1. The capacitance of an air-filled parallel-plate capacitor is 1.3 pF. If the separation of the plates is doubled and wax is inserted between them. The new capacitance is 2.6 Find the dielectric constant of the wax.

<table>
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Data:

$c_1 = 1.3\ \text{pF} = 1.3 \times 10^{-12}\ C$

$c_2 = 2.6\ \text{pF} = 2.6 \times 10^{-12}\ C$

$\epsilon_r = ?$

SOLUTION:

air-filled parallel-plate capacitor

$$C_1 = \frac{A\,\epsilon_0}{d} \dots\dots(i)$$

the separation of the plates is doubled and wax
wax-filled parallel-plate capacitor

$$C_2 = \frac{A\,\epsilon_0\ \epsilon_r}{2d} \dots\dots(ii)$$

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Dividing equation (ii) by (i)

$$\frac{C_2}{C_1} = \frac{\frac{A\,\epsilon_0\ \epsilon_r}{2\,d}}{\frac{A\,\epsilon_0}{d}}$$

$$\frac{C_2}{C_1} = \frac{A\,\epsilon_0\ \epsilon_r}{2d} \times \frac{d}{A\,\epsilon_0}$$

$$\frac{C_2}{C_1} = \frac{\epsilon_r}{2}$$

$$\frac{2.6 \times 10^{-12}}{1.3 \times 10^{-12}} = \frac{\epsilon_r}{2}$$

$$2 \times 2 = \epsilon_r$$

$$4 = \epsilon_r$$

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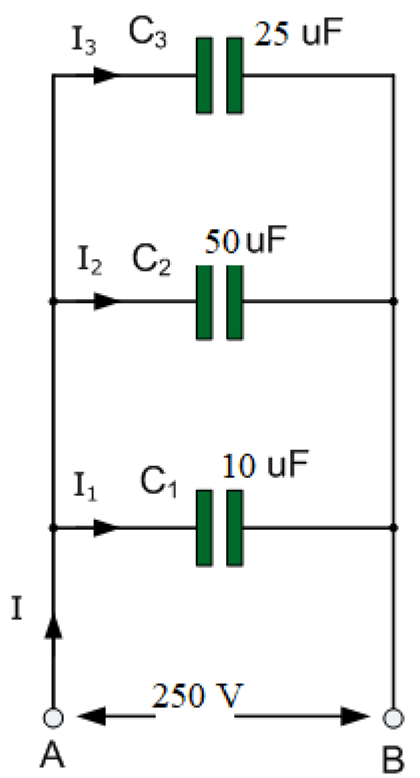
</table>

2. Three capacitors have capacitances 10, 50 and 25 μF respectively as shown in figure. Calculate (i) charge on each when connected in parallel to a 250 V supply (ii) total capacitance and (iii) potential difference across each when connected in series.



Data:

$c_1 = 10\ \mu\text{F} = 10 \times 10^{-6}\ F$

$c_2 = 50\ \mu\text{F} = 50 \times 10^{-6} F$

$c_3 = 25\ \mu\text{F} = 25 \times 10^{-6}\ F$

V = 250 V

$\epsilon_e = ?$

SOLUTION:

Three capacitor are connected in parallel capacitor

$$C_e = C_1 + C_2 + C_3$$

$$C_e = 10 + 50 + 25$$

$$C_e = 85\ \mu F$$

$$C_e = 85 \text{ x } 10^{-6}\ F$$

Total charge

$$Q_e = C_e \times V$$

$$Q_e = 85 \text{ x } 10^{-6} \times 250$$

$$Q_e = 2.125 \text{ x } 10^{-4}\ C$$

Charge on capacitor C_1

$$Q_1 = C_1 V$$

$$Q_1 = 10 \text{ x } 10^{-6} \times 250$$

$$Q_1 = 2.5 \text{ x } 10^{-3}\ C$$

Charge on capacitor C_1

$$Q_2 = C_2 V$$

$$Q_2 = 50 \text{ x } 10^{-6} \times 250$$

$$Q_1 = 12.5 \text{ x } 10^{-3}\ C$$

Charge on capacitor C_1

$$Q_3 = C_3 V$$

$$Q_3 = 25 \text{ x } 10^{-6} \times 250$$

$$Q_3 = 6.25 \text{ x } 10^{-3}\ C$$

Three capacitors are connected in series capacitor

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_e} = \frac{1}{10} + \frac{1}{25} + \frac{1}{50}$$

