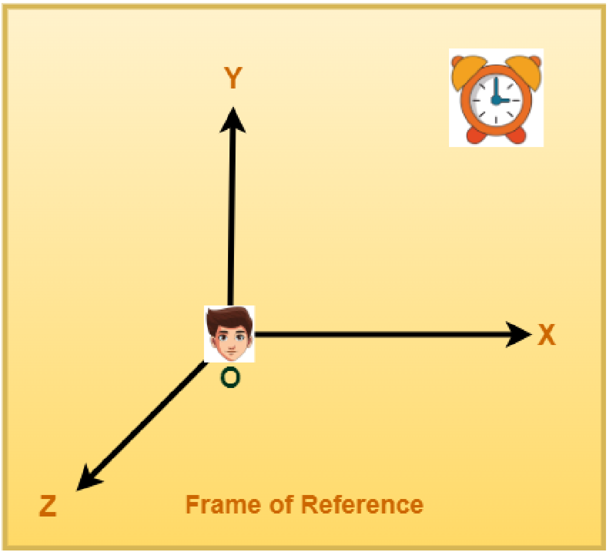
# UNIT 24 RELATIVITY

## INTRODUCTION

Relativity, a fundamental theory in physics developed by Albert Einstein, revolutionized our understanding of space, time, and energy. It consists of two main parts: Special Relativity and General Relativity. Special Relativity, and the idea that time and space are relative, varying according to the observer's motion. General Relativity, introduced extends these ideas to include gravity, describing it not as a force but as the curvature of caused mass and energy.

## FRAMES OF REFERENCE

*A frame of reference is a set of coordinate axes for which measurements are taken. Mathematically, it consists of a reference point (called origin) upon which three mutually orthogonal lines are fixed. The position of a body can be located by three Cartesian coordinates. This rectangular coordinate system can represent position, displacement, velocity and acceleration of the system. An observer has a fixed location in the coordinate axes system. Taken together, the observer and the coordinates axes constitute a frame of reference.*




## NON-INERTIAL FRAME OF REFERENCE

**An accelerated frame of reference is called the non-inertial frame. For example, a rotating frame or a frame moving with increasing or decreasing velocity is non-inertial.**

## INERTIAL FRAME OF REFERENCE

**An inertial frame of reference is that which moves with a constant velocity with respect to its surroundings**

**OR**

**An inertial frame of reference is that in which Newton's first law of motion is applicable.**

## PROPERTIES

1. Inertial frame of reference has got zero linear acceleration and zero angular acceleration.
2. All frames of reference are completely equivalent for all physical phenomena.
3. The law of addition of velocity and the law of conservation of momentum remains same in all the frames that move with uniform velocity.

## THE EARTH AS FRAME OF REFERENCE

In the case of the Earth, the acceleration associated with orbital and rotational motions is small enough to be ignored in most experiments, we shall consider the Earth and objects moving with constant velocity relative to it to be the inertial frame of reference.

## TYPE OF FRAME OF REFERENCES

| Criteria | Inertial frame of Reference | Non-Inertial frame of Reference |
|---|---|---|
| **Definition** | A frame of reference with a constant | A frame of reference with an accelerating motion. |
| **Motion of objects** | Objects in uniform motion appear to follow straight lines or constant velocities. | Objects may appear to accelerate or experience fictitious forces. |
| **Newton's first law** | Newton's first law (Law of inertia) is valid in this frame. | Newton's first law is not valid due to accelerating motion. |
| **Appearance of forces** | Real forces are observed and can be directly measured. | Fictitious forces (e.g., centrifugal force, coriolis force) may appear due to the acceleration of the frame. |
| **Equations of motion** | Newton's laws of motion hold true in this frame. | Additional terms or transformations may be required to account of the frame. |
| **Examples** | A person inside a moving train, an object in free fall. | A person inside a spinning carousel, an object in circular motion. |
| **Application** | Often used in analyzing motion and dynamics in | Important in understanding phenomena in rotating systems, general relativity, and |

## RELATIVE MOTION AND THE GALILEAN TRANSFORMATION EQUATION

The Galilean transformation equations describe the relationship between the coordinates of an event in two different inertial frames of reference. These equations were formulated by Galileo and are applicable when the relative velocities involved are much smaller than the speed of light.

There is no absolute inertial reference frame. Therefore, the results of an experiment performed in a vehicle moving with uniform velocity must be identical to those performed in a stationary vehicle. The formal statement of this result is called the principle of Galilean relativity:

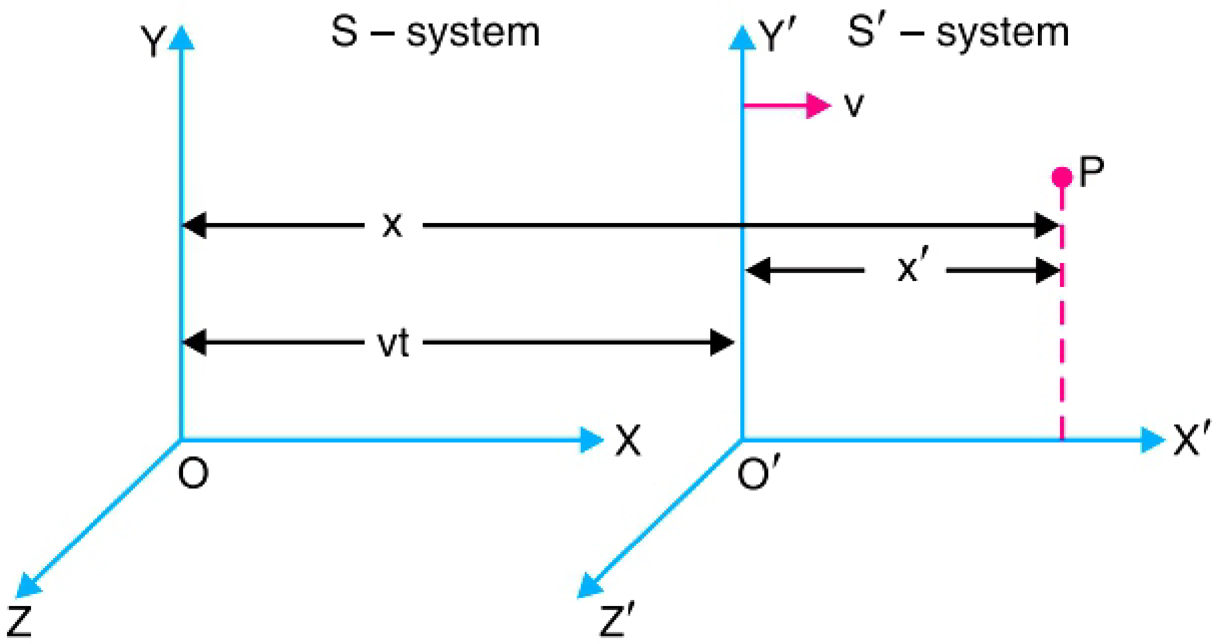
**The laws of mechanics must be the same in all inertial frames of reference**

