UNIT 9

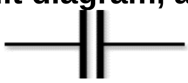
CAPACITORS

CAPACITORS
DEFINITION:

A system of two isolated conductors separated by air or any insulating material , used to store electric charge is called capacitor.

CIRCUIT SYMBOL:

In an electrical circuit diagram, a capacitor is represented by the symbol .



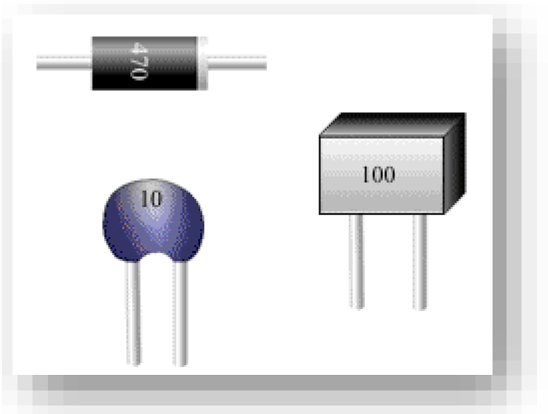
PRINCIPLE OF CAPACITOR:

The potential of a charged conductor is reduced without affecting the charge on it by placing another earth-connected conductor or an oppositely charged conductor near it.

TYPES OF CAPACITORS:

Following are some important types of capacitors.

1. **Multi-plate capacitors**
2. **Variable capacitors**
3. **Electrolytic capacitors.**



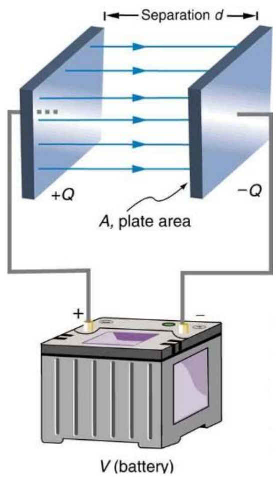
USES OF CAPACITORS:

Following are some important uses of capacitors.

1. **A capacitor is used to store energy in an electrostatic field.**
2. **A capacitor can be used to produce electric fields.**
3. **A capacitor can provide time varying current.**
4. **A capacitor can provide time varying voltage.**
5. **capacitors are fundamental components of tuning circuits**

CAPACITANCE
DEFINITION:

Capacitance is defined as the amount of charge Q stored in a capacitor per volt of potential difference V between the plate of a capacitor



MATHEMATICAL EXPRESSION:

A capacitor can be charged by connecting a battery across it. The battery transfers equal and opposite charges to the two plates . If
Q = charge stored by a capacitor
V = potential difference across it.

Then it is found experimentally that “q” is directly proportional to potential difference “V”

$$Q \alpha V$$

$$Q = CV \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (i)}$$

Where C, the proportionality constant, is called the capacitance of a capacitor.

UNIT OF CAPACITANCE:

The unit of capacitance is coulombs per volt, and to Faraday, it is called a farad(F)

From equation 1

$$1 \text{ farad} = 1 \text{ F} = 1 \ \frac{C}{V}$$

Capacitance is always a positive quantity

The sub-multiple of farad are

micro farad = μF = 10^{-6} F

micro farad = $\mu\mu F$ = pF = 10^{-12} F .

TYPES OF CAPACITOR

FILM CAPACITORS

Film capacitors are capacitors which use a thin plastic film as the dielectric. This film is made extremely thin using a sophisticated film drawing process. They are available in almost any value and voltage as high as 1500 volts.

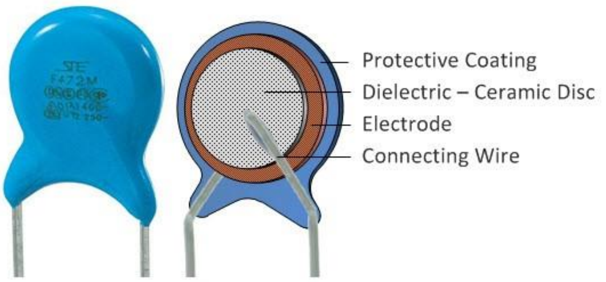
These capacitors are popular among power electronics enthusiasts. They are used in almost all the power electronic devices, x-ray machines, phase shifters, and pulsed lasers. Even the switched power supply uses a film capacitor for power factor correction

CERAMIC CAPACITOR

The ceramic capacitor is a type of capacitor used in high frequency circuits ranging from audio to radio frequency (RF).

