CHAPTER = 11 OSCILLATIONS

BOOK NUMERICAL

1 The period of oscillation of an object in an ideal spring and mass system is 0.50 s and the amplitude is 5.0 cm. what is the speed at the equilibrium point? and the acceleration at the point of maximum extension of the spring.

<u>DATA</u> T = 0.50 s $x_0 = 5.0\ cm = 0.05\ m$ $v = ?$ a = ? SOLUTIONS $\omega = \frac{2\pi}{T}$ $\omega = \frac{2\ (3.142)}{0.50} = 12.56\ s^{-1}$	$v = x_0\ \omega$ $v = 0.05 \times 12.56$ $v = 0.05 \times 12.56$ $v = 6.28\ m/s$ $a = x_0\ \omega^2$ $a = (0.05)\ (12.56)^2$ $a = 7.88\ m/s^2$

2 A sewing machine needle moves with a rapid vibratory motion, like SHM, as it sews a seam. Suppose the needle moves 8.4 mm from its highest to its lowest position and it makes 24 stitches in 9.0s. What is the maximum needle speed?

<u>DATA</u> $x = 8.4\ mm = 8.4 \times 10^{-3}\ m$ $x_0 = \frac{x}{2} = \frac{8.4 \times 10^{-3}}{2}$ $x_0 = 4.2 \times 10^{-3}\ m$ N = 24 stiches $t = 9.0$ $v = ?$ SOLUTIONS $T = \frac{time\ taken}{number\ of\ vibration}$ $T = \frac{9.0}{24} = 0.375\ s$	$\omega = \frac{2\pi}{T}$ $\omega = \frac{2\ (3.142)}{0.375}$ $\omega = 16.75\ s^{-1}$ $v = x_0\ \omega$ $v = 4.2 \times 10^{-3} \times 16.75$ $v = 0.07\ m/s$

3 An ideal spring with a spring constant of 15 N/m is suspended vertically. A body of mass 0.60 kg is attached to the upstretched spring and released.

(a) What is the extension of the spring when the speed is a maximum?

(b) What is the maximum speed?

<u>DATA</u>

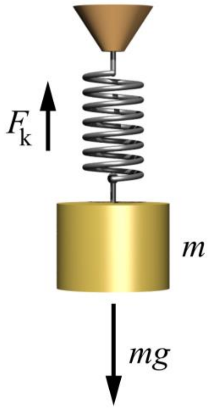
k = 15 N/m

$m = 0.60\ kg$

x_0 =?

v =?

SOLUTIONS



$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{15}{0.6}}$$

$$\omega = 5\ s^{-1}$$

$$spring\ force = weight$$

$$k\, x_0 = m\, g$$

$$x_0 = \frac{m\, g}{k}$$

$$x_0 = \frac{0.60 \times 9.8}{15}$$

$$x_0 = 0.392\ s^{-1}$$

$$v = x_0\, \omega$$

$$v = 0.392 \times 5$$

$$v = 1.96\ m/s$$

4 A body is suspended vertically from an ideal spring of spring constant 2.5 *N/m*. the spring is initially in its relaxed position. The body is then released and oscillates about its equilibrium position. The motion is described by Y = (4.0cm) sin[(0.70rad/s) t]. What is the maximum kinetic energy of the body?

<u>DATA</u>

k = 2.5 N/m

v =?

SOLUTIONS

Given equation

$$y = (4.0\ cm)\ sin\,(0.70\ rad/s \times t)$$

$$y = x_0\ sin\,(\omega\ \ t)$$

$$x_0 = 4.0\ cm = 0.04\ m$$

$$\omega = 0.70\ rads^{-1}$$

$$\text{kinetic energy} = \frac{1}{2}\ k\, x_0^2$$

$$kinetic\ energy = \frac{1}{2}\ (2.5)\,(0.04)^2$$

$$kinetic\ energy = 2 \times 10^{-3}\ J$$

5 The period of oscillation of a simple pendulum does not depend on the mass of the bob. By contrast the period of a mass-spring system does depend on mass. Explain the apparent condition.