$$\frac{1}{C_e} = \frac{5 + 2 + 1}{50}$$

$$\frac{1}{C_e} = \frac{8}{50}$$

$$\frac{C_e}{1} = \frac{50}{8}$$

$$C_e = 6.25\ \mu F$$

$$C_e = 6.25 \text{ x } 10^{-6}\ F$$

Total charge

$$Q_e = C_e V$$

$$Q_e = 6.25 \text{ x } 10^{-6} \times 250$$

$$Q_e = 1.56 \text{ x } 10^{-3}\ C$$

Pd across capacitor C_1

$$V_1 = \frac{Q_e}{C_1} = \frac{1.56 \text{ x } 10^{-3}}{10 \text{ x } 10^{-6}}$$

$$V_1 = 156.25 \text{ V}$$

Pd across capacitor C_2

$$V_2 = \frac{Q_e}{C_2} = \frac{1.56 \text{ x } 10^{-3}}{25 \text{ x } 10^{-6}}$$

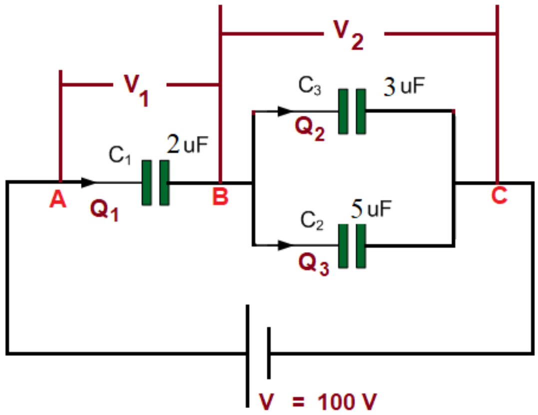
$$V_2 = 62.4 \text{ V}$$

Pd across capacitor C_3

$$V_2 = \frac{Q_e}{C_2} = \frac{1.56 \text{ x } 10^{-3}}{50 \text{ x } 10^{-6}}$$

$$V_2 = 31.2 \text{ V}$$

3 Three capacitors are connected through a potential difference of 100 volts as shown in figure. Find the charges and potential difference across each capacitor



SOLUTION:

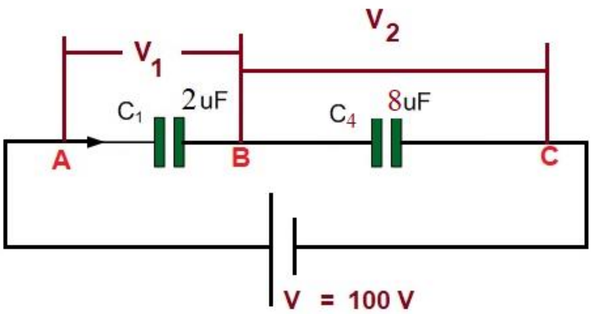
C_2 *and* C_3 *are in parallel*

$$C_4 = C_2 + C_3$$

$$C_4 = 3 + 5 = 8\ \mu F$$

$$C_4 = 8 \times 10^{-6}\ F$$

New circuit is



C_1 *and* C_4 *are in series*

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_4}$$

$$\frac{1}{C_e} = \frac{1}{2} + \frac{1}{8} = \frac{4+1}{8} = \frac{5}{8}$$

$$C_e = \frac{8}{5} = 1.6\ \mu F = 1.6 \times 10^{-6} F$$

Total Charge

$$Q_e = C_e\ V = 1.6 \times 10^{-6} \times 100$$

$$Q_e = 1.6 \times 10^{-4}\ C$$

Pd V_1 across capacitor C_1

$$V_1 = \frac{Q_e}{C_1} = \frac{1.6 \times 10^{-4}}{2 \times 10^{-6}}$$

$$V_1 = 80\ \text{V}$$

Pd V_2 across capacitor C_2

$$V_2 = \frac{Q_e}{C_4} = \frac{1.6 \times 10^{-4}}{8 \times 10^{-6}}$$

$$V_2 = 20\ \text{V}$$

Pd V_3 across capacitor C_3

$$V_3 = 20\ \text{V}$$

Charge on C_1

$$Q_1 = C_1\ V_1$$

$$Q_1 = 2 \times 10^{-6} \times 80$$

$$Q_1 = 1.6 \times 10^{-4}\ C$$

Charge on C_2

$$Q_2 = C_2\ V_2$$

$$Q_2 = 3 \times 10^{-6} \times 20$$

$$Q_2 = 6 \times 10^{-5}\ C$$

Charge on C_3

$$Q_3 = C_3\ V_2$$

$$Q_3 = 5 \times 10^{-6} \times 20$$

$$Q_3 = 10 \times 10^{-5}\ C$$

4 **Capacitor is charged through a large non-reactive resistance by a battery of constant voltage V. For this arrangement, if the capacitor has a capacitance of 10 μ F and the resistance is 1 M Ω, calculate the time taken for the capacitor to receive 90% of its final charge**