The values of the ceramic capacitors are typically between the 1nF and 1μF.

Ceramic capacitors typically utilize barium titanate as their dielectric material



ELECTROLYTIC CAPACITOR

Generally, the electrolyte capacitors are used when the large capacitor values are required. The thin metal film layer is used for one electrode and for the second electrode (cathode) a semi-liquid electrolyte solution in jelly or paste. The dielectric plate is a thin layer of oxide

The uses of electrolytic capacitors are generally in the DC power supply circuit because they are large in capacitance and small in reducing the ripple voltage.

PARALLEL PLATE CAPACITOR

1. CAPACITANCE WITH AIR BETWEEN IT’S PLATES:

Consider a parallel plate capacitor having air as a dielectric between its plates. The two metallic plates of equal area A are separated by a distance d, as shown in figure. One plate carries a charge +Q and other carries a charge –Q respectively.

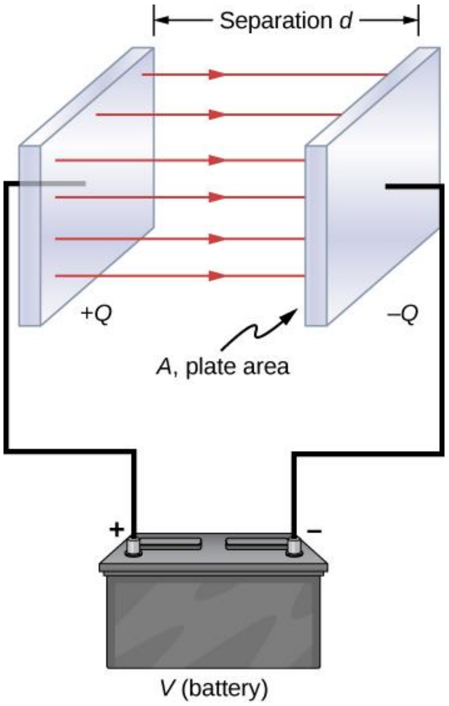
The surface charge density is defined as the charge stored per unit area (A)

$$\sigma = \frac{Q}{A}$$

The electric field is uniform between the plates and is zero elsewhere. According to Gauss's law, the value of electric field between the plates is

$$E = \frac{\sigma}{\epsilon_o}$$

or $$E = \frac{q}{A\epsilon_o} \qquad [\sigma = \frac{q}{A}]$$



Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals (E d); therefore,

$$V = E\,d$$

$$V = \frac{q}{A\epsilon_o}d \qquad \text{(1)}$$

Substituting this result into capacitance equation, we find the capacitance is

$$C = \frac{q}{V}$$

$$C = \frac{q}{\left(\frac{q}{A\epsilon_o}\right)d} \qquad \left\{V = \left(\frac{q}{A\epsilon_o}\right)\right\}$$

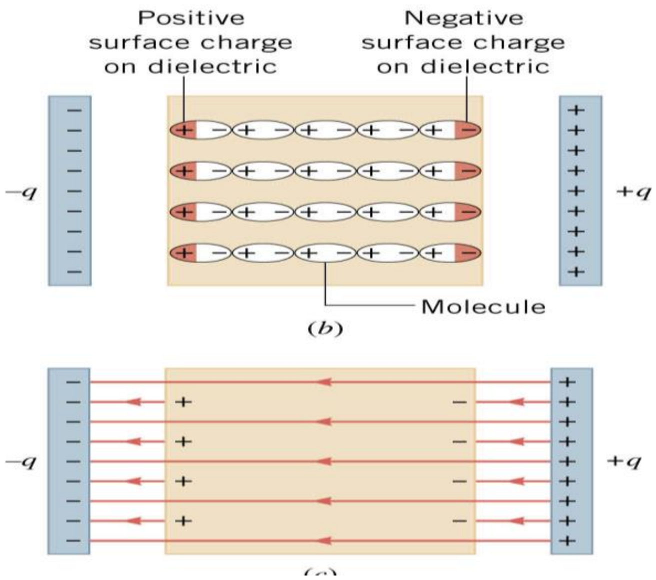
$$C = q \times \frac{A\epsilon_o}{qd}$$

$$C = \frac{A\epsilon_o}{d}$$

That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.

