CHAPTER = 13 **BOOK NUMERICAL**

PHYSICAL OPTICS

A monochromatic light of wavelength 6900A⁰ is used to illuminate two parallel 1 slits. On a screen that is 3.30 m away from the slits, interference fringes are observed. The distance between adjacent bright fringes in the centre of the pattern is 1.80 cm. what is the distance between the slits.

$$\lambda = 6900 \text{ A}^0 = 6900 \times 10^{-10} \text{ m}$$

$$L = 3.30 m$$

$$\Delta y = 1.80 \ cm = 1.80 \times 10^{-2} m$$

$$d = ?$$

SOLUTIONS

$$\Delta y = \frac{\lambda L}{d}$$
$$d = \frac{\lambda L}{\Delta y}$$

$$d = \frac{\lambda L}{\Delta v}$$

$$d = \frac{(6900 \times 10^{-10})(3.30)}{1.80 \times 10^{-2}}$$

$$d = 1.256 \times 10^{-4} m$$

2 A Michelson interferometer is adjusted so that a bright fringe is appears on the screen. As one of the mirrors is moved 25.8 micrometer, 92 bright fringes are counted on the screen. What is the wavelength of light used in the interferometer?

$$\Delta x = 25.8 \,\mu m$$

$$\Delta x = 25.8 \times 10^{-6} m$$

$$m = 92$$

$$\lambda = ?$$

SOLUTIONS

$$\Delta x = m \frac{\lambda}{2}$$

$$\lambda = \frac{2 \Delta x}{m}$$

$$\lambda = \frac{2 \Delta x}{m}$$

$$d = \frac{2 (25.8 \times 10^{-6})}{92}$$

$$d = 5.608 \times 10^{-7} m$$

$$d = 5608 \times 10^{-10} \ m$$

$$d = 5608 \times A^0 m$$

- Why must the film be thin? Why don't we see the interference effect when looking through a window or at a poster covered by a plate of glass, even if the glass is optically flat.
- Ans For a film to display interference effects, it needs to be very thin, typically on the order of the wavelength of the light interacting with it. This is because interference effects arise from the constructive and destructive interference of light waves reflecting off the front and back surfaces of the thin film. As thin film becomes thicker the increased distance allows more light from different areas (which are reflecting at different angles) to mix to your eye. Once you have more than a wavelength or two of distance between boundaries the mixing of the various wavelengths becomes more random. This effectively cancels out any visible interference effect and you no longer see specific colors.
- Newton 's rings are formed by the light of 400 nm wavelength. Determine the change in air film thickness between the third and sixth bright fringe. If the radius of curvature of the curved surface is 5.0 m what is the radius of the third bright ringe?

DATA

 $\overline{\lambda = 400 \ nm} = 400 \times 10^{-9} m$

R = 5.0 m

 $\Delta t = t_6 - t_3$

 $r_3 = ?$

SOLUTIONS

We know that air film thickness for

bright rings

$$2 t_N n = \left(N - \frac{1}{2}\right) \lambda$$

Thickness of the third bright ring (N=3)

$$2 t_3 (1) = (3 - 0.5)(400 \times 10^{-9})$$

$$2 t_3 = (2.5)(400 \times 10^{-9})$$

$$t_3 = 500 \times 10^{-9} m$$

Now,

$$2 t_N n = \left(N - \frac{1}{2}\right) \lambda$$

Thickness of the six bright ring (N=6)

$$2 t_3 (1) = (6 - 0.5)(400 \times 10^{-9})$$

$$2 t_3 = (5.5)(400 \times 10^{-9})$$

$$t_3 = 1100 \times 10^{-9} m$$

thickness between the third and sixth bright fringe

$$\Delta t = t_6 - t_3$$

$$\Delta t = 1100 \times 10^{-9} - 500 \times 10^{-9}$$

$$\Delta t = 600 \times 10^{-9} \, m$$

the radius of the third bright ringe

$$r_3 = \sqrt{\left(N - \frac{1}{2}\right) \lambda R}$$

$$r_3 = \sqrt{\left(3 - \frac{1}{2}\right) (400 \times 10^{-9}) (5.0)}$$

$$r_3 = 2.23 \times 10^{-3} m$$

(a) At a spot where the film thickness is 910.0 nm, which wavelengths are missing in the reflected light?

(b) Which wavelengths are strongest in the visible light?

DATA

$$n = 1.50$$

$$t = 910.0 nm$$

$$t = 910.0 \times 10^{-9} m$$

$$\lambda = ?$$

SOLUTIONS

We know that air film thickness for dark

rings

$$2tn = m\lambda$$

$$2 t n = m \lambda$$

$$\lambda = \frac{2 t n}{m}$$

Our eye detects wavelengths 400 nm to

700 nm

FOR m=4

$$\lambda_1 = \frac{2 (910.0 \times 10^{-9}) (1.50)}{4}$$

$$\lambda_1 = 6.285 \times 10^{-7} m$$

FOR m=5

$$\lambda_2 = \frac{2 (910.0 \times 10^{-9}) (1.50)}{5}$$
 $\lambda_2 = 5.46 \times 10^{-7} m$

$$\lambda_2 = 5.46 \times 10^{-7} m$$

FOR m=6

$$\lambda_3 = \frac{2 (910.0 \times 10^{-9}) (1.50)}{6}$$

$$\lambda_3 = 4.55 \times 10^{-7} m$$

We know that air film thickness for bright rings(constructive interference

$$2 t n = \left(m + \frac{1}{2}\right) \lambda$$

$$2 t n = (m + 0.5) \lambda$$

$$\lambda = \frac{2 t n}{(m + 0.5)}$$

FOR m=4

$$\lambda_1 = \frac{2 (910.0 \times 10^{-9}) (1.50)}{(4+0.5)}$$

$$\lambda_1 = 6.067 \times 10^{-7} m$$

FOR m=5

$$\lambda_2 = \frac{2 (910.0 \times 10^{-9}) (1.50)}{(5 + 0.5)}$$

$$\lambda_2 = 4.964 \times 10^{-7} m$$

FOR m=6

$$\lambda_3 = \frac{2 (910.0 \times 10^{-9}) (1.50)}{(6 + 0.5)}$$

$$\lambda_3 = 4.200 \times 10^{-7} m$$

What is the difference between deviation and diffraction? What do diffraction and interference have in common?

