PROBLEMS

1. **In a hydraulic press a force of 20 N is applied to a piston of area 0.20 m². The area of the other piston is 2.0 m². What is (a) the pressure transmitted through the fluid; (b) the force on the piston?**

   Data:

   $F_1 = 20$ N
   $A_1 = 0.20$ m^2
   $A_2 = 2.0$ m^2
   $P_1 = ?$
   $F_2 = ?$

   Solution:

   For P_1.
   By the definition of pressure.

   $$P = F / A$$

   Or

   $$P_1 = F_1 / A_1$$
   $$P_1 = 20 / 0.2$$
   $$P_1 = 100 \text{ Pa}$$

   For F_2.
   By the mechanical advantage of hydraulic press.

   $$M.A = A_2 / A_1 = F_2 / F_1$$
   $$2.0 / 0.20 = F_2 / 20$$
   $$10 = F_2 / 20$$
   $$F_2 = 10 \times 20$$
   $$F_2 = 200 \text{ N}$$

2. **The pressure in a water pipe in the ground floor of a building is 4×10^5 Pa but three floors up it is only 2×10^5 Pa. What is the height between the ground floor and the third floor? The water in the pipe may be assumed to be stationary; density of water = 1×10^3 Kg/m^3.**

   Data:

   $P_1 = 4 \times 10^5$ Pa
   $P_2 = 2 \times 10^5$ Pa
   $h = ?$
   $\rho = 1\times10^3$ Kg/m^3

   Solution:

   Change in pressure.

   $$\Delta P = P_1 - P_2$$
   $$\Delta P = 4 \times 10^5 - 2\times10^5$$
   $$\Delta P = 2\times10^5$$

   Now by pressure of fluid.

   $$\Delta P = \rho\, g\, h$$
   $$2\times10^5 = (1 \times 10^3)\ (9.8)\ h$$

$$h = 2 \times 10^5 / (1 \times 10^3)(9.8)$$
$$h = 2 \times 10^5 / 9800$$
$$h = 20.4 \text{ m}$$

3. **The small piston of hydraulic press has an area of 10.0 cm^2. If the applied force is 50.0 N, what must the area of the large piston to exert a pressing force of 4800N?**

   Data:

   $A_1 = 10.0 \text{ cm}^2$
   $F_1 = 50.0 \text{ N}$
   $A_2 = ?$
   $F_2 = 4800\text{N}$

   Solution:

   By the mechanical advantage of hydraulic press.

$$M.A = A_2 / A_1 = F_2 / F_1$$
$$A_2 / 10.0 = 4800 / 50.0$$
$$A_2 / 10.0 = 96$$
$$A_2 = 96 \times 10.0$$
$$A_2 = 960 \text{ cm}^2$$

4. **The mechanical advantage of a hydraulic jack is 420. Find the weight of the heaviest automobile that can be lifted by an applied force of 55 N.**

   Data:

   $M.A = 420$
   $W = F_2 = ?$
   $F_1 = 55 \text{ N}$

   Solution:

   By the mechanical advantage of hydraulic press.

$$M.A = A_2 / A_1 = F_2 / F_1$$
$$420 = F_2 / 55$$
$$F_2 = 420 \times 55$$
$$F_2 = 23100 \text{ N}$$

5. **A flat-bottom river barge is 30 ft wide, 85 ft long and 15 ft. deep, (a) how many m^3 of water will displace while the top stays 1 m above the water? (b) What load in tons will the barge contain under these conditions if the empty barge weighs 160 tons in dry dock?**

   Data:

   $w = 30 \text{ ft}$
   $l = 85 \text{ ft}$
   $d = 15 \text{ ft}$
   $V = ? \ (m^3)$
   Top of barge above water = 1 m = 1 x 3 ft = 3 ft
   Height of barge in water = h = d – top of barge above water = 15 – 3 = 12 ft
   Load of barge carry = m_{carry} = ? (Tons)
   Mass of empty barge = m_b = 160 tons