Data:

$C = 10\ \mu\text{F} = 10 \times 10^{-6}\ F$

$R = 1\ \text{M}\,\Omega = 1 \times 10^{6}\ \Omega$

SOLUTION:

(a)

$Q = 90\%\ Q_0$

$Q = 0.9\ Q_0$

$\tau = R\,C$

$\tau = (10 \times 10^{-6})(1 \times 10^{-6})$

$\tau = 10$

$Q = Q_0 \left\{1 - e^{-\left(\frac{t}{RC}\right)}\right\}$

$0.9\ Q_0 = Q_0 \left\{1 - e^{-\left(\frac{t}{10}\right)}\right\}$

$0.9 = 1 - e^{-(0.1\ t)}$

$e^{-(0.1\ t)} = 1 - 0.9$

$e^{-(0.1\ t)} = 0.1$

Taking $\log_e$ (ln) on both the side

$ln\left(e^{-(0.1\ t)}\right) = ln(0.1)$

$-0.1 \times t\ ln\,(e) = ln(0.1)$

$-0.1 \times t = -2.3$

$t = 23\ seconds$

5 **What capacitance is required to store energy of 10 kWh at a potential difference of 1000 V?**

Data:

$U = 10\ \text{KWh}$

$U = 10 \times 1000\ \left(\frac{\text{J}}{s}\right) 3600\ \text{s}$

$U = 3600000\ \text{J} = 3.6 \times 10^{7}\ J$

$V = 1000\ \text{V}$

SOLUTION:

$U = \frac{1}{2}\ C\,V^2$

$3.6 \times 10^{7} = \frac{1}{2}\ C\,(1000)^2$

$2 \times 3.6 \times 10^{7} = C \times 1 \times 10^{6}$

$\frac{7.6 \times 10^{7}}{1 \times 10^{6}} = C$

$76\ F = C$

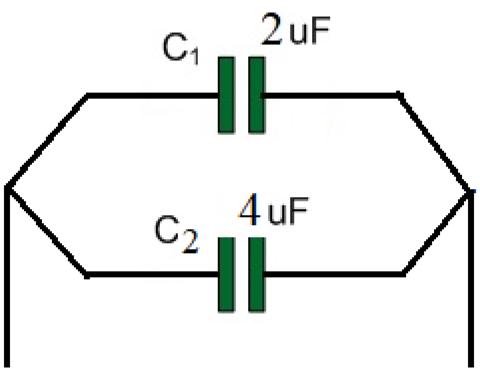
6 **A 2.0 µF capacitor and a 4.0 µF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.**

Data:

$c_1 = 2.0\ \mu F = 2.0 \times 10^{-6}\ F$

$c_2 = 4.0\ \mu F = 4.0 \times 10^{-6} F$

V = 300 V



SOLUTION:

C_1 *and* C_2 *are in parallel*

$$C_4 = C_2 + C_2$$

$$C_4 = 2 + 4 = 6\ \mu F$$

$$C_4 = 6 \times 10^{-6}\ F$$

$$U = \frac{1}{2}\ C V^2$$

$$U = \frac{1}{2}\ (6 \times 10^{-6})\ (300)^2$$

$$U = \frac{1}{2}\ (6 \times 10^{-6})\ (90000)$$

$$U = 0.27\ J$$

7 **A 12.0-V battery is connected to a capacitor, resulting in 54.0 µC of charge stored on the capacitor. How much energy is stored in the capacitor?**

Data:

$V = 12\ V$

$C = 54.0\ \mu F = 54.0 \times 10^{-6} F$

U

SOLUTION:

$$U = \frac{1}{2}\ C V^2$$

$$U = \frac{1}{2}\ (54.0 \times 10^{-6})\ (12)^2$$

$$U = \frac{1}{2}\ (54.0 \times 10^{-6})\ (144)$$

$$U = 3.88 \times 10^{-3}\ J$$

WORKED EXAMPLE

1 **A parallel plate capacitor consists of two plates with an area of 0.01 m^2 each, separated by a distance of 0.002 m. The capacitor is filled with a dielectric material having a relative permittivity (ϵ_r) of 4. Calculate the capacitance of this capacitor**

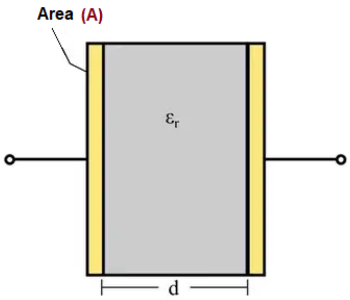
Data:

$A = 0.01\ m^2$

$d = 0.002\ m$

$\epsilon_r = 4$

$C = ?$



SOLUTION:

$$C = \frac{A\,\epsilon_0\ \epsilon_r}{d}$$

$$C = \frac{(0.01)\,(8.854 \times 10^{-12})\ (4)}{0.002}$$

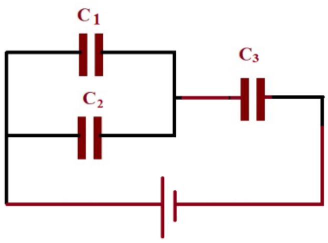
$$C = \frac{3.54 \times 10^{-13}}{0.002}$$

$$C = 1.77 \times 10^{-10}$$

$$C = 177 \times 10^{-12}$$

$$C = 177 \times pF$$

2 **Find the equivalent capacitance and charge on each of the capacitor shown in the diagram**



Data:

$C_1 = 2\mu F$

$C_2 = 4\mu F$

$C_3 = 6\mu F$

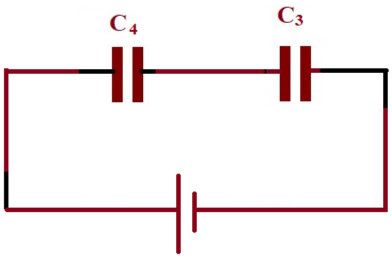
Solutions:

C_1 and C_2 are in parallel

$\therefore C_4 = C_1 + C_2$

$C_4 = 2 + 4$

$C_4 = 6\mu F$



Now, C_4 and C_3 are in series,

$$\therefore \frac{1}{Ce} = \frac{1}{C'} + \frac{1}{C_3}$$

$$\frac{1}{Ce} = \frac{1}{6} + \frac{1}{6}$$

$$\frac{1}{Ce} = \frac{1+1}{6}$$

$$\frac{1}{Ce} = \frac{2}{6} = \frac{1}{3}$$

$$Ce = 3\ \mu F$$

SELF ASSESSMENT QUESTIONS

1 **The capacitance of a capacitor formed by two parallel metal plates each 200 cm^2 in area separated by a dielectric 4 mm thick is 0.0004 microfarads. A potential difference of 20,000 V is applied. Calculate the total charge on the plates.**

Data:

$A = 200\ cm^2 = 200 \times 10^{-4}\ m^2$

$d = 4\ mm = 4 \times 10^{-3}\ m$

$C = 0.0004\ \mu F = 4.0 \times 10^{-10}\ F$

$V = 20,000 V$

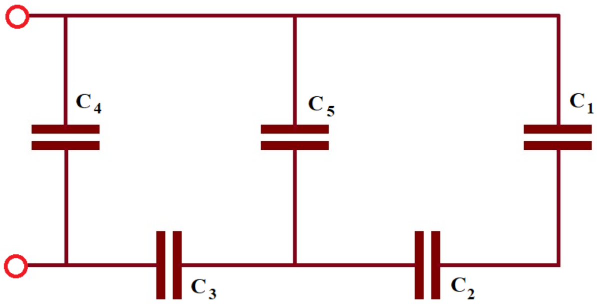
SOLUTION:

$Q = C\ V$

$Q = (4.0 \times 10^{-10})\ (20,000)$

$Q = 8.0 \times 10^{-6}\ C$

2 Find the equivalent capacitance of the circuit shown in the figure and all capacitors have the same capacitance of 15 μF



Data:

$C_1 = C_2 = C_3 = C_4 = C_5 = 15\ \mu F$

$C_e = ?$

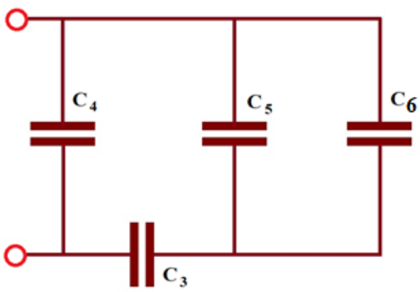
SOLUTION:

$C_5 and\ C_6\ are\ in\ series$

$$C_7 = \frac{C_1 \times C_2}{C_1 + C_2}$$

$$C_7 = \frac{15 \times 15}{15 + 15} = \frac{225}{30} = 7.5\ \mu F$$

New circuit is

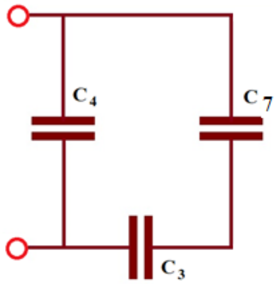


$C_5\ and\ C_6\ are\ in\ paralell$

$C_6 = C_5 + C_6$

$C_6 = 15 + 7.5 = 22.5\ \mu F$

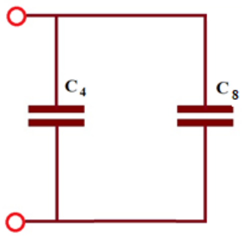
New circuit is



$C_3\ and\ C_7\ are\ in\ series$

$$C_8 = \frac{C_3 \times C_7}{C_3 + C_7}$$

$$C_8 = \frac{15 \times 22.5}{15 + 22.5} = \frac{337.5}{37.5} = 9.0\ \mu F$$

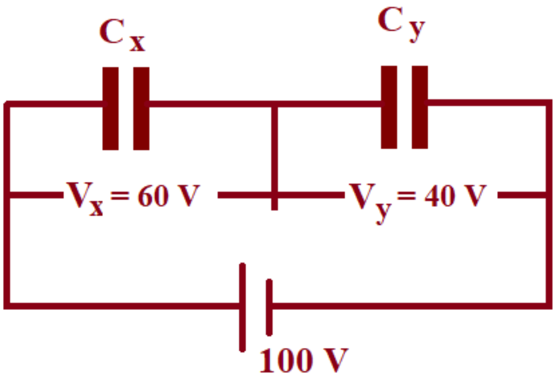


$C_4\ and\ C_8\ are\ in\ series$

$C_e = C_4 + C_8$

$C_6 = 15 + 9 = 24\ \mu F$

3 **Two capacitors X and Y are connected in series across a 100 V supply and it is observed that the potential difference between them are 60 V and 40 V respectively. A capacitor of 2 µF capacitance is now connected in parallel with X and the potential difference across Y rises to 90 volts. Calculate the capacitance of X and Y**



SOLUTION:

Capacitor x and y are connected in series

$$Q = C_x V_x$$

$$Q = C_y V_y$$

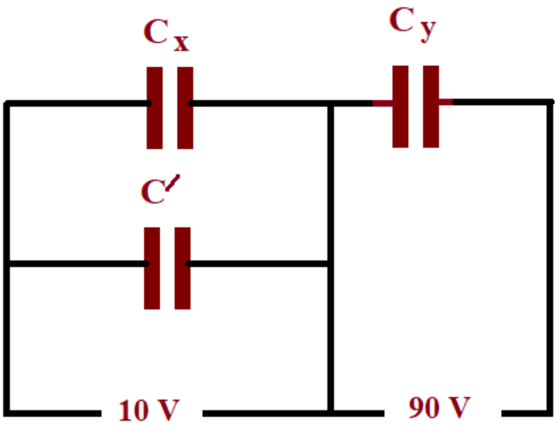
In series combination charges are equal on each capacitor

$$C_x V_x = C_y V_y$$

$$C_x = \frac{40}{60} C_y$$

$$C_x = \frac{2}{3} C_y \;\ldots\ldots\ldots\ldots (i)$$

When 2 µF capacitor is connected, Let C be the equivalent capacitance of C_x and C' Connected in parallel