DIFFERENCE BETWEEN DEVIATION AND DIFFRACTION

DEVIATION	DIFFRACTION
	Diffraction is the slight bending of light as it passes around the edge of an object. The amount of bending depends on the relative size of the wavelength of light to the size of the opening
The deviation obeys Snell's law, which is the ratio of the angle of incident and angle of refraction is a constant for any given wavelength of wave.	Huygens' principle (wavefront is composed of an infinite number of circularly propagating wavelets) combined with Fresnel's principle of interference provided a clear explanation of light's diffraction pattern when it passes through a single or multiple slit

DIFFRACTION AND INTERFERENCE HAVE IN COMMON

INTERFERENCE	DIFFRACTION
interference is a wave phenomenon that occurs when a wave encounters an obstacle	diffraction is wave phenomena that occur when a wave encounters an obstacle.
	The diffraction involves the superposition of waves. When two or more waves interact, they can either constructively or destructively interfere, leading to a pattern of bright and dark regions.

Is it possible to increase the orders of maxima for a given energy spectrum from a diffraction grating?

We want to increase the orders of maxima in a diffraction grating, we can consider the following approaches:

1. Change the Grating element (d): The grating element (distance between adjacent slits or rulings) affects the angular positions of the diffraction maxima. Decreasing the grating element (making d smaller) will spread out the maxima, allowing higher-order maxima to appear.

$$d \sin \theta = m \lambda$$

2. Use Higher Wavelength Light: The order of maxima is given by the equation

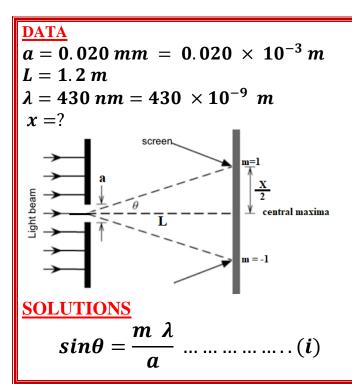
$$d \sin \theta = m \lambda$$

where (m) is the order, (λ) is the wavelength, (d) is the grating element, and (θ) is the angle of diffraction. Using light with a longer wavelength λ will result in larger angles of diffraction (θ) allowing higher-order maxima to be observed.

- 3. Increase the Angle of Incidence: The angle of incidence (θ)affects the angle of diffraction. By increasing (θ), you can shift the diffraction pattern and potentially observe higher-order maxima.
- 8 Describe what happens to a single slit diffraction pattern as the width of the slit is slowly decreased.

Ans The width of the single slit in a diffraction pattern has the following effects:

- 1. **Spatial Resolution**: As the slit width decreases, the diffraction pattern becomes wider. This increased width reduces the spatial resolution, making it harder to distinguish between closely spaced objects or features.
- 2. **Intensity**: The intensity of the central bright fringe in the diffraction pattern is inversely proportional to the slit width. Decreasing the slit width results in a brighter central fringe.
- 3. **Fringe Spacing**: The spacing between the fringes in the diffraction pattern is inversely proportional to the slit width. Decreasing the slit width leads to a greater separation between the fringes.
- 4. **Diffraction Angle**: The angle at which the diffraction fringes occur is inversely proportional to the slit width. Decreasing the slit width results in a larger diffraction angle.
- 9. The diffraction pattern from a single slit of width 0.020 mm is viewed on a screen. If the screen is 1.20 m from the slit and light of wavelength 430 nm is used. What is the width of the central maximum?



If angle θ is very small Sin $\theta = \tan \theta$, equation (i) can be written as $tan \theta = \frac{m \lambda}{a} \qquad \left\{ tan \theta = \frac{\frac{x}{2}}{L} \right\}$ $\frac{x}{2L} = \frac{m \lambda}{a}$ $x = \frac{m \lambda}{a} \times 2L$ $x = \frac{(1) \ 430 \times 10^{-9}}{0.020 \times 10^{-3}} \times 2 \ (1.2)$ $x = 0.0516 \ m = 5.16 \ cm$

Number of lines ruled = 500 lines

$$S = 2.54 \text{ cm} = 2.54 \times 10^{-2} \text{ m}$$

$$\lambda = 54.6 \text{ nm} = 546 \times 10^{-9} \text{ m}$$

Order of image m = 3

Angular deviation $\theta = ?$

SOLUTIONS

The grating element is given by

$$d = \frac{length of grating}{number of lines ruled}$$

$$d = \frac{2.54 \times 10^{-2}}{8000}$$

$$d = 3.175 \times 10^{-6} m$$

For a diffraction grating:

$d Sin \theta = m\lambda$

$$3.175 \times 10^{-6} Sin\theta = (3) \times 546 \times 10^{-9}$$

$$3.175 \times 10^{-6} Sin\theta = 1638 \times 10^{-9}$$

$$Sin\theta = \frac{1638 \times 10^{-9}}{3.175 \times 10^{-6}}$$

$$Sin\theta = (0.5159)$$

$$\theta = Sin^{-1}(0.5159)$$

$$\theta = 31^{\circ}$$

Angular deviation in the third order is 31°.

How many lines per centimeter are there in a grating which gives 1st order 11 spectra at an angle of 30° when the wavelength of light is 6 x 10⁻⁵ cm?