Weight density of water = ρ_w = 62.4 Lb / ft^3

Solution:

For "V".
By the definition of volume.

$$V = w \times l \times h$$
$$V = 30 \times 85 \times 12$$
$$V = 30600\ ft^3 = 3.06 \times 10^4\ ft^3 = 3.1 \times 10^4\ ft^3$$

In m^3.
We know that.
1 ft^3 = 0.0283 m^3

$$V = (3.1 \times 10^4) \times 0.0283 = 877.3\ m^3$$

For "m_{carry}".
Upthrust on barge.

$$F_{up} = \rho\ g\ V$$

Where ρ g = weight density = ρ_W

$$F_{up} = \rho_W\ V$$
$$F_{up} = 62.4 \times 3.1 \times 10^4$$
$$F_{up} = 1934400\ Lb = 1.93 \times 10^6\ Lb$$

Weight of empty barge.
1ton = 2000 Lb

$$W_b = 160 \times 2000 = 3.2 \times 10^5\ Lb$$

Weight carried by barge in water.

$$F_{carry} = F_{up} - W_b$$
$$F_{carry} = 1.93 \times 10^6 - 3.2 \times 10^5$$
$$F_{carry} = 1610000\ Lb = 1.61 \times 10^6\ Lb$$

Now

$$m_{carry} = 1.61 \times 10^6 / 2000$$
$$m_{carry} = 805\ tons$$

6. A canal lock gate is 20 m wide and 10 m deep. Calculate the thrust acting on it assuming that the water in the canal is in level with the top of the gate. Density of water is 1000 kg/m^3

Data:

w = 20 m
d = h = 10 m
F = ?
Density of water = ρ = 1000 kg / m^3

Solution:

By the definition of pressure.

$$P = F / A$$

Therefore.

$$F = P\ A$$

Or

$$F = P\ (w \times h) \text{ --------(1)}$$

For "P".
As the water in canal is in level with the top of gate, therefore.

$$P = (P_{top} + P_{bot}) / 2$$

Where.

$$P_{top} = \rho g \text{ (height of water at top of gate)}$$
$$P_{top} = 1000 \times 9.8 \times 0 = 0 \text{ Pa}$$
$$P_{bot} = \rho g \text{ (height of water at bottom of gate)}$$
$$P_{bot} = 1000 \times 9.8 \times 10 = 9.8 \times 10^4 \text{ Pa}$$

Hence.

$$P = (0 + 9.8 \times 10^4) / 2$$
$$P = 9.8 \times 10^4 / 2 = 4.9 \times 10^4 \text{ Pa}$$

Now equation (1) becomes.

$$F = (4.9 \times 10^4)(20 \times 10)$$
$$F = (4.9 \times 10^4)(200)$$
$$F = 9.8 \times 10^6 \text{ N}$$

7. **A tank 4 m long, 3 m wide and 2 m deep is filled to the brim with paraffin (density 800 kg/m^3). Calculate the pressure on the base? What is the thrust on the base?**

<u>Data:</u>

l = 4 m
w = 3 m
d = h = 2 m
P = ?
F = ?

<u>Solution:</u>

For "P".
By the pressure of fluid.

$$P = \rho g h$$
$$P = 800 \times 9.8 \times 2$$
$$P = 15680 \text{ Pa}$$

For "F".
By the definition of pressure.

$$P = F / A$$
$$F = P A = P (l \times w)$$
$$F = 15680 (4 \times 3)$$
$$F = 188160 \text{ N}$$

8. **A rectangular boat is 4.0 m wide, 8.0 m long, and 3.0 m deep. (a) How much water will it displace if the top stays 1.0 m above the water? (b) What load will the boat contain under these conditions if the empty boat weighs 8.60×10^4 N in dry dock? (Weight density of water = 9800 N/m^3)**

<u>Data:</u>

w = 4.0 m
l = 8.0 m
d = 3.0 m
V = ?
Top of the boat above the water = 1.0 m
Height of boat in water = h = d – top of boat above water = 3.0 – 1.0 = 2 m

