BOOK NUMERICALS:

Numerical 1.
Two spherical raindrops of equal size are falling through air at a velocity of 0.08 m/s. If the drops join together forming a large spherical drop, what will be the new terminal velocity?

Data.

v_t = 0.08 m/s

v_T = ?

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Solution.

We know that

$$v_t = \frac{2r^2\rho g}{9\eta}$$

Also

$$v_T = \frac{2R^2\rho g}{9\eta}$$

By dividing

$$\frac{v_T}{v_t} = \frac{R^2}{r^2}$$

$$\frac{v_T}{v_t} = \left(\frac{R}{r}\right)^2 ----(1)$$

When two similar spheres combine together to form a single large sphere, then.

$$V_1 + V_2 = V$$

$$\frac{4}{3}\pi r^3 + \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$2r^3 = R^3$$

$$\frac{R^3}{r^3} = 2$$

$$\frac{R}{r} = (2)^{\frac{1}{3}}$$

Now equation (1).

$$\frac{v_T}{v_t} = \left(\frac{R}{r}\right)^2$$

$$\frac{v_T}{0.08} = \left[(2)^{\frac{1}{3}}\right]^2$$

$$v_T = 0.08\ (2)^{\frac{2}{3}}$$

$$v_T = 0.08\ (1.587)$$

$$v_T = 0.126 = 0.13\ m/s$$

Numerical 2.
Calculate the viscous drag on a drop of oil of 0.1 mm radius falling through air at this terminal velocity. (Viscosity of air = 1.8x10 Pa. s; density of oil = 850 kg/m³)

Data.

r = 0.1 mm = 0.1 x 10^{-3} m
η = 1.8 x 10^{-5} Pa. s
ρ = 850 kg / m^3
F_d = ?
v_t = ?

Solution.

For "v_t".
We know that

$$v_t = \frac{2r^2\rho g}{9\eta}$$

$$v_t = \frac{2(0.1\ x\ 10^{-3})^2(850)(9.8)}{9(1.8\ x\ 10^{-5})}$$

$$v_t = \frac{2(1\ x\ 10^{-8})(850)(9.8)}{1.62\ x\ 10^{-4}}$$

$$v_t = \frac{1.66\ x\ 10^{-4}}{1.62\ x\ 10^{-4}}$$

$$v_t = \ 1.024\ m/s$$

For "F_d".

By Stoke`s law

$$F_d = 6\ \pi\ \eta\ r\ v_t$$

$$F_d = 6\ (3.142)(1.8\ x\ 10^{-5})(0.1\ x\ 10^{-3})(1.024)$$

$$F_d = 3.48\ x\ 10^{-8}\ N$$

Numerical 3.

What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume 300 m^3. Assume air density remains constant.

Data.

A = ?

v = 3.0 m / sec

t = 15 minutes = 15 x 60 = 900 sec

V = 300 m^3

SOLUTION.

By the rate of volume of fluid.

$$\frac{V}{t} = Av$$

$$A = \frac{V}{vt}$$

$$A = \frac{300}{3\ x\ 900}$$

$$A = \frac{300}{2700}$$

$$A = 0.11\ m^2$$

Numerical 4.
Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0 cm diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6 cm diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

Data.

v_1 = 0.50 m/s
d_1 = 4.0 cm = 0.04 m
$r_1 = d_1 / 2$ = 0.04 / 2 = 0.02 m
P_1 = 3.0 atm = 3.0 (1.01 x 10^5) = 3.03 x 10^5 Pa
v_2 = ?
d_2 = 2.6 cm = 0.026 m
$r_2 = d_2 / 2$ = 0.026 / 2 = 0.013 m
h = 5.0 m

Solution.

For "v_2".
By the equation of continuity.

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$v_2 = \frac{\pi\, r_1^2 v_1}{\pi\, r_2^2}$$

$$v_2 = \frac{r_1^2 v_1}{r_2^2}$$

$$v_2 = \frac{(0.02)^2 (0.5)}{(0.013)^2}$$

$$v_2 = \frac{(4\ x\ 10^{-4})(0.5)}{1.69\ x\ 10^{-4}}$$

$$v_2 = \frac{2\ x\ 10^{-4}}{1.69\ x\ 10^{-4}}$$

$$v_2 = 1.18 = 1.2\ \frac{m}{s}$$

For "P_2".