$$C = C' + C_x$$

$$C = 2 + C_x \ldots\ldots\ldots (ii)$$

Now C and C_y are in series, let the charge Q' on C and Q'_y on C_y

$$Q' = Q'_y$$

$$C\,(10) = C_y\,(90)$$

$$C = C_y\,(9)$$

$$C = 9\,C_y \;\ldots\ldots\ldots (iii)$$

Comparing equation (ii) and (iii), we get

$$2 + C_x = 9\,C_y$$

Substituting the expression for C_x from equation (i) in above equation, we get

$$2 + \frac{2}{3} C_y = 9\,C_y$$

$$6 + 2C_y = 27\,C_y$$

$$6 = 27\,C_y - 2C_y$$

$$6 = 25\,C_y$$

$$0.24\,\mu F = C_y$$

Putting C_y value in equation (i)

$$C_x = \frac{2}{3}(0.24)$$

$$C_x = 0.16\,\mu F$$

4 A 2200 µF capacitor is charged to a potential difference of 9 .0 V then discharged through a 100 k Ω resistor. Calculate (a) the initial charge stored by the capacitor, (b) the time constant of the circuit, (c) calculate the potential difference after a time of 300s equal to the time constant.

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Data:

$C = 2200\ \mu\text{F} = 2200 \times 10^{-6}\ F$

$V = 9.0\ \text{V}$

$R = 100\ \text{k}\,\Omega = 100 \times 10^{3}\ \Omega$

SOLUTION:

(a)

$Q_0 = C\,V$

$Q_0 = (2200 \times 10^{-6})\,(9.0)$

$Q_0 = 19800 \times 10^{-6}\ C$

$Q_0 = 1.98 \times 10^{-2}\ C$

(b)

$\tau = R\,C$

$\tau = (100 \times 10^{3})(2200 \times 10^{-6})$

$\tau = 220\ \text{second}$

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$Q = Q_0\ e^{-\left(\frac{t}{R\,C}\right)}$

$Q = (1.98 \times 10^{-2})\ e^{-\left(\frac{300}{220}\right)}$

$Q = (1.98 \times 10^{-2})\ e^{-(1.3636)}$

$Q = (1.98 \times 10^{-2})\ (0.2557)$

$Q = 5.06 \times 10^{-3}\ C$

(c)

$V = \frac{Q}{C}$

$V = \frac{5.06 \times 10^{-3}}{2200 \times 10^{-6}}$

$V = 2.3\ V$

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5 An air-capacitor of capacitance 0.005 µ F is connected to a direct voltage of 500 V, is disconnected and then immersed in oil with a relative permittivity of 2.5. Find the energy stored in the capacitor before and after immersion.

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Data:

$C = 0.005\ \mu F = 5.0 \times 10^{-9}\ F$

$V = 500\ \text{V}$

$\epsilon_r = 2.5$

$U = ?$

SOLUTION:

$Q = C\,V$

$Q = (5.0 \times 10^{-9})\,(500)$

$Q = 2.5 \times 10^{-6}\ C$

Energy stored before immersion

$U = \frac{1}{2}\ C'\ V^2$

$U = \frac{1}{2}\ (5 \times 10^{-9})\ (500)^2$

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$U = 6.25 \times 10^{-4}\ J$

After immersion in oil, capacitance will increase

$C' = \epsilon_r\ C$

$C' = (2.5)\,(5.0 \times 10^{-9}) = 12.5 \times 10^{-9}\ F$

Pd decreases with the factor of oil permittivity

$V' = \frac{V}{\epsilon_r} = \frac{500}{2.5} = 200\ V$

Energy stored after immersion

$U = \frac{1}{2}\ C'\ V^2$

$U = \frac{1}{2}\ (12.5 \times 10^{-9})\ (200)^2$

$U = 2.5 \times 10^{-4}\ J$

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ADDITIONAL PROBLEMS

1 **Two parallel plates of an air-filled capacitor are 1.0 mm apart. What must the plate area be of the capacitance is 1 nano-farad?**

Data:	**SOLUTION:**
$d = 1.0\ mm = 1.0 \times 10^{-3}\ m$ $A = ?$ $C = 1.0\ nF = 1.0 \times 10^{-9}\ F$ $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N\,m^2}$	$C = \frac{A\,\epsilon_0}{d}$ $A = \frac{C \times d}{\epsilon_0}$ $A = \frac{(1.0 \times 10^{-9})(1.0 \times 10^{-3})}{8.85 \times 10^{-12}}$ $A = 0.11\ m^2$

2 **A parallel plate capacitor has the plates 10cm x 10cm separated by a distance 2.5 cm. It is initially filled with air. What will be the increase in the capacitance if a dielectric slab of the same area and thickness 2.5cm is placed between the two plates? $\epsilon_r = 2$.**