Consider two inertial frames **S** and $S'$ (as shown in figure). The $S'$ frame moves with a constant velocity along the common $x$ and $x'$ axes, where is measured relative to **S**. We assume the origins of **S** and $S'$ coincide at $t = 0$ and an event occurs at point **P** in space at some instant of time. An observer in **S** describes the event with space–time coordinates ($x$, $y$, $z$, $t$), whereas an observer in $S'$ uses the coordinates ( $x'$, $y'$, $z'$, $t'$ ) to describe the same event. As we see from the geometry in Figure 2, the relationships among these various coordinates can be written

$$x' = x - vt$$
$$y' = y$$

$$z' = z$$

*These equations are the **Galilean space–time transformation equations.** Note that time is assumed to be the same in both inertial frames*



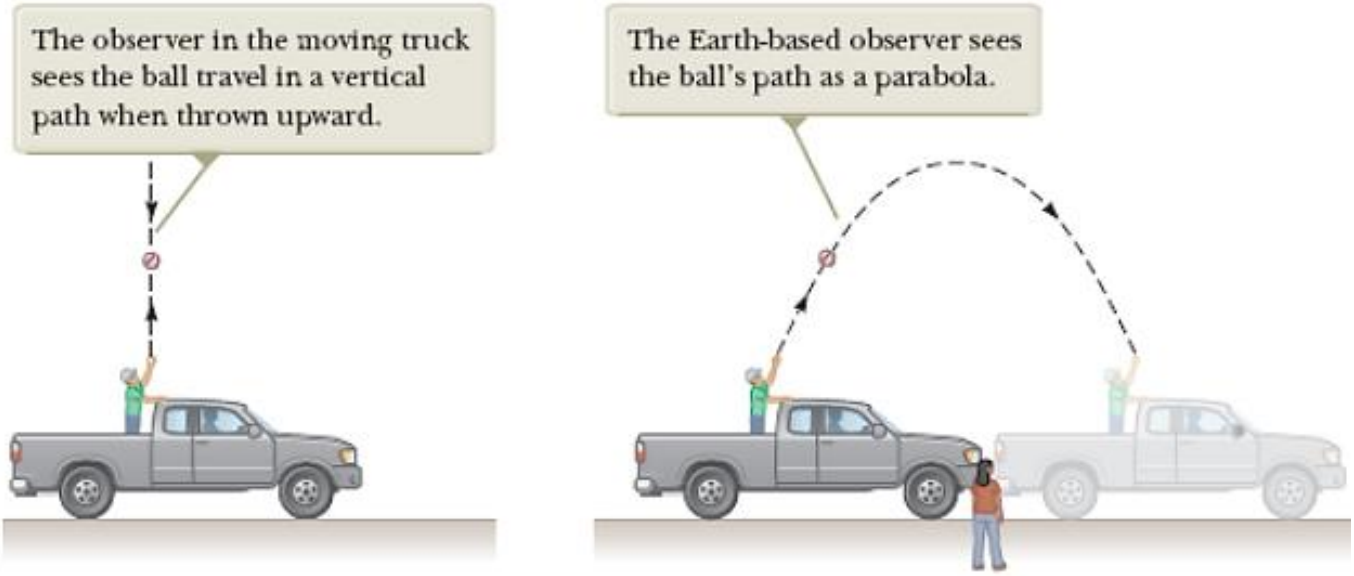

## ***THE POSTULATES OF SPECIAL THEORY OF RELATIVITY***

The special theory of relativity is based on two essential assumptions, commonly known as postulates.