Load of boat carry = W_{carry} = ?
Weight of empty boat = $W_b = 8.60 \times 10^4$ N
Weight density of water = $\rho_W = 9800\ N/m^3$

Solution:

For "V".
By the definition of volume.

$$V = w \times l \times h$$
$$V = 4.0 \times 8.0 \times 2.0$$
$$V = 64.0\ m^3$$

For "W_{carry}".
Upthrust on boat.

$$F_{up} = \rho\ g\ V$$

Where ρ g = weight density = ρ_W

$$F_{up} = \rho_W\ V$$
$$F_{up} = 9800 \times 64.0$$
$$F_{up} = 6.272 \times 10^5\ N$$

Now

$$W_{carry} = F_{up} - W_b$$
$$W_{carry} = 6.272 \times 10^5 - 8.60 \times 10^4$$
$$W_{carry} = 5.41 \times 10^5\ N$$

9. A hot air balloon has a volume of 2200 m^3. The density of air at temperature of 20 °C is 1.205 kg/m^3. The density of the hot air inside the balloon at a temperature of 100 °C is 0.946 kg/m^2. How much weight can the hot air can lift?

Data:

$V = 2200\ m^3$
$T_1 = 20\ ^oC$
$P_1 = 1.205\ kg/m^3$
$T_2 = 100\ ^oC$
$\rho_2 = 0.946\ kg/m^3$
W_{lift} = ?

Solution:

W_{lift} = upthrust of cold air – thrust of hot air

$$W_{lift} = F_{up} - F$$
$$W_{lift} = \rho_1\ g\ V - \rho_2\ g\ V$$
$$W_{lift} = (\rho_1 - \rho_2)\ g\ V$$
$$W_{lift} = (1.205 - 0.946) \times 9.8 \times 2200$$
$$W_{lift} = 0.259 \times 21560$$
$$W_{lift} = 5584\ N$$

10 **A spherical balloon has a radius of 7.15 m and is filled with helium. How large a cargo can it lift, assuming that the skin and structure of the balloon have a mass of 930 kg? Neglect the buoyant force on the cargo volume itself.**

Data:

$r = 7.35\ m$
$m_{cargo} = ?$
$m_{ballon} = 930\ kg$
$\rho_{He} = 0.179\ kg / m^3$
$\rho_{air} = 1.29\ kg / m^3$

Solution:

$$F_{up} = W_{cargo} + W_{ballon} + W_{He}$$

$$\rho_{air}\ g\ V = m_{cargo}\ g + m_{ballon}\ g + \rho_{He}\ g\ V$$

$$\rho_{air}\ V = m_{cargo} + m_{ballon} + \rho_{He}\ V$$

Where
$V = 4/3\ \pi\ r^3$
Therefore

$$\rho_{air}\ (4/3\ \pi\ r^3) = m_{cargo} + m_{ballon} + \rho_{He}\ (4/3\ \pi\ r^3)$$

$$m_{cargo} = \rho_{air}\ (4/3\ \pi\ r^3) - \rho_{He}\ (4/3\ \pi\ r^3) - m_{ballon}$$

$$m_{cargo} = (\rho_{air} - \rho_{He})\ (4/3\ \pi\ r^3) - m_{ballon}$$

$$m_{cargo} = [(1.29 - 0.179) \times 4/3 \times (3.142)\ (7.35)^3] - 930$$

$$m_{cargo} = [(1.111) \times 4/3 \times (3.142)\ (397.1)] – 930$$

$$m_{cargo} = (5540 / 3) - 930$$

$$m_{cargo} = 1846 - 930$$

$$m_{cargo} = 916\ kg = 920\ kg$$

Worked example#6.1

A hydraulic system consists of two connected cylinders A and B. Cylinder A has a piston with a radius of 5cm and cylinder B has a piston with a radius of 10cm. A force of 200N is applied to cylinder A. Calculate the force exerted by cylinder B.