2. WHEN AN INSULATOR AS DEILECTRIC :

A dielectric is a non conducting material, such as rubber, glass, or waxed paper , when a dielectric inserted between plates of a capacitor, the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor ϵ_r is called the dielectric constant. If the dielectric inserted between the plates the electric field between plates of a charged capacitor polarizes the molecules of the dielectric as shown in figure.



Due to this polarization the potential on plates decreases and a capacitor connected to battery accumulates more charge, hence its capacitance increases.
The capacitance in this case is given by

$$C_0 = \frac{\epsilon_o \ \epsilon_r \ A}{d}$$

$$C_0 = \epsilon_r \left(\frac{\epsilon_o \ A}{d}\right)$$

$$C_0 = \epsilon_r \ C$$

This is the expression for capacitance when an insulator is used as a dielectric between plates.

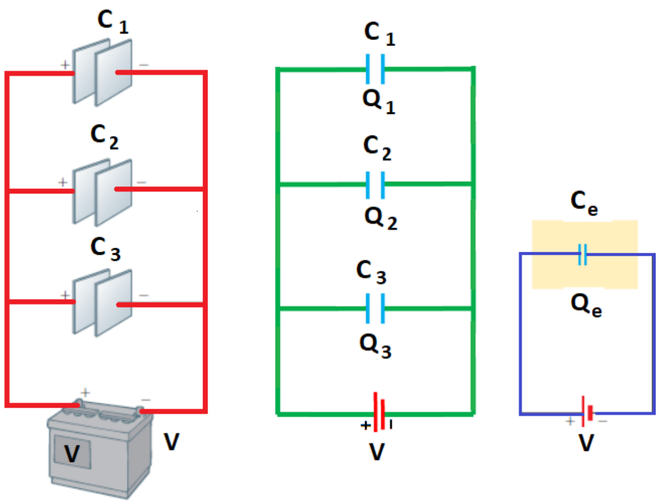
CAPACITORS CONNECTED IN PARALLEL:

DEFINITION:

When one plate of all capacitors is connected to a common terminal of the battery and the other plates of all capacitors are connected to the other terminal, such a circuit is called a parallel combination.

EQUIVALENT CAPACITANE :

Consider three capacitors of capacitances C_1, C_2 an C_3 connected in parallel as shown.



The individual Potential difference across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.
Now,
charged stored by capacitor C_1 is $Q_1 = C_1V$ ———— (i)
Charged stored by capacitor C_2 is $Q_2 = C_2 V$ ———— (ii)
Charged stored by capacitor C_3 is $Q_3 = C_3 V$ ———— (iii)
Consider an equivalent capacitor of capacitance Ce, if the applied potential difference is V, and the total charge stored is Q, then

$$Q = C_eV \quad ———— \quad (iv)$$

But

$$Q = Q_1 + Q_2 + Q_3$$

Substituting the values of Q, Q_1, Q_2 and Q_3 from equations (i).(ii), (iii), and (iv) into the above equation, we have