## **POSTULATE 1(PRICIPLE OF RELATIVITY)**

**The laws of mechanics must be the same in all inertial frames of reference**

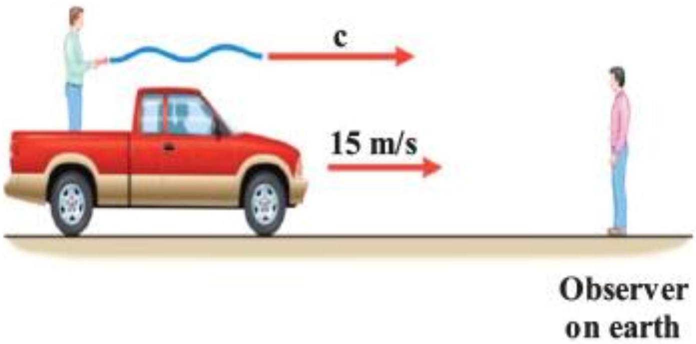
Consider a vehicle moving at a constant speed. Inside the vehicle, a passenger throws a ball straight up into the air. If we ignore any effects from the air, the passenger inside the moving vehicle sees the ball move up and then back down in a straight line, just as it would appear if someone standing still on the Earth threw a ball upwards. This means that the laws of physics that govern the motion of the ball, including gravity and equations for constant acceleration, work the same way whether the vehicle is moving or at rest. Both observers, the one inside the vehicle and the one on the ground, see the ball go up and come back down. However, they see the path of the ball differently. The person on the ground sees the ball's path as a curved shape (a parabola), while the person in the vehicle sees it as a simple up-and-down motion. As shown in figure. Additionally, the person on the ground thinks that the ball has a horizontal component of velocity, which is the same as the vehicle's velocity. For the outside observer, the distance traveled along the parabolic path is longer than the path straight down but the time for the fall is the same. For the outside observer, the average velocity is greater.




## POSTULATE II (CONSTANCY OF SPEED OF LIGHT)

**The speed of light in vacuum has the same value $c = 3 \times 10^8\ m/s$ in all inertial reference frames, regardless of velocity of the observer or of the source emitting the light.**

Consider again the same example of throwing a ball in moving vehicle. As shown in figure, But this time, instead of throwing a ball, we shine a flashlight. If we were to do the same thing with light as we did with the ball, common sense might suggest that the speed of light would increase if the vehicle is moving in the same direction as the light according to Galilean Transformation Equation: v = c + u (if we replace ball's speed v' with speed of light c). In fact, such an increase in the speed of light has never been found. In fact, in experiments carried out to test for the effect of the movement of the source on the speed of light (Michelson-Morley), the results indicate that the speed of light is completely unaffected by the motion of the source. It appears that the speed of light in a vacuum is constant regardless of relative motion.




In order to correct the Galilean equation of velocity addition, we have to consider velocity of light too. This correction was done by Hendrik Lorentz. The resultant equations are known as Lorentz transformation equations.

## RELATIVE MOTION: LORENTZ TRANSFORMATION EQUATIONS:

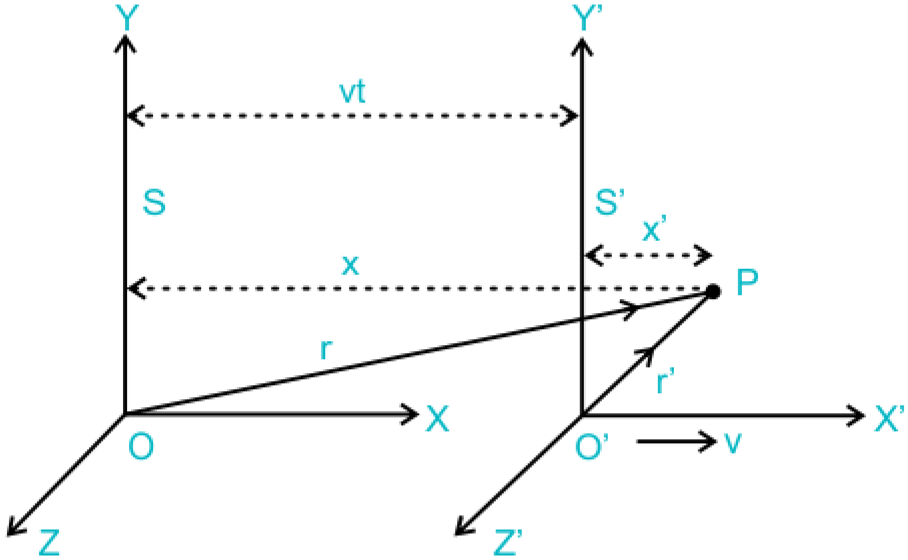
We have to introduce new transformation equations which are consistent with the new concept of the invariance of light velocity in free space.
The Lorentz transformation equations for time and space coordinates (x, y, z, t) can be Written as follows

$$t' = \gamma \left(t - \frac{v\,x}{c^2}\right)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$




where t and $(x, y, z)$ are the time and space coordinates in the original frame, $t'$ $and$ $(x', y', z')$ are the time and space coordinates in the frame moving with velocity $v$ relative to the original frame as shown in Figure, c is the speed of light, and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

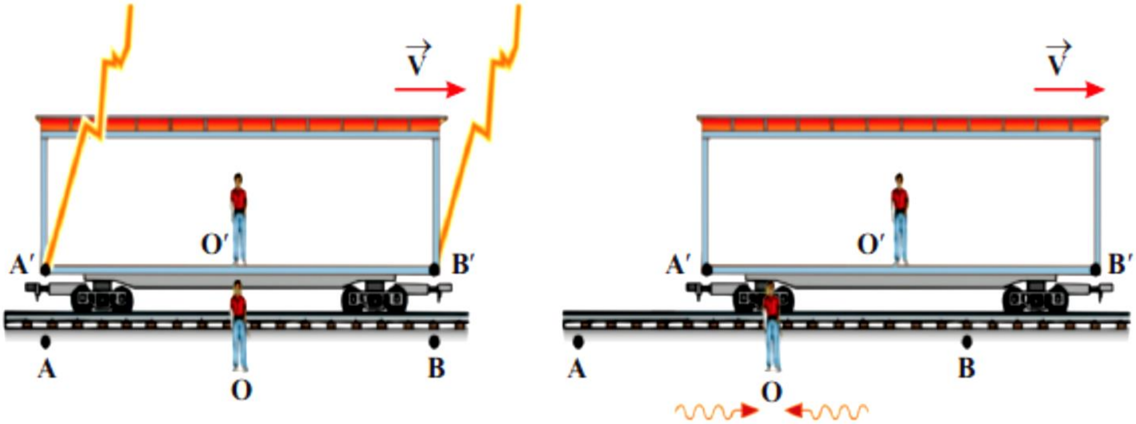
## CONSEQUENCES OF SPECIAL THEORY OF RELATIVITY

In Relativity, there is no such thing as an absolute length or absolute time interval. Here are some of the most important consequences of special relativity:

### SIMULTANEITY AND THE RELATIVITY OF TIME

A basic premise of Newtonian mechanics is that a universal time scale exists that is the same for all observers. Newton and his followers took simultaneity for granted. In his special theory of relativity, Einstein abandoned this assumption. Einstein devised the following thought experiment to illustrate this point.