Data:	(ii) Capacitance with with a dielectric medium
$A = 10cm \times 10\ cm = 100\ cm^2$ $A = 100\ cm^2 = 100 \times 10^{-4}\ m^2$ $C = 1.0\ nF = 1.0 \times 10^{-9}\ F$ $d = 2.5\ cm = 2.5 \times 10^{-2}\ m$ $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N\,m^2}$ $\varepsilon_r = 2$ **SOLUTION:** (i) Capacitance with air as medium $C_1 = \frac{A\,\epsilon_0}{d}$ $C_1 = \frac{100 \times 10^{-4} \times 8.85 \times 10^{-12}}{2.5 \times 10^{-2}}$ $C_1 = 3.54 \times 10^{-12}\ F = 3.54\ pF$	$C_2 = \frac{A\,\epsilon_0\ \epsilon_r}{d}$ $C_2 = \frac{100 \times 10^{-4} \times 8.85 \times 10^{-12} \times 2}{2.5 \times 10^{-2}}$ $C_2 = 7.08 \times 10^{-12}\ F = 7.08\ pF$ Increase in capacitance, C $C = C_2 - C_1$ $C = 7.08 \times 10^{-12} - 3.54 \times 10^{-12}$ $C = (7.08 - 3.54)\,10^{-12}$ $C = 3.54 \times 10^{-12}\ F$ $C = 3.54\ pF$

3 **A capacitor of 100 pF is charged to a potential difference of 50V. If the plates are then connected in parallel to another capacitor and it is found that the potential difference between its plates falls to 35 volts, what is the capacitance of the second capacitor?**

Data:

$C_1 = 100\,pF = 100 \times 10^{-12}\ F$

$V = 50\,V$

$C_2 = ?$

$V' = 35\ V$

SOLUTION:

Charge on C_1

$$Q_1 = C_1 V$$
$$Q_1 = 100 \times 10^{-12}\,(50)$$
$$Q_1 = 5000 \times 10^{-12}\ C$$

When charged C_1 and uncharged C_2 are connected in parallel (after removing the battery), the p.d is $V' = 35$ V; and the charge on C_1, now is:

$$Q_1' = C_1 V'$$
$$Q_1' = 100 \times 10^{-12}\,(35)$$
$$Q_1' = 3500 \times 10^{-12}\ C$$

Let the charge transferred to C_2 is q'_2, then

$$Q_1 = Q_1' + Q_2'$$
$$Q_2' = Q_1 - Q_1'$$
$$Q_2' = 5000 \times 10^{-12} - 3500 \times 10^{-12}$$
$$Q_2' = 1500 \times 10^{-12}\ C$$

The p.d. across C_2 is $V' = 35$ V

Hence capacitance C_2

$$C_2 = \frac{Q_2'}{V'}$$
$$C_2 = \frac{1500 \times 10^{-12}}{35}$$
$$C_2 = 42.8 \times 10^{-12}\ F$$
$$C_2 = 42.8\ pF$$

4 **A parallel plate capacitor consists of two plates with an area of 150 cm² each, separated by a distance of 20 mm. The capacitor is filled with a dielectric material having a relative permittivity (ϵ_r) of 2. Calculate the capacitance of this capacitor**

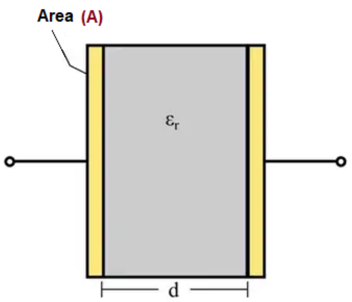
Data:

$A = 150\ cm^2 = 150 \times 10^{-4} m^2$

$d = 20\ mm = 20 \times 10^{-3}\ m$

$\epsilon_r = 2$

$C = ?$



SOLUTION:

$$C = \frac{A\,\epsilon_0\,\epsilon_r}{d}$$
$$C = \frac{(150 \times 10^{-4})\,(8.854 \times 10^{-12})\,(2)}{20 \times 10^{-3}}$$
$$C = \frac{2.656 \times 10^{-13}}{20 \times 10^{-3}}$$
$$C = 1.32 \times 10^{-11}$$
$$C = 13.2 \times 10^{-12}$$
$$C = 13.2 \times pF$$