$$C_eV = C_1V + C_2V + C_3V$$

$$C_e\cancel{V} = \cancel{V}(C_1 + C_2 = C_3)$$

$$C_e = C_1 + C_2 + C_3$$

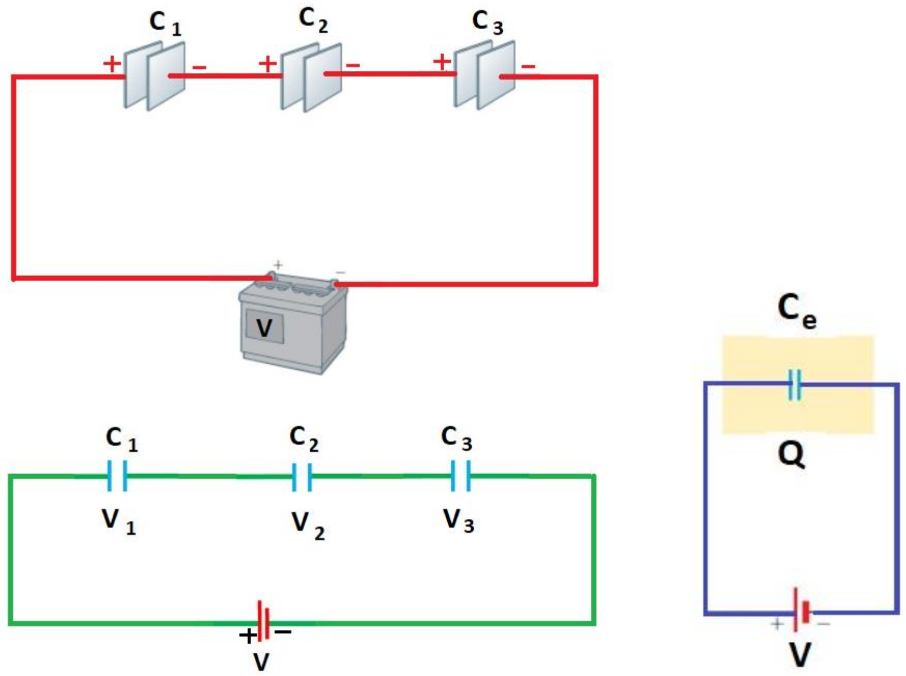
CAPACITORS CONNECTED IN SERIES:

DEFINITION:

When capacitors are connected end-to-end with only one terminal common between them, such a circuit is called a series combination.

EQUIVALENT CAPACITANE :

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in series as shown.



When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_3 through the battery. The result of this that all the right plates gain of negative charges of –Q and all the left plates have charges of +Q.

By applying the definition of capacitance, we have

$$V = \frac{q}{C_e}$$

where V is the potential difference between the terminal of the battery and C_e is the equivalent capacitance.

In general, the total potential difference across any number of capacitors connected is series is the sum of the potential differences across the individual capacitors.

$$V = V_1 + V_2 + V_3 \qquad \text{——————} \qquad (i)$$

Where V_1, V_2 and V_3 are the potential differences across capacitance C_1, C_2 and C_3 are given by

$$V_1 = \frac{Q}{C_1} \qquad V_2 = \frac{Q}{C_2} \qquad V_3 = \frac{Q}{C_3}$$

substituting these expressions into equation (i), we have

$$\frac{Q}{C_e} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

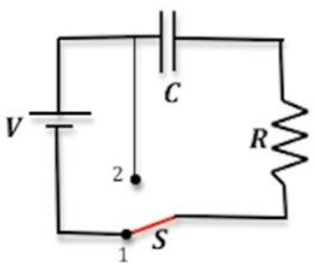
$$Q \times \frac{1}{C_e} = Q \times \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

canceling q, we arrive at the relationship

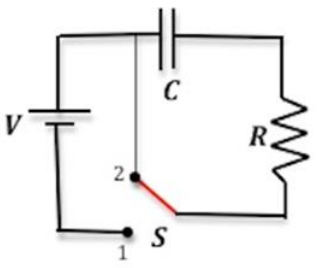
$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

CHARGE AND DISCHARGE OF A CAPACITOR THROUGH A RESISTOR

Consider a circuit having a capacitance C and a resistor R which are connected in series with a battery of potential difference V through a switch as shown in figure



(a)



(b)

When a switch is at position 1 as shown in figure (a), the capacitor begins to store charge. If any time during charging, the charge Q flowing through a circuit and Q is the charge on a capacitor, then sum of the potential difference between the palate of a capacitor and across the resistor is equal to the potential difference from the battery. As the current stop flowing when the capacitor is fully charged, when Q = Q_0, then

$$Q_0 = CV$$

Experiments shows that the charging of a capacitor exhibits the experimental behavior, we can write its equation.

$$Q = Q_0\left(1 - e^{-\frac{t}{RC}}\right) \ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Where Q_0 represent the final charge on the capacitor.