A boxcar moves with uniform velocity, and two lightning bolts strike its ends as illustrated in Figure (a), leaving marks on the boxcar and the ground. The marks on the boxcar are labeled $A'$ $and$ $B'$, and those on the ground are labeled $A$ $and$ $B$. An observer $O'$ moving with the boxcar is midway between $A'$ $and$ $B'$, and a ground observer $O$ is midway between $A$ $and$ $B$. The events recorded by the observers are the striking of the boxcar by the two lightning bolts.




## FROM OBSERVER O's RESPECTIVE

The light signals emitted from *A* and *B* at the instant at which the two bolts strike later reach observer ***O*** at the same time as indicated in Figure (b). This observer realizes that the signals traveled at the same speed over equal distances and so concludes that the events at *A* and *B* occurred simultaneously.

## FROM OBSERVER *O´*'s RESPECTIVE

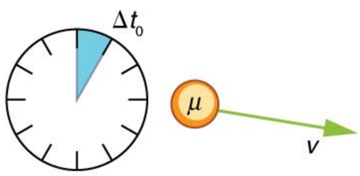
Now consider the same events as viewed by an observer ***O´***. By the time the signals have reached observer *O*, an observer ***O´***. has moved as indicated in Figure (b). Therefore, the signal from ***B´***. has already swept past ***O´***., but the signal from ***A´***. has not yet reached ***O´***.. In other words, ***O´***. sees the signal from ***B´***. before seeing the signal from ***A´***. According to Einstein, *the two observers must find that light travels at the same speed*. Therefore, observer ***O´*** concludes that one lightning bolt strikes the front of the boxcar *before* the other one strikes the back.

This thought experiment demonstrates that the two events that appear to be simultaneous to observer *O* do *not* appear to be simultaneous to observer***O´***. In other words,

> **two events that are simultaneous in one reference frame are in general not simultaneous in a second frame moving relative to the first.**

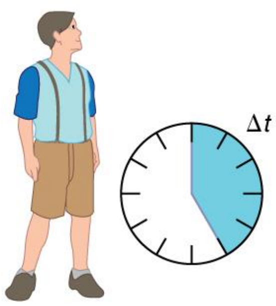
## TIME DILATION

The time interval, between two events occurring at a given point in the moving frame S' appears to be longer to the observer in the stationary frame S. This effect is called time dilation.




The time interval $\Delta t$ measured by an observer moving to the clock is longer than the time interval $\Delta t_0$ measured by an observer at rest to the clock ( $\Delta t > \Delta t_0$ ). This effect is known as time dilation.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

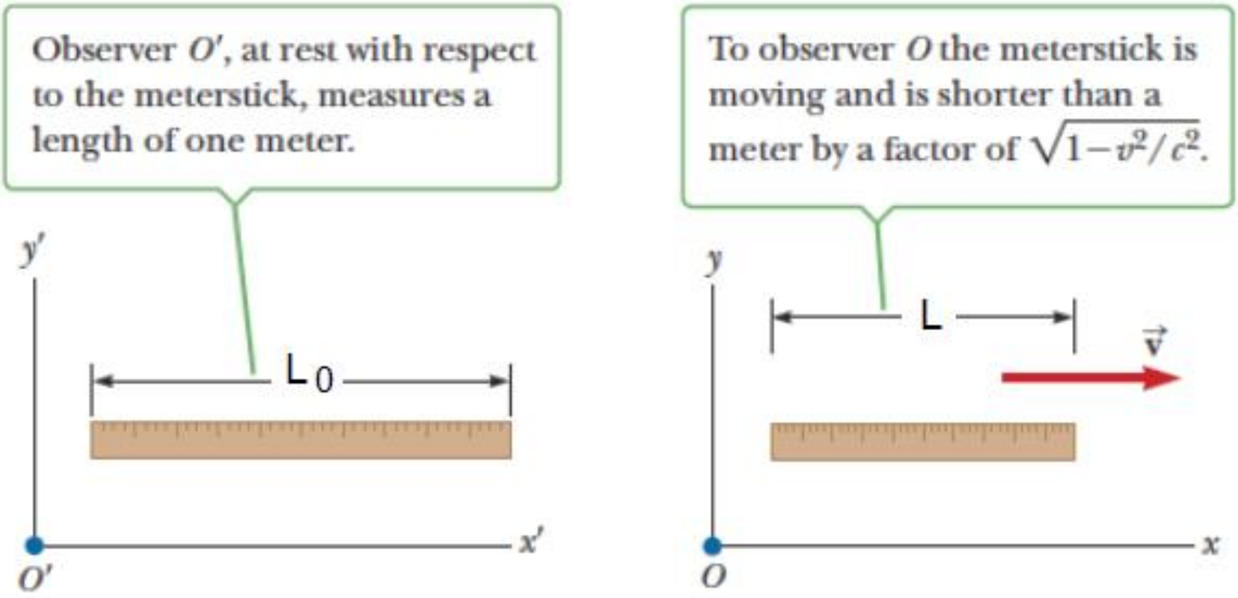



The time interval $\Delta t_0$,in above equation is called the proper time. In general, proper time is the time interval between two events measured by an observer who sees the events occur at the same point in space. Proper time is always measured with a single clock at rest in the frame in which the events take place. The time dilation effect says that Clock moving relative to an observer are measured by that observer to run more slowly ( as compared to clock at rest )