DATA:

$$m = 1$$

$$\theta = 30^{\circ}$$

$$\lambda = 6 \times 10^{-5} cm = 6 \times 10^{-7} m$$

(i)
$$d = ?$$

(ii) No. of lines
$$/ \text{ cm} = ?$$

SOLUTION:

According to diffraction grating equation

$$dsin\,\theta\,=\,m\lambda$$

$$d=\frac{\mathrm{m}\,\lambda}{\sin\theta}$$

$$d = \frac{1 \times 6 \times 10^{-7}}{\sin 30^{\circ}}$$

$$d = 1.2 \times 10^{-6} m$$

Now,

No .of lines =
$$\frac{\text{Length of grating}}{d}$$

No of lines =
$$\frac{1 \text{ cm}}{d}$$

No. of lines =
$$\frac{1 \times 10^{-2}}{1.2 \times 10^{-6}}$$

$$No.of\ lines = 8333$$

No. of lines =
$$8333$$
 lines / cm

12 Light of wavelength 450 nm is incident on a diffraction grating on which 5000 lines/cm have been ruled. Determine

- (i) How many orders of spectra can be observed on either side of spectra?
- (ii) Determine the angle corresponding to each order.

DATA

 $\lambda = 450 \text{ nm} = 450 \times 10^{-9} \text{ m}$

Number of lines ruled = 5000 lines/cm

Order of image m = ?

Angular deviation $\theta = ?$

SOLUTIONS

The grating element is given by

$$d = \frac{length of grating}{number of lines ruled}$$

$$d = \frac{1 \text{ cm}}{5000}$$

$$d = \frac{1 \times 10^{-2}}{5000}$$

$$d = 2 \times 10^{-6} m$$

For a diffraction grating:

$$d \sin \theta = m\lambda$$
 (keeping $\sin \theta = 90$)

$$2 \times 10^{-6} Sin90 = m \times 450 \times 10^{-9}$$

$$2 \times 10^{-6} (1) = m \times 450 \times 10^{-9}$$

$$m = \frac{2 \times 10^{-6}}{450 \times 10^{-9}}$$

$$m = 4.4 = 4$$

For m=1

$$Sin\theta = \frac{m\lambda}{d}$$

$$Sin\theta = \frac{(1)(450 \times 10^{-9})}{2 \times 10^{-6}}$$

$$Sin\theta = (0.225)$$

$$\theta = Sin^{-1}(0.225)$$

$$\theta = 13^{0}$$

For
$$m=2$$

$$Sin\theta = \frac{m\lambda}{d}$$

$$Sin\theta = \frac{(2)(450 \times 10^{-9})}{2 \times 10^{-6}}$$

$$Sin\theta = (0.450)$$

$$\theta = Sin^{-1}(0.450)$$

$$\theta = 26.74^{\circ}$$

For m=3

$$Sin\theta = \frac{m\lambda}{d}$$

$$Sin\theta = \frac{(3)(450 \times 10^{-9})}{2 \times 10^{-6}}$$

$$Sin\theta = (0.675)$$

$$\theta = Sin^{-1}(0.675)$$

$$\theta = 42.45^{0}$$

For m=4

$$Sin\theta = \frac{m\lambda}{d}$$

$$Sin\theta = \frac{(4)(450 \times 10^{-9})}{2 \times 10^{-6}}$$

$$Sin\theta = (0.90)$$

$$\theta = Sin^{-1}(0.90)$$

$$\theta = 64.15^{0}$$

5th order diffraction is not possible

Why does a crystal act as a three-dimensional grating for X-rays but not for visible light?

Ans The diffraction of x-rays by a crystal is a result of the periodic arrangement of atoms within the crystal lattice. The spacing between the atoms in the crystal lattice is on the order of a few angstroms, which is similar to the wavelength of x-rays (typically around 1-10 angstroms). When x-rays encounter a crystal, act as a three-dimensional grating for X-rays, they interact with the atoms in the lattice and diffract at specific angles, creating a diffraction pattern.

On the other hand, the wavelength of visible light is on the order of 400-700 nm, which is much larger than the spacing between atoms in a crystal lattice. When visible light encounters a crystal, it does not interact with the atoms in the same way as x-rays, and does not diffract to the same degree. Therefore, a crystal does not act as a three-dimensional grating for visible light, the diffraction pattern is not as noticeable as x-rays.

A beam of X-rays of wavelength 0.071 nm is diffracted by a diffracting plane of rock salt with distance between the atomic planes are 1.98 A° Find the glancing angle for the second-order diffraction

DATA:

 $\lambda = 0.071 \, nm = 0.071 \times 10^{-9} \, m$

 $d = 1.98 A^{\circ}$

 $d = 1.98 \times 10^{-10} \text{ m}.$

m = 2

 $\theta = 3$

SOLUTION:

According to Bragg's law,

 $2d \sin \theta = m\lambda$

$$\sin\theta = \frac{m\,\lambda}{2\times d}$$

$$\sin\theta = \frac{2 \times 0.071 \times 10^{-9}}{2 \times 1.09 \times 10^{-10}}$$

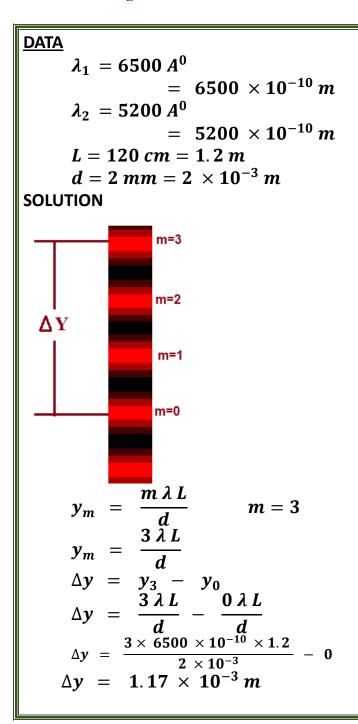
 $Sin\theta = 0.3585$

$$\theta = Sin^{-1} (0.3585)$$

 $\theta = 21^{0}$

WORKED EXAMPLES

In a Young's double slit experiment a beam of light consisting of two wavelengths, $6500\,\mathrm{A^0}$ and $5200\,\mathrm{A^0}$ is used to obtain interference fringes on a screen 120 cm away from two slits 2mm mm apart. (i) Find the distance of the third bright fringe on the screen from the central maxima for the wavelength $6500\,\mathrm{A^0}$ (ii) What is the least distance from the central maximum where the bright fringes due to both wavelengths coincide?