Data:

Radius of cylinder A = r_1 = 5cm

Radius of cylinder B = r_2 = 10cm

Force on cylinder A = F_1 = 200N

Force on cylinder A = F_1 = 200N

Force on cylinder B = F_2 =?

Solution:

According to Pascal's law

$$P_1 = P_2$$

Or

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{F_1}{\pi\, r_1^2} = \frac{F_2}{\pi\, r_2^2}$$

$$\frac{F_1}{r_1^2} = \frac{F_2}{r_2^2}$$

$$F_2 = \frac{F_1\, r_2^2}{r_1^2}$$

$$F_2 = \frac{(200)(10)^2}{(5)^2} = 800\text{N}$$

Worked example#6.2

When a crown of mass 14.7kg is submerged in water, an accurate scale reads only 13.4kg. Is the crown made of gold? The density of gold is 19.3gm/cm^3 and the density of water is 1gm/cm^3.

Data:

Mass of crown = m_1 = 14.7kg

Mass of crown in water = m_2 = 13.4kg

Density of gold = ρ_g = 19.3gm/cm^3

Density of water = ρ_w = 1gm/cm^3

Density of water = ρ_c =?

Solution:

The relative density of the crown is

$$\frac{\rho_c}{\rho_w} = \frac{W_1}{W_1 - W_2}$$

$$\frac{\rho_c}{\rho_w} = \frac{m_1 g}{m_1 g - m_2 g}$$

$$\frac{\rho_c}{\rho_w} = \frac{m_1}{m_1 - m_2}$$

$$\frac{\rho_c}{1} = \frac{14.7}{14.7 - 13.4}$$

$$\rho_c = 11.3 gm/cm^3$$

The crown is not made of gold.

Worked example#6.3

What volume of helium is needed if a balloon is to lift a load of 185kg? The density of helium is 0.179kg/m^3 and the density of air is 1.29kg/m^3.

Data:

Mass of the load = m_1 = 185kg

Mass of helium = m_2

Density of the air = ρ_a = 1.29kg/m^3

Density of helium = ρ_h = 0.179kg/m^3

The volume of helium = V =?

Solution:

The buoyant force on the balloon must be equal to the weight of the air displaced

$$F_b = W$$

$$\rho_a g V = (m_1 + m_2)g$$

$$\rho_a V = (185 + \rho_h V)$$

$$\rho_a V - \rho_h V = 185$$

$$V(\rho_a - \rho_h) = 185$$

$$V(1.29 - 0.179) = 185$$

$$1.111V = 185$$

$$V = \frac{185}{1.111} = 166.5 m^3$$

Worked example#6.4

A solid, square pine wood raft measures 4.5 m on a side and it is thick.

(a) Determine whether the raft floats in water and (b) if so, how much of the raft is beneath the surface in the distance h?

Data:

Length of the side = L = 4.5m

Thickness = d = 0.35 m

Solution:

(a) To determine whether the raft floats, we will compare the weight of the raft to the maximum possible buoyant force and see whether there could be enough buoyant force to balance the weight. If so, then the value of the distance h can be obtained by utilizing the fact that the floating raft is in equilibrium, with the magnitude of the buoyant force equalizing the raft's weight.

The weight of the raft is

$$\mathbf{W = mg}$$

$$\mathbf{W = \rho V g}$$

$$\mathbf{W = \rho\,(L \times L \times d)\,g}$$

$$\mathbf{W = 550 \times (4.5 \times 4.5 \times 0.35) \times 9.8}$$

$$\mathbf{W = 550 \times 7.1 \times 9.8 = 38269N}$$

The maximum buoyant force on the raft is

$$\mathbf{F_b = \rho g V}$$

$$\mathbf{F_b = 1000 \times 9.8 \times 7.1 = 69580N}$$

Since the maximum possible buoyant force exceeds the 38269 N weight of the raft, the raft will float only partially submerged at a distance h beneath the water.