By the Bernoulli`s equation.

$$(P_1 - P_2) = \frac{1}{2}\rho\, v_2^2 - \frac{1}{2}\rho\, v_1^2 + \rho\, g\, h_2 - \rho\, g\, h_1$$

$$P_1 - P_2 = \frac{1}{2}\rho\, v_2^2 - \frac{1}{2}\rho\, v_1^2 + \rho\, g\, h$$

$$P_1 - P_2 = \rho\, (\frac{1}{2}\, v_2^2 - \frac{1}{2}\, v_1^2 + g\, h)$$

$$3.03\, x\, 10^5 - P_2 = 1000\left[\frac{1}{2}(0.12)^2 - \frac{1}{2}(0.5)^2 + 9.8(5)\right]$$

$$3.03\, x\, 10^5 - P_2 = 1000\left[\frac{1}{2}(0.0144) - \frac{1}{2}(0.25) + 49\right]$$

$$3.03\, x\, 10^5 - P_2 = 1000\, [7.2\, x\, 10^{-3} - 0.125 + 49]$$

$$3.03\, x\, 10^5 - P_2 = 1000\, (48.9)$$

$$3.03\, x\, 10^5 - P_2 = 4.89\, x\, 10^4$$

$$P_2 = 3.03\, x\, 10^5 - 4.89\, x\, 10^4$$

$$P_2 = 2.541\, x\, 10^5\, Pa$$

In atm.

$$P_2 = \frac{2.541\, x\, 10^5}{1.01\, x\, 10^5}$$

$$P_2 = 2.51\, atm$$

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Numerical 5.
What is the volume rate of flow of water from a 1.85 cm diameter faucet if the pressure head is 12 m?

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Data.

V / t = ?
d = 1.85 cm = 0.0185 m
r = d / 2 = 0.0185 / 2 = 9.25 x 10^{-3} m
h = 12 m

SOLUTION.
By the volume rate of flow of fluid.

$$\frac{V}{t} = A\,v ----(1)$$

For "A".

$$A = \pi\, r^2$$

$$A = 3.142\,(9.25\,x\,10^{-3})^2$$

$$A = 2.69\,x\,10^{-4}\,m^2$$

For "v".
By the energy transformation.

$$K.E = P.E$$

$$\frac{1}{2}\,m\,v^2 = mgh$$

$$v = \sqrt{2\,g\,h}$$

$$v = \sqrt{2\,(9.8)\,(12)}$$

$$v = 15.33\,\frac{m}{s}$$

Now equation (1).

$$\frac{V}{t} = (2.69\,x\,10^{-4}\,)(15.33)$$

$$\frac{V}{t} = 4.13\,x\,10^{-3}\,\frac{m^3}{s}$$

Numerical 6.
The stream of water emerges from a faucet 'neck down' as it falls. The cross-sectional area is 1.2 cm^2 and 0.35 cm^2. The two levels are separated by a vertical distance of 45 mm as shown in the figure. At what rate does water flow from the tap?

Data.

$A_1 = 1.2\ cm^2$
$A_2 = 0.35\ cm^2$
h = 45 mm = 4.5 cm
V / t = ?

Solution.

By the volume rate of flow of fluid.

$$\frac{V}{t} = A\,v$$

$$\frac{V}{t} = A_2\,v - - - - (1)$$

For "v".
By using the formula.

$$v = \sqrt{\frac{2\,g\,h\,A_2^2}{A_1^2 - A_2^2}}$$

$$v = \sqrt{\frac{2\,(980)\,(4.5)\,(0.35)^2}{(1.2)^2 - (0.35)^2}}$$

$$v = \sqrt{\frac{2\,(980)\,(4.5)\,(0.1225)}{(1.44) - (0.1225)}}$$

$$v = \sqrt{\frac{1080.45}{1.3175}}$$

$$v = \sqrt{820.07}$$

$$v = 28.63\ \frac{cm}{s}$$

Now equation (1).

$$\frac{V}{t} = A_2\, v$$

$$\frac{V}{t} = (1.2)(28.63)$$

$$\frac{V}{t} = 34.4\ \frac{cm^3}{s}$$

Numerical 7.
Water leaves the jet of a horizontal hose at 10 m/s. If the velocity of water within the hose is 0.40 m/s, calculate the pressure within the hose. The density of water is 1000 kg/m^2 and the atmospheric pressure is 100000 Pa.

Data.

v_1 = 10 m / s

v_2 = 0.40 m / s

P_2 = ?

ρ = 1000 kg / m^3

P_1 = 100000 Pa

Solution.