(b) suppose the mth bright fringe due to the wavelength $\lambda_1=6500\,A^0$ coincide with the nth bright fringe of wavelength $\lambda_2=5200\,A^0$ then, $\gamma_m=\gamma_n$

$$\frac{m \lambda_1 L}{d} = \frac{y_n}{d}$$

$$\frac{m \lambda_1 L}{d} = \frac{n \lambda_2 L}{d}$$

$$\frac{m \times 6500 \times 10^{-10} L}{d}$$

$$\frac{m}{n} = \frac{\frac{5200 \times 10^{-10}}{6500 \times 10^{-10}}}{\frac{m}{n}} = \frac{4}{5}$$

Minimum value of m = 4 and n = 5

$$y_{m} = \frac{m \lambda L}{d} \qquad m = 4$$

$$y_{4} = \frac{4 \times 6500 \times 10^{-10} \times 1.2}{2 \times 10^{-3}}$$

$$y_{4} = 1.56 \times 10^{-3} m$$

$$y_m = \frac{m \lambda L}{d}$$
 $m = 5$
 $y_5 = \frac{5 \times 5200 \times 10^{-10} \times 1.2}{2 \times 10^{-3}}$
 $y_5 = 1.56 \times 10^{-3} m$

A soap bubble in air is of thickness 320 nm. If it is illuminated with white light at near normal incidence, what color will appear to be in reflected light? (Refractive index of soap bubble n= 1.50).

DATA

$$t = 320 nm = 320 \times 10^{-9} m$$

 $n = 1.5$
 $\lambda = ?$

SOLUTION



$$2 t n = \left(m + \frac{1}{2}\right) \lambda$$

$$\lambda = \frac{2 t n}{\left(m + \frac{1}{2}\right)}$$

For m=0

$$\lambda = \frac{2 \times 320 \times 10^{-9} \times 1.5}{\left(0 + \frac{1}{2}\right)}$$

$$\lambda = 1920 \times 10^{-9} m$$

For m=1
$$\lambda = \frac{2 \times 320 \times 10^{-9} \times 1.5}{\left(1 + \frac{1}{2}\right)}$$

$$\lambda = 640 \times 10^{-9} m$$

For m=2

$$\lambda = \frac{2 \times 320 \times 10^{-9} \times 1.5}{\left(2 + \frac{1}{2}\right)}$$

$$\lambda = 384 \times 10^{-9} m$$

For m=3

$$\lambda = \frac{2 \times 320 \times 10^{-9} \times 1.5}{\left(3 + \frac{1}{2}\right)}$$

$$\lambda = 274 \times 10^{-9} m$$

We note that only the maxima, for m= I and m=2 lies in the visible region and the colors for 640 nm 384 nm are nearly red and violet

DATA

$$D_{11} = 0.527 cm$$

$$= 0.527$$

$$\times 10^{-2} m$$

$$r_{11} = \frac{0.527 \times 10^{-2}}{2}$$

$$= 0.2635 \times 10^{-2} m$$

$$D_{4} = 0.336 cm$$

$$= 0.336$$

$$\times 10^{-2} m$$

$$r_{4} = \frac{0.336 \times 10^{-2}}{2}$$

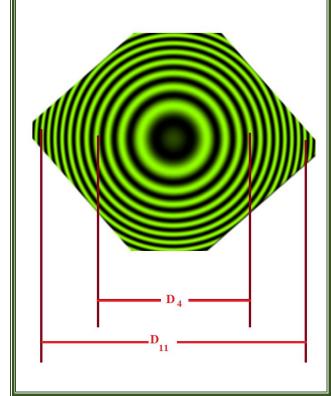
$$= 0.168$$

$$\times 10^{-2} m$$

$$R = 100 cm = 1 m$$

$$\lambda = ?$$

SOLUTION



Radius of the 11th bright ring is given by

$$r_{11}^2 = \left(N - \frac{1}{2}\right) \lambda$$

 $r_{11}^2 = (11 - 0.5) \lambda$
 $r_{11}^2 = (10.5) \lambda \dots \dots \dots \dots (i)$

Radius of the 4th bright ring is given by

$$r_4^2 = \left(N - \frac{1}{2}\right) \lambda$$

 $r_4^2 = (4 - 0.5) \lambda$
 $r_4^2 = (3.5) \lambda \dots \dots \dots (ii)$

Subtracting equation (i) from (ii)

$$r_{11}^2 - r_4^2 = (10.5) \lambda$$
 $- (3.5) \lambda$
 $(0.2635 \times 10^{-2})^2$
 $- (0.168 \times 10^{-2})^2$
 $= (10.5 - 3.5) \lambda$

$$4.120825 \times 10^{-6}$$

$$= (10.5 - 3.5) \lambda$$

$$4.120825 \times 10^{-6} = 7 \lambda$$

$$\frac{4.120825 \times 10^{-6}}{7} = \lambda$$

$$5.886 \times 10^{-7} m = \lambda$$

EXTRA SOLVED PROBLEM

- In a Young's double slit experiment the light has a wavelength of 6×10^{-5} cm.
- If the distance between the slits is 0.05 cm and the screen is 2m from the Slit. What is the distance between two successive bright fringes?

DATA:

Distance between slits and the screen

$$L = 2 m$$

Wavelength of light $\lambda = 6 \times 10^{-5}$ cm

$$\lambda = 6 \times 10^{-7} \text{ m}$$

Distance between two successive bright fringe = Δx

Separation of slits d = 0.05 cm

$$d = 0.05x 10^{-2} m$$

SOLUTION:

In Young's double slit experiment:

$$\Delta x = \frac{\lambda L}{d}$$

$$\Delta x = \frac{6 \times 10^{-7} \times 2}{0.05 \times 10^{-2}}$$

$$\Delta x = \frac{12 \times 10^{-7}}{0.05 \times 10^{-2}}$$

$$\Delta x = 2.4 \times 10^{-3} \text{ m}$$

Monochromatic light of wavelength of 6.0×10^{-7} m falls normally on a diffraction grating having 500 lines per millimeter. Calculate the angular deviation of the second order spectrum obtained on this side of the normal.