## LENGTH CONTRACTION

The measured distance between two points in space also depends on the frame of reference of the observer. **The proper length $L_0$ of an object is the length measured by someone at rest relative to the object.** The length $L$ of an object measured by someone in a reference frame that is moving with respect to the object is always less than the proper length. This effect is known as **length contraction.**

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$




*This is the general result of the special theory of relativity and applies to the length of objects as well as to distance. It is clear from this expression that $L < L_0$, thus length of an object contracts. Note that* ***length contraction takes place only along the direction of motion***

## MASS VARIATION:

According to theory of relativity mass of an object is not an absolute quantity; its value depends upon the frame where it is measured. The mass of a body, moving with velocity v relative to an observer, is larger than its mass at rest. It is given by

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where $v$ is the relative speed and $m_o$ mass of an object, measured by an observer at rest with respect to the object. It is called ***Proper Mass***. **m** mass of the object, measured by an observer from a frame moving with respect to object is called ***Relativistic Mass***.

It is clean from above equation that $m > m_o$. Thus, mass of an object depends upon

(i) the frame from where it is measured

(ii) the speed of frame relative to object

## Mass Energy Relationship:

According to Einstein's special theory of relativity, mass and energy are interchangeable. An object's mass m and the equivalent energy $E_0$ are related by:

$$E_0 = m\,c^2$$

where c is the speed of light. This equation gives the mass-energy $E_0$ that is associated with the object's mass m, regardless of whether the object is at rest or moving.

**E** is the total relativistic energy of the object. then

$$E = \gamma\, m\, C^2$$

where $\gamma$ is the Lorentz factor for the object's motion

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{C^2}}}$$

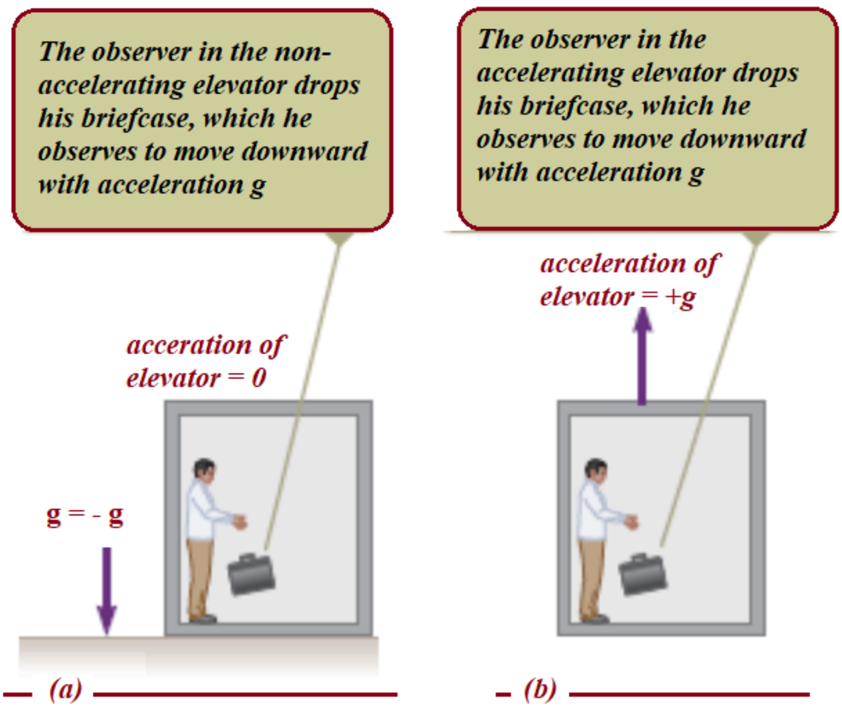
If the object is moving, it has additional energy in the form of kinetic energy K. The total energy **E** is the sum of its rest mass energy and its kinetic energy:

$$E = E_0 + K$$
$$K = E - E_0$$
$$K = \gamma\, m\, C^2 - m\, c^2$$
$$K = (\gamma - 1)\, m\, C^2$$

## GENERAL THEORY OF RELATIVITY:

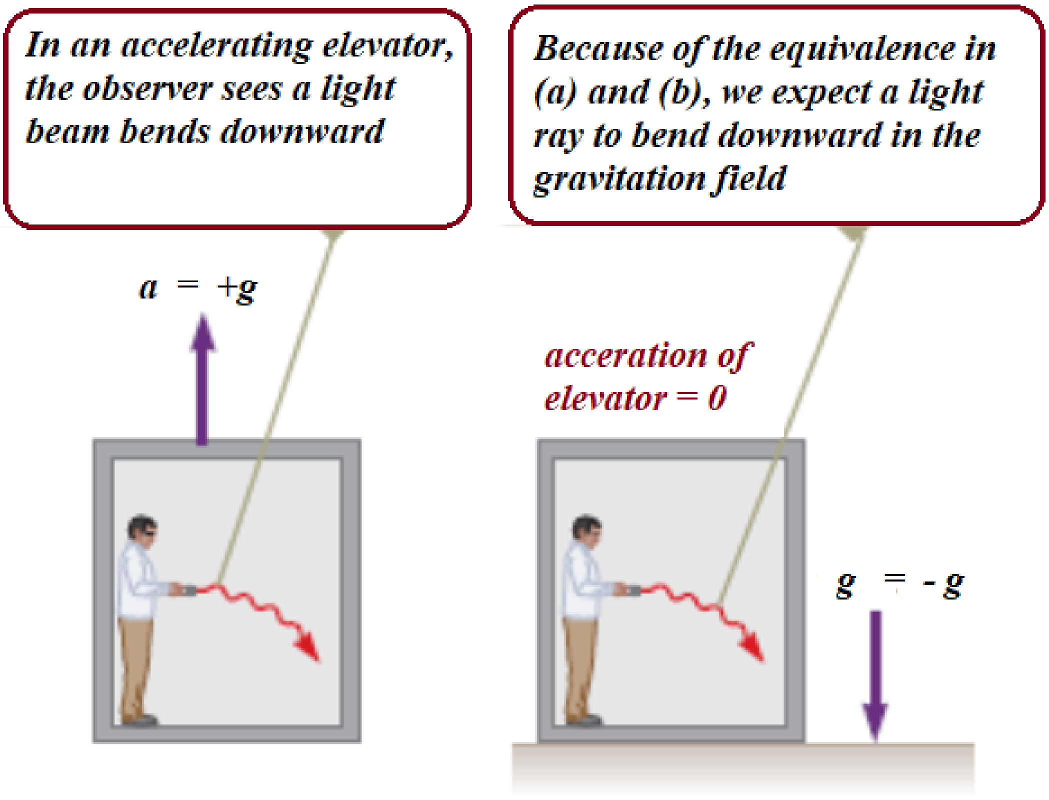
Einstein's **general theory of relativity** applies to accelerated frames of reference and gravitation. In fact, the theory provides a link between these two types of physical processes that lead to a new interpretation of gravity.