By the Bernoulli`s equation.

$$(P_1 \ - P_2) = \frac{1}{2}\rho\, v_2^2 \ - \frac{1}{2}\rho\, v_1^2 + \rho\, g\, h_2 - \ \rho\, g\, h_1$$

Also

$$(P_1 \ - P_2) = \frac{1}{2}\rho\, v_2^2 \ - \frac{1}{2}\rho\, v_1^2$$

$$(P_1 \ - P_2) = \frac{1}{2}\rho\, (v_2^2 \ - \ v_1^2)$$

$$(100000 \ - P_2) = \frac{1}{2}1000\ [(0.4)^2 - (10)^2]$$

$$(100000 \ - P_2) = 500\ [0.16 - 100]$$

$$(100000 \; - P_2) = 500 \; [-99.84]$$

$$(100000 \; - P_2) = -49920$$

$$P_2 = 100000 + 49920$$

$$P_2 = 1.49 \; x \; 10^5 \; Pa = 1.5 \; x \; 10^5 \; Pa$$

Numerical 8.
What is the maximum weight of an aircraft with a wing area of 50 m² flying horizontally, is the velocity of the air over the upper surface of the wing is 150 m/s and that the lower surface 140 m/s? The density of air is 1.29 kg/m³

Data.

W = ?
A = 50 m²
v_1 = 150 m / s
v_2 = 140 m / s
ρ = 1.29 kg / m³

Solution.
By the Bernoulli`s equation.

$$P_1 \; - P_2 = \frac{1}{2} \rho \, v_2^2 \; - \; \frac{1}{2} \rho \, v_1^2 \; - - - - (1)$$

We know that.

$$P = \frac{F}{A}$$

$$F = P \; A$$

or

$$W = \; \Delta P \; A$$

$$W = (P_1 - P_2) \; A$$

From equation (1)

$$W = (\frac{1}{2} \rho \, v_2^2 \; - \; \frac{1}{2} \rho \, v_1^2) \; A$$

$$W = \frac{1}{2} (1.29) \; (150^2 \; - 140^2) \; 50$$

$$W = \frac{1}{2}(1.29)\,(22500\ - 19600)\,50$$

$$W = \frac{1}{2}(1.29)\,(2900)\,50$$

$$W = \frac{1}{2}(187050)$$

$$W = 93525\,N = \ 9.35\,x\,10^{4}\,N$$

Numerical 9.-
A liquid flows through a pipe with a diameter of 0.50 m at a speed of 4.20 m/s. What is the rate of flow in L/min?

DATA.

d = 0.50 m

r = d/2 = 0.50 / 2 = 0.25 m

v = 4.20 m/s

V / t = ?

SOLUTION.

By the rate of volume of fluid.

$$\frac{V}{t} = Av$$

Where A = π r^2

$$\frac{V}{t} = \pi\,r^{2}\,v$$

$$\frac{V}{t} = (3.142)(0.25)^{2}\,(4.20)$$

$$\frac{V}{t} = (3.142)(0.0625)(4.20)$$

$$\frac{V}{t} = 0.8247\,\frac{m^{3}}{s}$$

But $1m^3$ = 1000 litre
And 1s = 1/60 minutes

$$\frac{V}{t} = 0.8247\ x\ 1000\ x\ 60$$

$$\frac{V}{t} = 49486.5 = 49500\ \frac{litre}{minute}$$

Numerical 10.
Calculate the average speed of blood flow in the major arteries of the body, which have a total cross-sectional area of about 2.1 cm². Use the data of an example.

Data.

$A_{areries} = A_2 = 2.1cm^2$

$r_{aorta} = r_1 = 1.2$ cm

$v_{aorta} = v_1 = 40$ cm / s

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$v_{arteries} = v_2 = ?$

SOLUTION.
By equation of continuity.

$$A_1\, v_1 = A_2\, v_2$$

$$v_2 = \frac{A_1\, v_1}{A_2}$$

$$v_2 = \frac{\pi\, r_1^2\, v_1}{A_2}$$

$$v_2 = \frac{(3.142)(1.2)^2\,(40)}{2.1}$$

$$v_2 = \frac{(3.142)(1.44)\,(40)}{2.1}$$

$$v_2 = \frac{181}{2.1}$$

$$v_2 = 86.2\ \frac{cm}{s}$$

WORKED EXAMPLES

Worked example 7.1

Q) Calculate the terminal velocity of a raindrop of radius 0.2cm. (Density of water 1000kg/m^3 and that of air 1kg/m^3)

DATA:

r = 0.2cm = 0.002m

Density of water = ρ = 1000kg/m^3

Density of air = σ = 1kg/m^3

Coefficient of viscosity = η = 10^{-3} Pa. s

Terminal velocity = v_t =?