Data:

Wavelength of light $\lambda = 6 \times 10^{-7}$ m

Number of lines ruled = 500 lines/mm

:. Grating element

$$d = \frac{\text{length of grating}}{\text{number of lines ruled}}$$

$$d = \frac{0.001}{500}$$

$$d = 2 \times 10^{-6} \text{ m}$$

Order of image m = 2

Angular deviation $\theta = ?$

SOLUTION:

For a diffraction grating:

$$d \sin\theta = m\lambda$$

$$2 \times 10^{-6} \text{ Sin}\theta = 2 \times 6.0 \times 10^{-7}$$

$$\therefore \sin \theta = \frac{2 \times 6 \times 10^{-7}}{2 \times 10^{-6}}$$

$$\sin \theta = \frac{2 \times 6 \times 10^{-7}}{2 \times 10^{-6}}$$

$$\sin\theta = (0.6)$$

$$\theta = \operatorname{Sin}^{-1}(0.6)$$

$$\theta = 36.86^{\circ}$$

Angular deviation in third order is **36.86**°.

Data:

Number of lines ruled = 300

lines/mm.

Angular deviation $\theta = 20^{\circ}$

Order of image m = 2

Wavelength of light used $\lambda = ?$

SOLUTION:

:. Grating element

$$d = \frac{\text{length of grating}}{\text{number of lines ruled}}$$

$$d = \frac{0.001}{300}$$
, $d = 3.33 \times 10^{-6} \text{ m}$

For a diffraction grating:

$$d \sin\theta = m\lambda$$

$$\lambda = \frac{d \sin \theta}{}$$

$$\lambda = \frac{3.33 \times 10^{-6} \times \sin 20^{0}}{2}$$

$$\lambda = \frac{d \sin \theta}{m}$$

$$\lambda = \frac{3.33 \times 10^{-6} \times \sin 20^{0}}{2}$$

$$\lambda = \frac{3.33 \times 10^{-6} \times 0.342}{2}$$

$$\lambda = 5.694 \times 10^{-7} \text{ m}$$

Light from a narrow slit passes through two parallel slits 2mm apart. The two 4 successive dark interference fringes on a screen 100mm away are 0.2mm apart. Calculate the wavelength of the light used. What would have been fringe separation for light wavelength 6000A° units?

Data:

$$d = 2 \text{ m.m}$$

$$=$$
 2 × 10⁻³ meter

$$\Delta x = 0.2 \text{ m.m}$$

$$= 0.2 \times 10^{-3} \text{ m}$$

$$L = 100 \text{ mm} = 100 \text{ x } 10^{-3} \text{ m}$$

$$\lambda = 2$$

Solution:

In young's double slits experiment, the fringe spacing is,

$$\Delta x = \frac{L\lambda}{d}$$

$$\lambda = \frac{\Delta x \ d}{I}$$

$$\lambda_1 = \frac{0.2 \times 10^{-3} \times 2 \times 10^{-3}}{100 \times 10^{-3}}$$
$$\lambda_1 = 4.0 \times 10^{-7} \text{ m}$$

$$\lambda_1 = 4.0 \times 10^{-7} \text{ m}$$

$$\Delta x = \frac{L\lambda}{d}$$

$$\Delta x = \frac{100 \times 10^{-3} \times 6000 \times 10^{-10}}{2 \times 10^{-3}}$$

$$\Delta x = \frac{6.0 \times 10^{-8}}{2 \times 10^{-3}}$$

$$\Delta x = 3 \times 10^{-5} \text{ m}$$

$$\Delta x = \frac{6.0 \times 10^{-8}}{2.10^{-3}}$$

$$\Delta x = 3 \times 10^{-5} \text{ m}$$

Two parallel slits are illuminated by light of two wave lengths one of which is 6.0×10^{-7} m. On a screen the fourth dark line of the known wave length coincides with the fifth bright line of the unknown wave length. Calculate the unknown wave length.

DATA:

Known wave length $\lambda_1 = 6.0 \times 10^{-7}$ m

Unknown wave length $\lambda_2 = ?$

Fourth dark line of known wave length coincides with the fifth bright line of unknown wave length.

SOLUTION:

Distance "y" of bright fringes from center of the screen is given by:

$$Y_{bright} = \frac{m\lambda L}{d}$$

:. Distance of fifth bright fringe of unknown wave length " λ_2 " from center of the screen will be:

$$Y_{bright(5)} = \frac{5\lambda_2 L}{d}$$

Similarly the distance of fourth dark fringe of known wave length " λ_1 " from center of the screen is given by:

$$Y_{dark} = \left(m + \frac{1}{2}\right) \frac{\lambda_1 L}{d}$$

$$Y_{dark(4)} = \left(3 + \frac{1}{2}\right) \frac{\lambda_1 L}{d}$$

$$Y_{dark(4)} = \frac{3.5 \ \lambda_1 L}{d}$$

Since fourth dark fringe of known wave length coincides with the fifth bright fringe of unknown wave length, hence both must be equal distance from center of the screen.

$$Y_{bright(5)} = Y_{dark(4)}$$

$$\frac{5 \lambda_2 L}{d} = \frac{3.5 \lambda_1 L}{d}$$

$$5\lambda_2 = 3.5 \lambda_1$$

$$\therefore \lambda_2 = \frac{3.5}{5} \times 6 \times 10^{-7}$$

$$\lambda_2 = 4.2 \times 10^{-7} \text{ m}$$

$$\lambda_2 = 4200 \text{ A}^{\circ}$$

Unknown wave length is

$$5.4 \times 10^{-7}$$
 m or 5400 A°.