He pointed out that no mechanical experiment (such as dropping an object) could distinguish between the two situations illustrated in Figures (a) and (b). In Figure (a), a person standing in an elevator on the surface of a planet feels pressed into the floor due to the gravitational force. If he releases his briefcase, he observes it moving toward the floor with acceleration $g = -g$ . In Figure (b), the person is in an elevator in empty space accelerating upward with $g = +g$. The person feels pressed into the floor with the same force as in Figure (a). If he releases his briefcase, he observes it moving toward the floor with acceleration $g$, exactly as in the previous situation.




In each situation, an object released by the observer undergoes a downward acceleration of magnitude $g$ relative to the floor. In Figure (a), the person is at rest in an inertial frame in a gravitational field due to the planet. In Figure (b), the person is in a no inertial frame

accelerating in gravity-free space. Einstein claims that these two situations are completely equivalent.
Suppose a light pulse is sent horizontally across the elevator as in Figure (c), in which the elevator is accelerating upward in space. From the point of view of an observer in an inertial frame outside the elevator, the light travels in a straight line while the floor of the elevator accelerates upward. According to the observer on the elevator, however, the trajectory of the light pulse bends downward as the floor of the elevator (and the observer) accelerates upward. Therefore, based on the equality of parts (a) and (b) of the figure, Einstein proposed that a beam of light should also be bent downward by a gravitational field as in Figure (d)




(c) (d)

Einstein's **general theory of relativity** has two postulates:

***All the laws of nature have the same form for observers in any frame of reference, whether accelerated or not.***

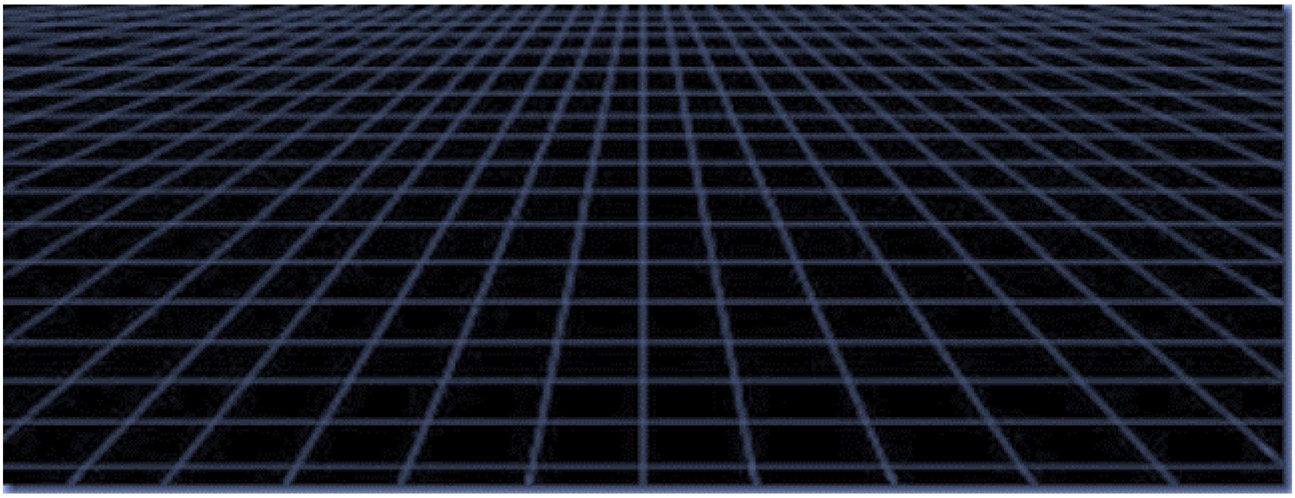
***In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in gravity-free space (the principle of equivalence).***