SOLUTION:

The terminal velocity of the raindrop is

$$v_t = \frac{2gr^2(\rho - \sigma)}{9\eta}$$

$$v_t = \frac{2 \times 9.8 \times 0.002^2 \times (1000 - 1)}{9 \times 10^{-3}}$$

$$v_t = 8.7 m/s$$

Worked example 7.2

The volume rate of an air conditioning system to be $3.84 \times 10^{-3} m^3/s$ and sent through an insulated, round conduit with a diameter of 18 cm. This calculation assumed laminar flow. (a) Was this a good assumption? (b) At what velocity would the flow become turbulent?

DATA:

Volume rate = Q = $3.84 \times 10^{-3} m^3/s$

Diameter = D = 18cm

r = D/2 = 9cm = 0.09m

η of air = 1.8×10^{-5} Pa. s

Density of air = ρ = 1.23kg/m^3

SOLUTION:

The volume rate is

$$Q = Av$$

$$Q = \pi r^2 v$$

$$v = \frac{Q}{\pi r^2}$$

$$v = \frac{3.84 \times 10^{-3}}{3.14 \times 0.09^2} = 0.15 m/s$$

Reynolds number is

$$R_e = \frac{2\rho v r}{\eta}$$

$$R_e = \frac{2 \times 1.23 \times 0.15 \times 0.09}{1.8 \times 10^{-5}} = 1845$$

Since the Reynolds number is 1835 < 2000 the flow is laminar and not turbulent. The assumption that the flow was laminar is valid.

To find the maximum speed of the air to keep the flow laminar, consider the Reynolds number

$$R_e \leq 2000$$

$$\frac{2\rho v r}{\eta} \leq 2000$$

$$v \leq \frac{2000 \times 1.8 \times 10^{-5}}{2 \times 1.23 \times 0.09} = 0.16 m/s$$

Significance:

When transferring a fluid from one point to another, it is desirable to limit turbulence. Turbulence results in wasted energy, as some of the energy intended to move the fluid is dissipated when eddies are formed. In this case, the air conditioning system will become less efficient once the velocity exceeds 0.16m / s since this is the point at which turbulence will begin to occur.

Worked example 7.3

The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4}cm, and blood flows through it at a speed of about 5×10^{-5}m/s. Estimate the number of capillaries that are in the body.

DATA:

The radius of the aorta = r_1 = 1.2cm = 0.012m

The velocity of the blood in the aorta = v_1 = 40cm/s = 0.4m/s

The radius of capillary = r_2 = 4×10^{-4}cm = 4×10^{-6}m

The velocity of the blood in capillary = v_2 = 5×10^{-4}m/s

Number of capillaries = N =?

SOLUTION:

According to the equation of continuity

$$A_1 v_1 = A_2 v_2 \quad (A_1 = \pi\, r_1{}^2, A_2 = \pi\, N\, r_2{}^2)$$

$$\pi\, r_1{}^2\, v_1 = \pi\, N\, r_2{}^2 v_2$$

$$r_1{}^2\, v_1 = N\, r_2{}^2\, v_2$$

$$N = \frac{r_1{}^2\, v_1}{r_2{}^2\, v_2}$$

$$N = \frac{0.012^2 \times 0.4}{(4 \times 10^{-6})^2 \times 5 \times 10^{-4}} = 7.2 \times 10^9$$

Worked example 7.4

Water leaves the jet of a horizontal hose at 10m /s if the velocity of water within the hose is 0.4 m/s, calculate the pressure P within the hose. (The density of water is 1000kg/m^3 and atmospheric pressure is 100000 Pa).

Data:

v_1 = 10m/s

v_2 = 0.4m/s

P_1 = 100000 Pa

P_2 =?

P = 1000kg/m^3

Solution:

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1{}^2 = P_2 + \frac{1}{2}\rho v_2{}^2$$

$$100000 + \frac{1}{2} \times 1000 \times 10^2 = P_2 + \frac{1}{2} \times 1000 \times 0.4^2$$

$$150000 = P_2 + 80$$

$$P_2 = 150000 - 80 = 1.49 \times 10^5\ \mathrm{Pa}$$

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