6. In double slits experiment the separation of the slits is 1.8 mm and the fringe spacing is 0.30 mm at a distance of 1200 mm from the slits; find the wavelength of light.

DATA:

Separation of slits

 $D = 1.8 \text{ mm} = 1.8 \times 10^{-3} \text{ m}$

Fringe spacing

 $\Delta x = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$

Distance between screen and slits

L = 1200 mm = 1.2 m

Wave length of light $\lambda = ?$

SOLUTION:

In Young's double slit experiment:

$$\Delta x = \frac{\lambda L}{d}$$

$$0.3 \times 10^{-3} = \frac{\lambda \times 1.2}{1.8 \times 10^{-3}}$$

$$\lambda = \frac{0.3 \times 10^{-3} \times 1.8 \times 10^{-3}}{1.2}$$

$$\therefore \lambda = 4.5 \times 10^{-7} \text{ m}$$

$$\lambda = 4500 \stackrel{\circ}{A}$$

7. Interference fringes were produced by two slits on a screen 0.8 m from them when the light of wave length 5.8×10^{-7} m was used. If the separation between the first and the fifth bright fringe 2.5 mm. Calculate the separation of the two slits.

DATA:

Distance between slits and the screen

$$L = 0.8 m$$

Wavelength of light $\lambda = 5.8 \times 10^{-7}$ m Distance between firs and fifth bright fringe = 2.5 mm = 2.5×10^{-3} m Separation of slits d = ?

SOLUTION:

Distance of the first bright fringe from centre of the screen is given by:

$$Y_{bright(1)} = \frac{\lambda L}{d}$$

Similarly distance of the fifth bright fringe is given by:

$$Y_{bright(5)} = \frac{5 \lambda L}{d}$$

But distance between the two fringes is 2.5 mm

$$Y_{bright(5)} - Y_{bright(5)} = 2.5 \times 10^{-3}$$
$$\therefore \frac{5 \lambda L}{d} - \frac{\lambda L}{d} = 2.5 \times 10^{-3}$$

$$\frac{4 \lambda L}{d} = 2.5 \times 10^{-3}$$
$$\therefore \frac{4 \times 5.8 \times 10^{-7} \times 0.8}{d} = 2.5 \times 10^{-3}$$

$$\therefore d = \frac{4 \times 5.8 \times 10^{-7} \times 0.8}{2.5 \times 10^{-3}}$$

$$\therefore \mathbf{d} = 7.424 \times 10^{-4} \,\mathrm{m}$$

$$d = 0.74 \text{ mm}$$

Separation of the slits is 0.74 mm.

8 How much should the moveable mirror of the Michelson's interferometer be moved in order to observe 400 fringes with reference to a point? The wavelength of the light used is 5890 $\,\mathrm{A}^{0}$.

DATA:

Distance moved $\Delta x = ?$

Number of fringes m = 400

Wave length light used $\lambda = 5890 \text{ } \mathring{A}$

SOLUTION:

Distance, through which the moveable mirror of a Michelson's interferometer is moved, is given by

$$\Delta x = m\left(\frac{\lambda}{2}\right)$$

$$\Delta x = \frac{400 \times 5890 \times 10^{-10}}{2}$$

$$\Delta x = 1.178 \times 10^{-4} \text{ m}$$

Or
$$\Delta x = 0.1178 \text{ mm}$$

- 9. When a light of wave length 6000 ⁰A falls on a diffraction grating. It produces a second order spectrum at an angle of 30⁰ from normal find:
 - (i) The grating element
 - (ii) the number of lines per millimeter ruled on it.

DATA:

$$\mathbf{m} = \mathbf{2}$$

$$\lambda = 6000 \text{ A}^0 = 6000 \times 10^{-10} \text{ m}$$

$$0 = 30^{\circ}$$

(i)
$$d = ?$$

(ii) No. of lines
$$/ m.m = ?$$

SOLUTION:

$$d \sin \theta = m\lambda$$

$$\mathbf{d} = \frac{m \, \lambda}{\sin \, \theta}$$

$$\mathbf{d} = \frac{2 \times 6000 \times 10^{-10}}{\sin 30^{\circ}}$$

$$d = 2.4 \times 10^{-6} \text{ m}$$

Given that length grating

$$d = 2.4 \times 10^{-6} \text{ m}$$

Now,

$$\mathbf{d} = \frac{\text{Length of grating}}{\text{No. of lines}}$$

No. of lines =
$$\frac{\text{Length of grating}}{d}$$

No. of lines =
$$\frac{1 \text{ mm}}{d}$$

No. of lines =
$$\frac{1 \times 10^{-3}}{2.4 \times 10^{-6}}$$

No. of lines =
$$0.41666 \times 10^3$$

No. of lines
$$= 570$$
 lines $/$ mm

10. If the diameter of the 10th bright Newton's ring is 0.005 m when the light of wavelength 5893 ⁰A is used. What is the radius of curvature of the planoconvex lens? Also calculate the thickness of the air film corresponding to this ring.

DATA:

$$\overline{\text{No. of bright ring: }}$$
 N = 10

$$\lambda = 5893 \times 10^{-10} \text{ m}$$

$$r = 0.005 m$$

$$\mathbf{R} = ?$$

$$t = ?$$

SOLUTION:

For Nth bright ring, r_N

$$\mathbf{r}_{\mathrm{N}} = \sqrt{\left(\mathbf{N} - \frac{1}{2}\right)\lambda \ \mathbf{R}}$$

Squaring both the sides we get,

$$\left(\mathbf{r}_{\mathbf{N}}\right)^{2} = \left(\mathbf{N} - \frac{1}{2}\right) \lambda \ \mathbf{R}$$

$$(0.005)^2 = \left(10 - \frac{1}{2}\right) 5893 \times 10^{-10} \text{ R}$$

$$(0.005)^2 = \frac{19}{2} \times 5893 \times 10^{-10} R$$

$$R = \frac{2.5 \times 10^{-5}}{9.5 \times 5893 \times 10^{-10}}$$

$$R = 4.465 \text{ m}$$

For thickness of the air film

As
$$t = \frac{r^2}{2R}$$

$$t = \frac{\left(0.005\right)^2}{2 \times 4.465}$$

$$t = 2.79 \times 10^{-6} \text{ m}$$

11. If the diameter of the 12th dark Newton's ring is 1.0 mm when the light of wavelength 5890 ⁰A is used. What is the radius of curvature of the plano-convex lens?