Time Constant

The product RC (having units of time) has a special significance; it is called the time constant of the circuit. The time constant is the amount of time required for the charge on a charging capacitor to rise to 63% of its final value. In other words, when t = RC,

If τ = t , equation (i) become,

$$Q = Q_0\left(1 - e^{-\frac{t}{t}}\right)$$

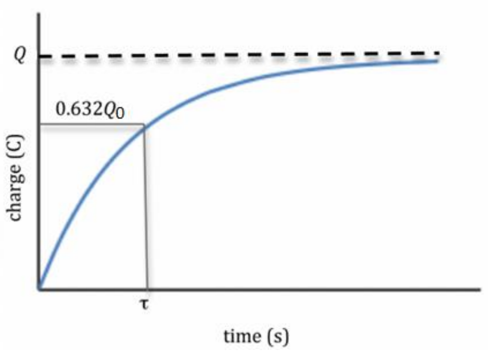
$$Q = Q_0\left(1 - e^{-1}\right)$$

$$Q = Q_0\left(1 - \frac{1}{e}\right)$$

$$Q = Q_0\left(1 - \frac{1}{2.718}\right)$$

$$Q = Q_0\left(1 - 0.368\right)$$

$$Q = 0.632\,Q_0$$

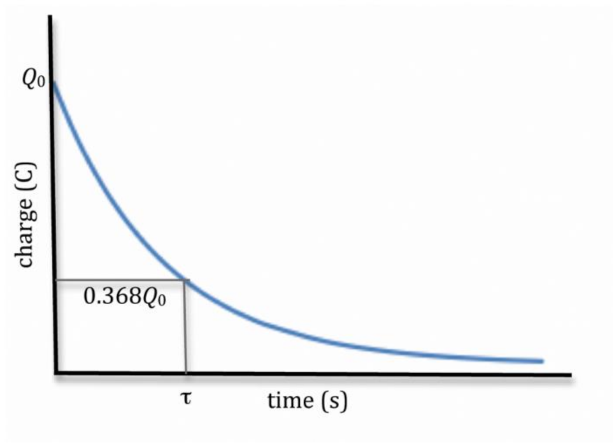


When the switch is moved to position 2, the circuit shows the discharging of the charge capacitor, the battery is now out of the circuit and the capacitor discharges itself through a resistor. This discharging process of the capacitor follows the following equation.

$$Q = Q_0\, e^{-\frac{t}{RC}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

where Q_0 represents the initial charge on the capacitor at the beginning of the discharge at t = 0: You can see from this expression that the charge decays exponentially when the capacitor

discharges, and that it takes an infinite amount of time to fully discharge.
When τ = t, equation (ii) become.



$$Q = Q_0\, e^{-\frac{t}{t}}$$

$$Q = Q_0\, e^{-\frac{t}{t}}$$

$$Q = Q_0\, e^{-1}$$

$$Q = \frac{Q_0}{e}$$

$$Q = \frac{Q_0}{2.178}$$

$$Q = 0.368\ Q_0$$

Time constant of RC circuit is thus also the time during which the charge on the capacitor falls from its maximum value to 0.368 of its maximum value.

ENERGY STORED IN A CAPACITOR

Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge *Q* and voltage *V* on the capacitor. We must be careful when applying the equation for electrical potential energy U = *Q* Δ*V* to a capacitor. Remember that U is the potential energy of a charge *Q* going through a voltage Δ*V*. But the capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage Δ*V* = 0 since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences Δ*V* = *V*, since the capacitor now has its full voltage *V* on it. The average voltage on the capacitor during the charging process is $\frac{V}{2}$, The equation can be modified in this case

$$U = Q\ \frac{V}{2}$$

$$U = \frac{1}{2}\, Q\, V \text{.................(i)}$$

Charge and voltage are related to the capacitance *C* of a capacitor by *Q* = *CV*,
Substituting Q = CV in equation (i) , we get

$$U = \frac{1}{2}\, (CV)\, V$$

$$U = \frac{1}{2}\ C\ V^2 \quad \text{........................(ii)}$$

Substituting $V = \frac{Q}{C}$ in equation (i) , we get

$$U = \frac{1}{2}\, Q\, \frac{Q}{C}$$

$$U = \frac{1}{2}\ \frac{Q^2}{C} \quad \text{..........................(iii)}$$

The above three equations express the energy stored in a capacitor.