The time period of a simple pendulum is given by the formula: $$T = 2\pi\sqrt{\frac{L}{g}}$$ As you can see, the time period of a simple pendulum does not depend on the mass of the pendulum bob. It only depends on the length of the pendulum and the acceleration due to gravity. the restoring force in a simple pendulum is provided by gravity, which is independent of the mass of the bob	The time period of mass-spring system is given by the formula: $$T = 2\pi\sqrt{\frac{m}{k}}$$ As you can see, the time period is directly proportional to the square root of the mass. This means that increasing the mass will increase the time period, and vice versa. The restoring force in a spring-mass system is provided by the spring, which depends on the mass.

6 What is the period of a simple pendulum of a 6.0 kg mass oscillating on a 4.0 m long string?

DATA	**SOLUTIONS**
m = 6.0 kg $L = 4.0\ m$ **T =?**	$$T = 2\pi\sqrt{\frac{L}{g}}$$ $$T = 2\pi\sqrt{\frac{4.0}{9.8}}$$ $$T = 4.01\ s$$

7 A pendulum of length 75 cm and mass 2.5 kg swings with a mechanical energy of 0.015 J . what is its amplitude?

DATA

$L = 75\ cm = 0.75\ m$

$m = 2.5\ kg$

$E = 0.015\ J$

$x_0 = ?$

SOLUTIONS

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}} \quad \left\{\omega = \sqrt{\frac{k}{m}}\right\}$$

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}}$$

$$\frac{k}{m} = \frac{g}{L}$$

$$k = \frac{g \times m}{L}$$

$$k = \frac{9.8 \times 2.5}{0.75}$$

$$k = 32.66\ N/m$$

$$E = \frac{1}{2}k x_0^2$$

$$0.015 = 0.5\ (32.66)\ x_0^2$$

$$\frac{0.015}{0.5\ (32.66)} = x_0^2$$

$$9.185 \times 10^{-4} = x_0^2$$

$$\sqrt{9.185 \times 10^{-4}} = x_0$$

$$0.03\ m = x_0$$

8 A pendulum of length L_1 has a period of T_1 = 0.950 s. the length of the pendulum is adjusted to a new value L_2 such that T_2 = 1.00 s what is the ratio L_2/L_1

DATA

$lenth\ of\ pendulu = L_1$

$T_1 = 0.950\ s$

$lenth\ of\ pendulu = L_2$

$T_2 = 1.00\ s$

$\frac{L_2}{L_1} = ?$

SOLUTIONS

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$(T)^2 = (2\pi)^2\left(\sqrt{\frac{L}{g}}\right)^2$$

$$\frac{g\ {T_1}^2}{4\pi^2} = L_1 \quad \ldots\ldots\ldots\ldots(i)$$

similarly

$$\frac{g\ {T_2}^2}{4\pi^2} = L_2 \quad \ldots\ldots\ldots\ldots(ii)$$

Dividing equation(ii) by (i), we get

$$\frac{L_2}{L_1} = \frac{\frac{g\ {T_2}^2}{4\pi^2}}{\frac{g\ {T_1}^2}{4\pi^2}}$$

$$\frac{L_2}{L_1} = \frac{{T_2}^2}{{T_1}^2}$$

$$\frac{L_2}{L_1} = \frac{(1.00)^2}{(0.950)^2}$$

$$\frac{L_2}{L_1} = 1.11$$

10 The amplitude of oscillation of a pendulum decays by a factor of 20.0 in 120 s. By what factor has its energy decayed in that time

DATA

$x_0 = maximum\ amplitude$

$x_0' = \frac{1}{20}\, x_0$ (reduced amplitude)

$Time = 120\ s$

SOLUTIONS

$$E = \frac{1}{2}\, k\, x_0^2 \ \ldots\ldots\ldots\ldots (i)$$

Reduced energy

$$E' = \frac{1}{2}\, k\, (x_0')^2$$

$$E' = \frac{1}{2}\, k \left(\frac{1}{20}\, x_0\right)^2$$

$$E' = \frac{1}{2}\, k \left(\frac{1}{400}\right) x_0^2$$

$$E' = \left(\frac{1}{400}\right) \frac{1}{2}\, k\, x_0^2$$

$$E' = \left(\frac{1}{400}\right) E$$

CHAPTER = 11 OSCILLATION

WORKED EXAMPLES

1 The spring used in one such device shown in Fig. has a spring constant of 606 N/m, and the mass of the chair is 12.0 kg. The measured oscillation period is 2.41 s. Find the mass of the astronaut.