DATA:

No. of dark ring: N = 12

$$\lambda = 5893 \times 10^{-10} \text{ m}$$

$$r = 1 mm = 1x 10^{-3} m$$

 $\mathbf{R} = ?$

SOLUTION:

For Nth bright ring, r_N

$$r_{N} = \sqrt{N \lambda R}$$

Squaring both the sides we get,

$$(r_N)^2 = N \lambda R$$

$$(1 \times 10^{-3})^2 = 12 \times 5893 \times 10^{-10} \text{ R}$$

$$R = \frac{1.0 \times 10^{-6}}{12 \times 5893 \times 10^{-10}}$$

$$R = 0.1414 \text{ m}$$

12. X-rays of wavelength 1.54 A⁰ are diffracted by a crystal whose plane are 2.81 ⁰A apart. Find the glancing angle for the first order

DATA:

$$\lambda = 1.54 \text{ A}^{\circ} = 1.54 \times 10^{-10} \text{ m}$$

$$d = 2.81 A^{\circ}$$

$$d = 2.81 \times 10^{-10} \text{ m}.$$

$$m = 1$$

$$\theta = ?$$

SOLUTION:

According to Bragg's law,

$$2d \sin \theta = m\lambda$$

$$\sin \theta = \frac{m \lambda}{2xd}$$

$$\sin \theta = \frac{1 \times 1.54 \times 10^{-10}}{2 \times 2.81 \times 10^{-10}}$$

$$Sin\theta = 0.274$$

$$\theta = Sin^{-1} (0.274)$$

$$\theta = 15.9$$

13. A parallel beam of X-ray diffraction by a crystal. The first order maxima is obtained when the glancing angle of incidence is 6.5° . if the distance the atomic plane of crystal is $2.8 \, \text{A}^{\circ}$. Calculate the wavelength of the radiation.

Data:

$$m = 5$$

$$\theta = 6.5^{\circ}$$

$$d = 2.8 A^0 = 2.8 \times 10^{-10} m.$$

$$\lambda = ?$$

SOLUTION:

According to Bragg's law,

$$2d\ sin\theta = m\lambda$$

$$\lambda = \frac{2d \sin \theta}{m}$$

$$\lambda = \frac{2 \times 2.8 \times 10^{-10} \times \sin 6.5^{\circ}}{5}$$

$$\lambda = \frac{-6.3393 \times 10^{-11}}{5}$$

$$d = 1.26 \ 10^{-11} \ m$$

$$\lambda = 0.126$$

14. A green light of wavelength $5400 \, A^0$ is diffracted by a grating having 2000 lines per centimeter, find the angular deviation for the 3^{rd} order.

Data:

Wavelength of light $\lambda = 5400 \text{ A}^{\circ}$ $\lambda = 5400 \times 10^{-10}$

m

Number of lines ruled = 2000

lines/cm

Order of image m = 3

Angular deviation $\theta = ?$

Solution:

$$\mathbf{d} = \frac{\text{Length of grating}}{\text{No. of lines}}$$

$$d = \frac{1 \times 10^{-2}}{2000} \, \text{m}$$

$$d = 0.5 \times 10^{-2} \times 10^{-3} \text{ m}$$

 $d = 5.0 \times 10^{-6} \text{ m}$

For a diffraction grating:

$$Sin\theta = m\lambda$$

$$5 \times 10^{-6} \operatorname{Sin}\theta = 3 \times 5400 \times 10^{-10}$$

$$\therefore \sin \theta = \frac{3 \times 5400 \times 10^{-10}}{5 \times 10^{-6}}$$

$$\sin\theta = \frac{1.62 \times 10^{-6}}{5 \times 10^{-6}}$$

$$Sin\theta = 0.324$$

$$\therefore \theta = \operatorname{Sin}^{-1} 0.324$$

$$\theta = 18.9^{\circ}$$

Angular deviation in third order is 18.9°.

15. Interference fringes were produced by two slits 0.25 mm apart on the screen 150 mm form the slits. If ten fringes occupy 3.275 mm. What is the wavelength of the light produced fringes.

DATA:

$$\overline{d} = 0.25 \text{ m.m} = 0.25 \times 10^{-3} \text{ m.}$$

$$L = 150 \text{ m.m} = 150 \times 10^{-3} \text{ m.}$$

Number of fringes = 10

then
$$\Delta y = \frac{3.275}{10}$$
 m.m

$$\Delta y = 0.3275 \text{ m.m}$$

$$\Delta y = 0.3275 \times 10^{-3} \text{ m}.$$

$$\lambda = ?$$

SOLUTION:

In young's double slits experiments, the fringe spacing is:

$$\Delta x = \frac{L\lambda}{d}$$

$$\lambda = \frac{\Delta X - d}{L}$$

$$\lambda = \frac{0.3275 \times 10^{-3} \times 0.25 \times 10^{-3}}{150 \times 10^{-3}}$$

$$\lambda = 5.45 \times 10^{-7} \text{ meter}$$

or
$$\lambda = \frac{5.45 \times 10^{-7}}{10^{-10}}$$

$$\lambda = 5.45 \times 10^3 \,\mathrm{A}^{\circ}.$$

$$\lambda = 5450 \text{ A}^{\circ}$$

If the radius of the 14th bright Newton's ring is 1 mm and the radius of curvature **16.** of the lens is 125 mm. calculate the wavelength of the light.

