r$ $2 = 1.8\,r$ $\frac{2}{1.8} = r$ $1.11\,\Omega = r$ Terminal p.d $V = I\ R = 1.8 \times 10 = 18\,V$

9 **Find the resistance at 50°C of a copper wire 2 mm in diameter and 3m long ($\rho = 1.60 \times 10^{-8}\,\Omega$. m, $\alpha = 0.0039$ °C^{-1})**

Data:	
$L = 3\text{ m}$ $R_t = ?$ $D = 2\text{mm}$ $r = \frac{D}{2} = \frac{2}{2} = 1\,mm = 1 \times 10^{-3} m$ $\rho = 1.6 \text{ x } 10^{-8}\,\Omega\text{ m}$ $\alpha = 0.0039$ °C-1 **SOLUTION:** $R_0 = \frac{\rho\ L}{A}$ $R_0 = \frac{\rho\ L}{\pi\ r^2}$	$R_0 = \frac{(1.6 \text{ x } 10^{-8})\ (3)}{(3.142)\ (1 \times 10^{-3})^2}$ $R_0 = 1.52 \times 10^{-2}\,\Omega.$ $R_t = R_0\ [1 + \alpha\,\Delta T]$ $R_t = 1.52 \times 10^{-2}\ [1 + (0.0039)\,(50 - 0)]$ $R_t = 1.52 \times 10^{-2}\ [1 + 0.195]$ $R_t = 0.018 = 1.8 \times 10^{-2}\,\Omega.$

10 A given battery has a l 2V emf and an internal resistance of 0.1.Ω Calculate its terminal voltage when connected to a I0 Ω load. If the internal resistance grows to 0.5.Ω, find the current and terminal voltage

Data: $E = 12\ V$ $r = 0.1\ \Omega$ $R = 10\ \Omega$ $I = ?$ $V = ?$ **SOLUTION:** Case 1: Internal Resistance of 0.1Ω $E = V + I\,r$ $E = IR + I\,r$ $E = (R + r)I$ $12 = (10 + 0.1)I$ $\frac{12}{10.1} = I$ $1.188\ A = I$	the terminal voltage $V_t = E - I\,r$ $V_t = 12 - (1.188 \times 0.1)$ $V_t = 11.8812\ V$ Case 1: Internal Resistance of 0.5 Ω $E = V + I\,r$ $E = IR + I\,r$ $E = (R + r)I$ $12 = (10 + 0.5)I$ $\frac{12}{10.5} = I$ $1.1428\ A = I$ the terminal voltage $V_t = E - I\,r$ $V_t = 12 - (1.1428 \times 0.5)$ $V_t = 11.4286\ V$

11 Calculate the resistance of a tungsten wire 20m long that has a radius of 2mm. If the two ends of the wire are connected to a source of 24V, what is the value of current?

Data: $L = 20\ m$ $D = 2mm$ $r = \frac{D}{2} = \frac{2}{2} = 1\ mm = 1 \times 10^{-3} m$ $\rho = 5.6 \text{ x } 10^{-8}\ \Omega\ m$ $V = 24\ V$ $I = ?$	**SOLUTION:** $R = \frac{\rho\ L}{A}$ $R = \frac{\rho\ L}{\pi\ r^2}$ $R = \frac{5.6 \text{ x } 10^{-8}\ (20)}{3.142\ (1 \times 10^{-3})^2} = 8.75 \times 10^{-2}\ \Omega$ $I = \frac{V}{R} = \frac{24}{8.75 \times 10^{-2}.} = 274.28\ A$