DATA

$k = 606\ N/m$

$m_{chair} = 12.0\ kg$

$T = 2.41\ s$

$m_{astronaut} = 12.0\ kg$

$m = m_{chair} + m_{astronaut}$

$m_{total} = 12.0 + m_{astronaut}$

$m_{astronaut} = m_{total} - 12.0$

SOLUTIONS

$$T = 2\pi \sqrt{\frac{m_{total}}{k}}$$

Squaring both the side

$$(T)^2 = (2\pi)^2 \left(\sqrt{\frac{m_{total}}{k}}\right)^2$$

$$T^2 = 4\,\pi^2\, \frac{m_{total}}{k}$$

$$\frac{k\ T^2}{4\,\pi^2} = m_{total}$$

$$m_{total} = \frac{k\ T^2}{4\,\pi^2}$$

$$m_{total} = \frac{606 \times (2.41)^2}{4\,(3.142)^2}$$

$$m_{total} = 89.15\ kg$$

$$m_{astronaut} = 89.15 - 12.0$$

$$m_{astronaut} = 77.15\ kg$$

2 **A block is kept on a horizontal table. The table is undergoing simple harmonic motion of frequency 3 Hz in a horizontal plane. The coefficient of static friction between the block and the table surface is 0.72. Find the maximum amplitude of the table at which the block does not slip on the surface.**

DATA f = 3 Hz μ = 0.72 T = 2.41 s x_0 = ? **SOLUTIONS** Maximum force of static friction is given as f= μR f= μmg In case that the body does not slip $newton's force = friction force$	$ma = \mu mg \quad (a = x_0 \omega^2)$ $m(x_0 \omega^2) = \mu mg$ $x_0 \omega^2 = \mu g \quad (\omega = 2\pi f)$ $x_0 (2\pi f)^2 = \mu g$ $x_0 \times 4\pi^2 f^2 = \mu g$ $x_0 \times 4 (3.142)^2 (3)^2 = (0.72)(9.8)$ $x_0 \times 355.305 = 7.056$ $x_0 = \frac{7.056}{355.305}$ $x_0 = 0.021\, m$ $x_0 = 2.1\, cm$

CHAPTER = 11 OSCILLATION

ADDITIONAL NUMERICAL

1 **Calculate the length of Second's pendulum at a place where g = 10.0 m/s²**

Data: Time period of a second's pendulumT = 2 s Value of "g" = 10 m/s² Length of second's pendulum L = ? **SOLUTION:** **The time period of a simple pendulum is given by:** $T = 2\pi\sqrt{\frac{L}{g}}$ $2 = 2\pi\sqrt{\frac{L}{10}}$	Squaring both sides we get: $4 = 4\pi^2 \frac{L}{10}$ L = $L = \frac{4\times 10}{4\pi^2}$ L = $\frac{40}{4\pi^2}$ L = 1.01 m

2. Compute the acceleration due to gravity on the surface of the moon where a simple pendulum 1.5 m long makes 100 vibration in 605 seconds

DATA:

Length of the pendulum L = 1.5m

Number of vibrations = 100

Time taken = 605 sec.

Time period $T = \frac{605}{100} = 6.05$ s

Acceleration due to gravity on the surface of moon

g_m = ?

SOLUTION:

Acceleration due to gravity on the surface of moon is given by:

$$T = 2\pi\sqrt{\frac{L}{g_m}}$$

Squaring both sides we get:

$$T^2 = 4\,\pi^2\,\frac{L}{g_m}$$

$$(6.05)^2 = 4\,(3.141)^2\,\frac{1.5}{g_m}$$

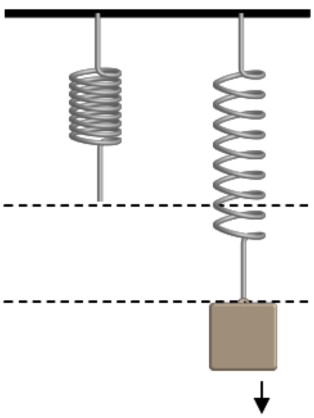
$$g_m = 4\,(3.141)^2\,\frac{1.5}{(6.05)^2}$$

$$g_m = \frac{59.2176}{36.6}$$

$$g_m = 1.6178\ ms^{-2}$$

3. A body hanging from spring is sest into motion and the period of oscillation is to be 0.8 s. After the body has come to rest, it is removed. How much shorter will the spring be when it comes to rest.