SHORT REASONING QUESTIONS

1: State the factors on which the capacitance of a parallel plate capacitor depends.

ANSWER: The capacitance (C) of a parallel plate capacitor depends on the following factors:

- Plate Area (A): The capacitance is directly proportional to the surface area of the plates. A larger plate area results in a higher capacitance.
- Plate Separation (d): The capacitance is inversely proportional to the distance between the plates. A smaller plate separation leads to a higher capacitance.
- Permittivity of the Dielectric (ε): If there is a dielectric material between the plates, its permittivity affects the capacitance. Different materials have different permittivity and a higher permittivity result in a higher capacitance.

2: **Explain what is meant by dielectric constant (relative permittivity). State two physical properties desirable in a material to be used as the dielectric in a capacitor.**

ANS: The dielectric constant or relative permittivity of a materiel indicates its ability to increase the capacitance of a capacitor compared to a vacuum. Two desirable properties for a dielectric material are a high dielectric constant for greater capacitance enhancement and low dielectric loss for efficient energy storage and minimal heat dissipation.

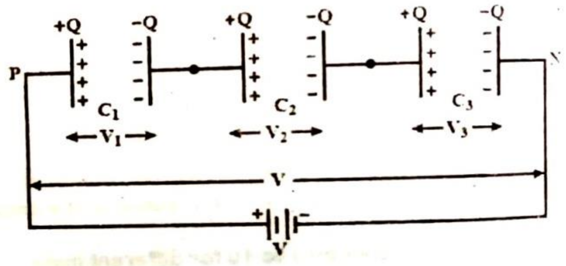
3: Drive expressions for the combined capacitance of two capacitors. (a) connected in series, (b) connected in parallel.

ANS:

SERIES COMBINATION OF CAPACITORS

Capacitors are said to be in series when only one terminal is common between them, they are arranged in plate to plate in series manner.

As shown in the figure.



Series combination of capacitors

Properties:

- Each capacitor store same charge q.
- Potential difference across each capacitor is different.

Derivation:

As shown capacitances C_1, C_2 and C_3 are connected in series. As q is constant due to single path of electricity flow.

$$V_1 = \frac{q}{C_1}$$

$$V_2 = \frac{q}{C_2}$$

$$V_3 = \frac{q}{C_3}$$

If these capacitors are replaced by a single equivalent capacitor C_e, which will store same charge q, then;

$$V = \frac{q}{C_e}$$

Here, $$V = V_1 + V_2 + V_3$$

There fore

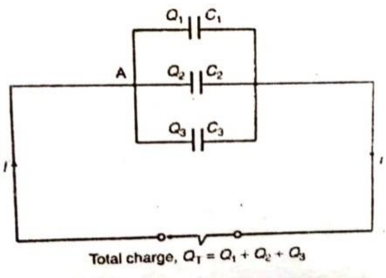
$$\frac{q}{C_3} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or

$$\frac{1}{C_3} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

PARALLEL COMBINATION OF CAPACITORS

Capacitors are said to be in parallel when one plate of all capacitors is connected to one terminal of battery, and all other plates to another terminal.



Properties:

- Each capacitor store different charge.
- Potential difference across each capacitor is same due to same terminal point of each capacitor.

Derivation:

As shown capacitances C_1, C_2 and C_3 are connected in parallel, now

If these capacitors are replaced by a single equivalent capacitor C_e which will store total charge Q_T, then

$$Q_T = C_eV$$

But

$$Q_T = Q_1 + Q_2 + Q_3$$

$$C_eV = C_1V + C_2V + C_3V$$

or

$$C_e = C_1 + C_2 + C_3$$

4: Derive an expression for the energy stored in a capacitor C when there is a potential difference V between the plates.

ANS(**See topic on page……****)**

QUESTION 5: A capacitor gets a charge of 50C when it is connected to a battery of emf 5 V. Calculate the capacity of the capacitor.

ANSWER:

Data:

Charge (Q) = 50 C

Voltage (V) = 5 V

Solution:

The capacitance (C) of a capacitor can be calculated using the formula:

C = V / Q

Now, let's calculate the capacitance:

C = 50 / 5

C = 10 F