## CLOCK IN THE PRESENCE OF GRAVITY

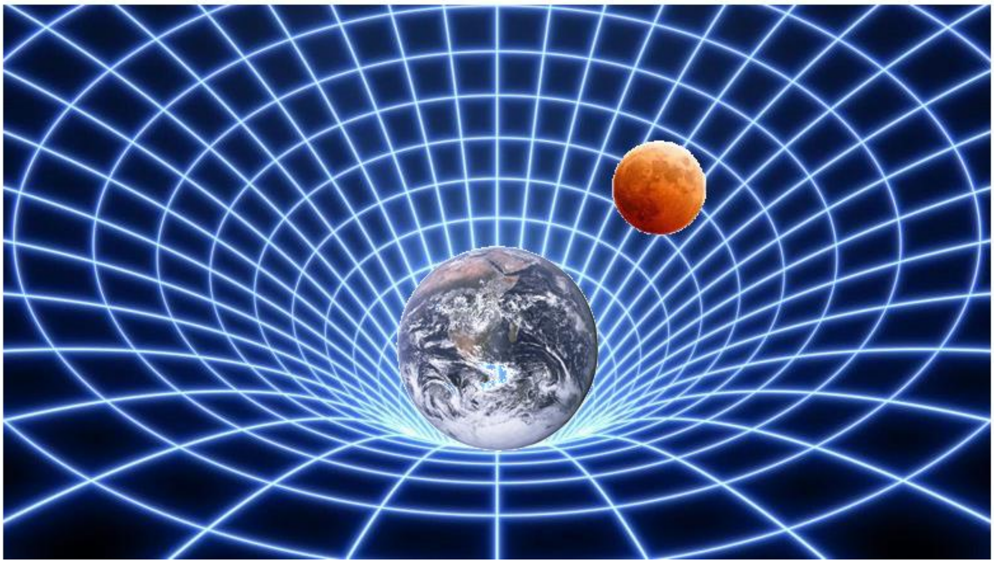
One interesting effect predicted by the general theory is that time is altered by gravity. A clock in the presence of gravity runs slower than one located where gravity is negligible. Consequently, the frequencies of radiation emitted by atoms in the presence of a strong gravitational field are *redshifted* to lower frequencies when compared with the same emissions in the presence of a weak field.

## SPACE-TIME CONTINUUM:

Space and time are unified into a single, four- dimensional entity known as space-time. In the absence of gravity or significant mass, space time is flat.

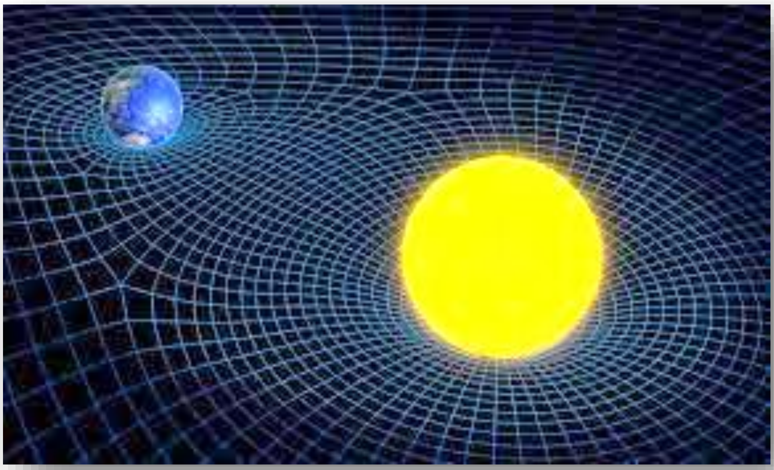


General relativity predicts that objects of mass and energy cause a curvature of the fabric of space - time, which Einstein united into a single entity called space-time. The greater the mass, the more it curves spacetime.



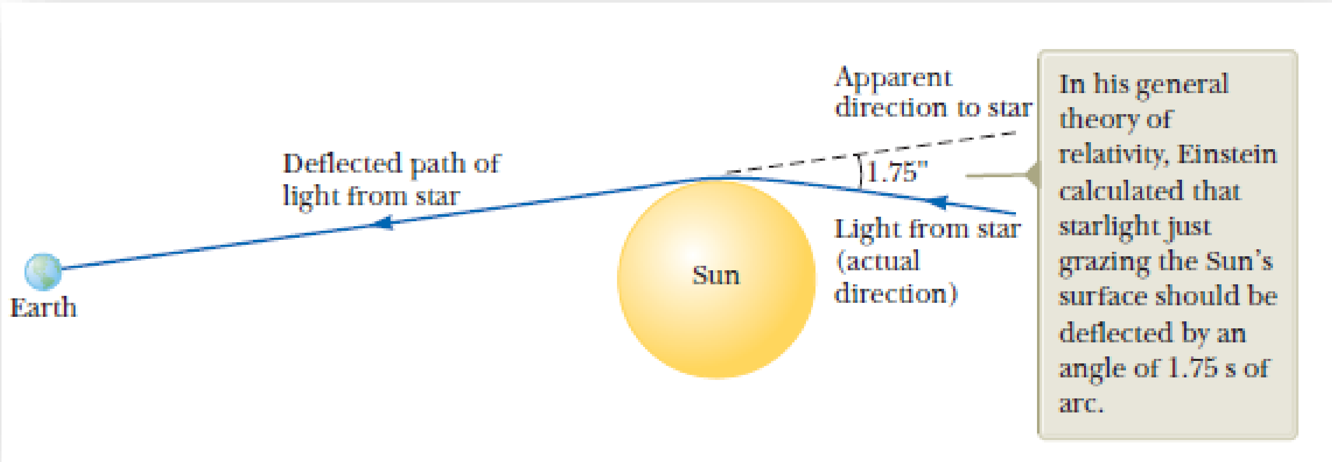
## CURVATURE OF SPACE–TIME

Massive objects, such as stars, planets, and galaxies, curve the space-time around them. The greater the mass or energy density, the greater the curvature of space-time. Objects move along paths dictated by the curvature of space-time, which we perceive as being influenced by gravity. For example, Earth orbits the Sun because the Sun's mass curves the space-time around it, causing Earth to follow a curved path.



## DEFLECTION OF LIGHT

The most dramatic test of general relativity came shortly after the end of World War I. Einstein's theory predicts that a star would bend a light ray by a certain precise amount. Sir Arthur Eddington mounted an expedition to Africa and, during a solar eclipse, confirmed that starlight bent on passing the Sun in an amount matching the prediction of general relativity (Fig). When this discovery was announced, Einstein became an international celebrity.