DATA:



T = 0.8 sec
x = ?
g = 9.8 m/sec^2

SOLUTION:

In case of a body attached with string, F = kx
As body is suspended by means of spring then

$$F = W$$

$$F = kx \qquad W = mg$$

$$kx = mg$$

$$x = \left(\frac{m}{k}\right) g$$

$$\frac{x}{g} = \left(\frac{m}{k}\right) \text{.................(i)}$$

We know that

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Squaring both sides

$$T^2 = \left(2\pi\sqrt{\frac{m}{k}}\right)^2$$

$$T^2 = 4\pi^2\,\frac{m}{k}$$

$$\frac{m}{k} = \frac{T^2}{4\pi^2} \text{ .... (ii)}$$

Comparing equation (i) and (ii)

$$\frac{x}{g} = \left(\frac{T^2}{4\pi^2}\right)$$

.

$$x = \left(\frac{T^2}{4\pi^2}\right) g$$

$$x = \frac{(0.5)^2}{4 \times 9.872} \text{ x } 9$$

$$x = \frac{0.25}{39.488} \text{ x } 9.8$$

x = 0.062 meter

x = 062 cm

4. A mass at the end of spring oscillate with simple harmonic motion with a period of 0.40 s ; find the acceleration when the displacement is 4.0 cm.

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DATA

T = 0.4 s
X = 4 cm = 0.04 m

SOLUTIONS

Time period is given by

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T}$$

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$$\omega = \frac{2\pi}{0.4}$$

$$\omega = 15.7\ s^{-1}$$

We know that,

$$a_x = -\omega^2 X$$

$$a_x = -(15.7)^2 (0.04)$$

$$a_x = -9.85\ m\ s^{-2}$$

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5. The period of vibration of a body of mass 25 g attached to a spring., Vibrates on a smooth horizontal surface. When it is displaced 10 cm to the right of its extreme position, the period of vibration is 1.57 second and the velocity at the end of this displacement is 0.4 m/s. determine the (a) spring constant (b) total energy (c) amplitude of the vibration.

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DATA:

Mass of body — m = 25 gm = 0.025 kg

Displacement — X = 10 cm = 0.10 m

Time period — T = 1.57 sec

Velocity — v = 0.4 m/s

Spring constant — k = ?

Total Energy — E = ?

Amplitude — X_0 = ?

Solution:

The period of a mass attached with an elastic spring executing S.H.M is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$ squaring both sides

$$T^2 = 4\pi^2 \frac{m}{k}$$

$$(1.57)^2 = 4\pi^2 \frac{0.025}{k}$$

$$\therefore \quad k = \frac{4\pi^2 \times 0.025}{(1.57)^2}$$

$$k = 0.4\ N/m$$

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Energy of the system at any instant is partially K.E and partially P.E, hence:

Total energy K = K.E + P.E

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$E = \frac{1}{2} \times 0.025 \times (0.4)^2 + \frac{1}{2} \times 0.4 \times (0.1)^2$$

$$E = 2 \times 10^{-3} + 2 \times 10^{-3}$$

∴ E = **<u>4 x 10^{-3} J</u>**

Energy of the system is **<u>4 x 10^{-3} J</u>**.

But $E = \frac{1}{2} k x_0^2$

$$4 \times 10\text{-}3 = \frac{1}{2} \times 0.4\ x_0^2$$

$$x_0^2 = \frac{4 \times 10^{-3} \times 2}{0.4} = 0.02$$

∴ x_0 = **<u>0.1414 m</u>**

Amplitude of oscillation is **<u>0.1414 m</u>**.

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6. Find the length of the second's pendulum on the planet Jupiter where the value of 'g' is 2.63 times that of the 'g' on the surface of earth.