(b) The buoyant force balances the raft's weight is

$$\mathbf{F_b = W}$$

$$\mathbf{\rho g V = W}$$

$$\mathbf{1000 \times 9.8 \times (4.5 \times 4.5 \times h) = 38269}$$

$$\mathbf{h = \frac{38269}{198450} = 0.19m}$$

Worked example#6.5

The base of the insect's leg is approximately spherical in shape, with a radius of about 2.1×10^{-5}m. The 3.2×10^{-6}kg mass of the insect is supported equally by its six legs. Estimate the angle for an insect on the surface of water. Assume the water temperature is 20°C.

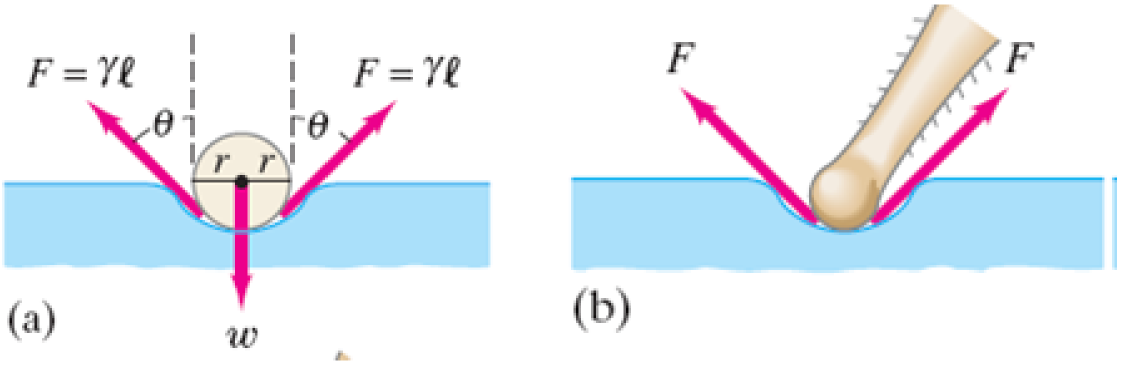
Data:

$r = 2.1 \times 10^{-5}$m

$m = 3.2 \times 10^{-6}$kg

Surface tension = γ = 0.072N/m

θ =?



Since the insect is in equilibrium, the upward surface tension force is equal to the effective pull of gravity downward on each leg. For each leg, we assume the surface tension force acts all around the circle of radius r, at an angle θ. Only the vertical component, γ Cosθ, acts to balance the weight mg. So, we set the length l equal to the circumference of the circle $l = 2\pi r$ Then the net upward force is due to surface tension is $F_y = \gamma l \cos\theta$. We set this surface tension force y equal to one-sixth the weight of the insect since it has six legs:

$$F_y = \frac{1}{6} w$$

$$\gamma l \cos\theta = \frac{1}{6} mg$$

$$\cos\theta = \frac{mg}{6\gamma l}$$

$$\cos\theta = \frac{mg}{6(2\pi r)\gamma} \qquad (l = 2\pi r)$$

$$\cos\theta = \frac{(3.2 \times 10^{-6})(9.8)}{6 \times 2(3.14)(2.1 \times 10^{-5}) \times (0.072)} = 0.55$$

$$\theta = \cos^{-1} 0.55 = 56.63°$$

Worked example#6.5

Water flows through a fire hose of diameter 6.5cm at a speed of 5.99m/s. Find the flow rate of the fire hose.

Data:

v = 5.99m/s

D = 6.5cm

r = D/2 = 3.25cm = 0.0325m

Flow rate = Q =?

Solution:

The flow rate of the hose pipe is

$$Q = Av$$

$$Q = \pi r^2 v$$

$$Q = 3.14 \times (0.0325)^2 \times 5.99$$

$$Q = 0.019866 m^3/s$$

$$Q = 0.019866 \times 1000 \times 60 = 1192 L/min$$