DATA:

Radius of 14^{th} ring $r_{14} = 1$ m.m = $1 \times$ $10^{-3} \, \text{m}$

Radius of curvature of the lens

 $o R = 126 \text{ m.m} = 126 \times 10^{-3} \text{ m}$

Number of ring N = 14

Wavelength of light $\lambda = ?$

Solution:

Radius of Nth Newton's ring is given by:

$$r_N = \sqrt{R \lambda \left(N - \frac{1}{2}\right)}$$

$$r_{14} = \sqrt{R \lambda \left(14 - \frac{1}{2}\right)}$$

$$1 \times 10^{-3} = \sqrt{125 \times 10^{-3} \lambda \left(14 - \frac{1}{2}\right)}$$
$$1 \times 10^{-3} = \sqrt{125 \times 10^{-3} \lambda \times \frac{27}{2}}$$

Squaring both sides we get:

$$(1 \times 10^{-3})^2 = 125 \times 10^{-3} \times 13.5\lambda$$

$$\therefore 1 \times 10^{-6} = 1.6875 \lambda$$

$$\lambda = \frac{1 \times 10^{-6}}{1.6875}$$

$$\lambda = 5.9259 \times 10^{-7} \text{ m}$$

if the diffraction grating produced first order spectrum of light of wavelenght of 6 \times 10⁻⁷ m at an angle of 20⁰ from the normal, calculate the number of lines per mm.

DATA:

$$m = 1$$

$$\lambda = 6 \times 10^{-7} \,\mathrm{m}$$

$$0 = 20^{\circ}$$

(a)
$$d = ?$$

(b) No. of lines
$$/ m.m = ?$$

SOLUTION:

According to grating equation

$$d \sin \theta = m\lambda$$

$$\mathbf{d} = \frac{m \, \lambda}{\sin \theta}$$

$$\mathbf{d} = \frac{1 \times 6 \times 10^{-7}}{\sin 20^{\circ}}$$

$$d = 17.54 \times 10^{-7} \text{ m}$$

Given that length grating

$$d = 1 \times 10^{-3} \text{ m}$$

Now.

$$\mathbf{d} = \frac{\text{Length of grating}}{\text{No. of lines}}$$

No. of lines=
$$\frac{\text{Length of grating}}{d}$$

No. of lines =
$$\frac{1 \times 10^{-3}}{17.54 \times 10^{-7}}$$

No. of lines =
$$0.057 \times 10^4$$

No. of lines
$$= 570$$
 lines $/$ mm

In a double slit experiment, eight fringes occupy 2.62 mm on a screen 145 mm away from the slits. The wave length of light is 545 nm. Find the slit separation.

DATA:

 $L = 145 \text{ m.m} = 145 \times 10^{-3} \text{ m.}$ Number of fringes = 8

then
$$\Delta x = \frac{2.62}{8}$$
 m.m

$$\Delta x = 0.3275 \text{ m.m}$$

$$\Delta x = 0.3275 \times 10^{-3} \text{ m}.$$

$$\lambda = 545 \text{ nm}$$

$$\lambda = 545 \times 10^{-9} \text{ m}$$

d = ?

SOLUTION:

In Young's double slit experiment:

$$\Delta x = \frac{\lambda L}{d}$$

$$0.3275 \times 10^{-3} = \frac{545 \times 10^{-9} \times 145 \times 10^{-3}}{d}$$

$$d = \frac{545 \times 10^{-9} \times 145 \times 10^{-3}}{0.3275 \times 10^{-3}}$$

$$d = \frac{7.9025 \times 10^{-8}}{0.3275 \times 10^{-3}}$$

$$d = 2.413 \times 10^{-4} \text{ m}$$

19 Green light of wavelength 5400 °A is a diffracted by grating having 2000 lines/cm. Compute the angular deviation of the third order image.

Data:

Wavelength of light $\lambda = 5400 \text{ A}^{\circ}$

$$\lambda = 5400 \times 10^{-10}$$

m

Number of lines ruled = 2000

lines/cm

Order of image m = 3

Angular deviation $\theta = ?$

Solution:

$$\mathbf{d} = \frac{\text{Length of grating}}{\text{No. of lines}}$$

$$d = \frac{1 \times 10^{-2}}{2000} \, \text{m}$$

$$d = 0.5 \times 10^{-2} \times 10^{-3} \text{ m}$$

$$d = 5.0 \times 10^{-6} \text{ m}$$

For a diffraction grating:

$$Sin\theta = m\lambda$$

$$5 \times 10^{-6} \text{ Sin}\theta = 3 \times 5400 \times 10^{-10}$$

$$\therefore \sin \theta = \frac{3 \times 5400 \times 10^{-10}}{5 \times 10^{-6}}$$

$$\sin\theta = \frac{1.62 \times 10^{-6}}{5 \times 10^{-6}}$$

$$Sin\theta = 0.324$$

$$\therefore \theta = \operatorname{Sin}^{-1} 0.324$$

$$\theta = 18.9^{\circ}$$

Angular deviation in third order is 18.9°.

A parallel beam of X-ray is diffracted by rock salt. The first order maximum is being obtained when the glancing angle of incidence is 6 degree and 5 minutes. If the distance between the atomic panes of the crystal is 2.81×10^{-10} m, calculate the wavelength of the radiation.

Data:

$$m = 1$$

 $\theta = 6^{\circ}5' = \left(6^{\circ} + \frac{5}{60}\right) = 6.08^{\circ}$
 $d = 2.81 \times 10^{-10} \text{ m.}$
 $\lambda = ?$

Solution:

According to Bragg's law, $2d \sin\theta = m\lambda$

$$\lambda = \frac{2d \sin \theta}{m}$$

$$\lambda = \lambda = \frac{2 \times 2.81 \times 10^{-10} \times \sin 6.08^{\circ}}{1}$$

$$\lambda = 5.62 \times 10^{-10} \times 0.106$$

$$d = 0.6 \times 10^{-10} \text{ m}$$