DATA: T = 2s L= ? g = 9.8 m/sec^2 **SOLUTION:** The time period of simple pendulum is given by, $T = 2\pi\sqrt{\frac{L}{g}}$ Squaring both sides, we get	$(T)^2 = 2\pi\left(\sqrt{\frac{L}{g}}\right)^2$ $T^2 = 4\pi^2 \times \frac{L}{g}$ $L = \frac{T^2 \times g}{4\pi^2}$ $L = \frac{(2)^2 \times 2.63}{4 \times (3.1415)^2}$ $L = 0.226\ m$

7. The time period of simple pendulum is 2 seconds on the surface of the earth. Calculate its time period on the surface of the moon where the acceleration due to gravity is on sixth that of earth.

The time period of a simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ $2 = 2\pi\sqrt{\frac{L}{g}}$ $\frac{2}{2\pi} = \sqrt{\frac{L}{g}}$ ....(i) Time period of the pendulum on moon is given by: $T_m = 2\pi\sqrt{\frac{L}{g_m}}$ $T_m = 2\pi\sqrt{\frac{L}{1/6\ g}}$	$T_m = 2\pi\sqrt{\frac{6\ L}{g}}$ $T_{moon} = 2\pi\sqrt{6}\ \sqrt{\frac{L}{g}}$ ....(ii) **Substituting the expression of $\sqrt{\frac{L}{g}}$. from equation (i) in equation (ii), we get:** $T_m = 2\pi\sqrt{6} \times \frac{2}{2\pi}$ $T_m = 2 \times \sqrt{6}$ $T_m = 2 \times 2.45$ **T_m = 4.9 second**

8. A wire hangs from a dark high tower so that upper end is not visible. How can we determine the length of the wire?

Ans. **According to the equation of time period of the simple pendulum**

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Re arrange the above equation for the length of a pendulum

$$L = \frac{g\,T^2}{4\,\pi^2} \quad \text{------------------------} \quad (i)$$

Let a spherical bob of known radius is attached to one end of a visible wire and oscillate it like a simple pendulum. Calculate the time period of the simple pendulum and then substitute its value in equation (i) to determine the length of the pendulum.

Finally, the length of wire can be obtained by subtracting the radius of the bob form the kwon length of the pendulum.

Length of wire = length of pendulum - radius of the hanging bob

9. A body of mass 32 g attached to elastic spring is performing SHM its velocity is 0.4 m/s when the displacement is 8 cm towards right. If the spring constant is 0.4 N/m. Calculate (i) total energy (ii) the amplitude of its motion.

DATA **Mass of body m = 32 gm = 0.032 kg** **Displacement x = 8 cm = 0.08 m** **Velocity v = 0.4 m/s** **Spring constant k = 0.4 N/m** **Total Energy E = ?** **Amplitude X_0 = ?** **SOLUTION:** **Energy of the system at any instant is partially K.E and partially P.E, hence:** Total energy K = K.E + P.E $E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$ $E = [\frac{1}{2} \times (0.032 \times (0.4)^2] + [\frac{1}{2} \times 0.4 \times (0.08)^2]$	$E = 2.56 \times 10^{-3} + 1.28 \times 10^{-3}$ ∴ $E = \mathbf{3.84 \times 10^{-3}\ J}$ Energy of the system is **3.84×10^{-3} J**. But $E = \frac{1}{2} k x_0^2$ $3.84 \times 10^{-3} = \frac{1}{2} \times 0.4\ x_0^2$ $x_0^2 = \frac{3.84 \times 10^{-3} \times 2}{0.4} = 0.02$ ∴ $x_0^2 = \mathbf{0.0192}$ $x_0 = \sqrt{0.0192}$ $x_0 = 0.1385$ $x_0 = 13.85$ cm Amplitude of oscillation is **0.1385 m**.or 13.85 cm

10 The periodic time of a body executing S.H.M. is 2 s. After how much time interval from t =0 will its displacement be half the amplitude?

DATA T = 2 s $x = \frac{x_0}{2}$ SOLUTIONS Angular velocity = ω = 2π/T = 2π/2 = π rad/s	Displacement of a particle performing S.H.M. is given by x = a sin (ωt + α) ∴ 1/2 a = a sin (πt + 0) ∴ 1/2 =sin (πt) ∴ $\pi t = \sin^{-1}(1/2) = \pi/6$ ∴ t = 1/6 s Ans: After 1/6 s the displacement will be half the amplitude