# SHORT REASONING QUESTIONS

**1. Show that for values of v << c, Lorentz transformation reduces to the Galilean transformation.**

Ans The Lorentz transformation equations are given by

$$t' = \gamma\left(t - \frac{v\,x}{c^2}\right)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

where $\gamma$ is the Lorentz factor for the object's motion

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

If the relative velocity $v$ is very small, approaching zero $(v \ll C)$

$$\gamma = \frac{1}{\sqrt{1-\frac{(0)^2}{C^2}}}$$

$$\gamma = \frac{1}{\sqrt{1-0}}$$

$$\gamma = \frac{1}{1}$$

$$\gamma = 1$$

$\gamma = 1$ *and relatice velocity is very smaller than the speed of light* $(v = 0)$

Substituting these values in Lorentz transformation equations, we get

$$t' = \gamma\left(t - \frac{v\,x}{c^2}\right) = 1\left(t - \frac{(0)\,x}{c^2}\right) = t$$

$$t' = t$$

$$x' = \gamma(x - vt) = 1(x - vt)$$

$$x' = (x - vt)$$

$$y' = y$$

$$z' = z$$

The above results show that the Lorentz transformation becomes equivalent to the Galilean transformation when the relative velocities involved are much smaller than the speed of light

2 If a particle could move with the velocity of light, how much K.E. would it possess?

Ans a particle with mass can't reach the speed of light. At Fermi lab, for example, protons are accelerated close to the speed of light, and it takes a huge amount of energy.
The rest mass does not change - by definition, it is the mass, or equivalent energy, of a particle At rest. The total energy is the particle's rest mass energy plus its kinetic energy. Einstein discovered that the total energy of a particle moving at speeds close to the speed of light (relativistic speeds) is given as

$$E = \frac{m_0\,c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \frac{m_0\,c^2}{\sqrt{1 - \frac{c^2}{c^2}}}$$

$$E = \frac{m_0\,c^2}{\sqrt{1 - 1}}$$

$$E = \frac{m_0\,c^2}{0} = energy\ undefine$$

The above calculation shows that as v approaches c, the denominator approaches zero, so the total energy becomes undefined.

**3 Explain the difference between Special and General Relativity in simple terms**

*The Key Differences between Special and General Relativity*

**1 Space-time and inertial frame**

The general theory of relativity and special theory of relativity is that general theory of relativity deals with the space-time continuum whereas, Special relativity only deals with the inertial frames.

### 2. The Role of Acceleration

Special relativity deals with the behavior of objects moving at constant velocities in the absence of gravitational forces. On the other hand, general relativity takes into account the effects of acceleration due to gravity and treats it as a manifestation of the curvature of spacetime.

### 3. Gravitational Time Dilation

Time dilation is a fascinating concept in special relativity, which states that time can pass at different rates for observers in relative motion. However, in general relativity, gravitational time dilation comes into play. This phenomenon occurs when there is a difference in the strength of the gravitational field at different locations.

### 4 Differentiate between Inertial Frames of Reference and Non- Inertial Frames of Reference.

| S No | Inertial Frames of Reference | Non- Inertial Frames of Reference |
|---|---|---|
| 1 | An inertial frame of reference is one in which an object either remains at rest or moves at a constant velocity | A non-inertial frame of reference is one that is accelerating or rotating |
| 2 | Newton's laws of motion hold true in inertial frames, without any additional forces. | Newton's laws do not hold true in non-inertial frames without introducing additional forces |
| 3 | Inertial frames do not experience acceleration. | Non-inertial frames are characterized by acceleration. |
| 4 | There are no fictitious forces acting on objects. The only forces present are real forces (like gravity, friction, etc.). | Objects may appear to accelerate without any real force acting on them, leading to the introduction of fictitious forces (also known as pseudo-forces) |
| 5 | Examples include a stationary observer on the surface of the Earth (ignoring its rotation for simplicity) or an observer in deep space moving at a constant velocity. | Common examples include a rotating carousel, an accelerating car, or an elevator that is speeding up or slowing down. |

### 5 Why can't any object move at the speed of light

1. ***Mass and Energy Relationship***: According to the equation $E = mc^2$, as an object with mass accelerates towards the speed of light, its relativistic mass increases. This means that it requires more and more energy to continue accelerating. As the object approaches the speed of light, its mass approaches infinity, and thus it would require an infinite amount of energy to reach the speed of light, which is impossible.

2. ***Time Dilation***: As an object moves closer to the speed of light, time for that object slows down relative to an outside observer. If an object were to reach the speed of light, time would effectively stop for it. This creates paradoxes and contradictions in the laws of physics as we understand them.

3. ***Length Contraction***: Objects moving at relativistic speeds experience length contraction, meaning they appear shorter in the direction of motion. At the speed of light, this contraction would theoretically reduce the length of the object to zero, which is not physically meaningful.

**6 What are the limitations in the Galilean Transformation Equation, and how did Lorentz solve it?**

Ans **Limitations of Galilean Transformation:**

***Assumption of Absolute Time***:

The Galilean Transformation assumes that time is the same for all observers, regardless of their relative motion.

***Invariance of Velocity***:

Galilean assumes that velocities add linearly. For example, if one observer measures a speed of $v$ and another observer moves at speed $u$, the relative speed should simply be $(v + u)$. This assumption fails at speeds approaching the speed of light.

***Incompatibility with Electromagnetism***:

The equations derived from Galilean Transformation do not account for the behavior of light and electromagnetic waves, which travel at a constant speed $c = 3 \times 10^8\ m/s$ in all inertial frames, regardless of the motion of the source or observer.

**LORENTZ'S SOLUTION:**

**1** ***Relativity of Time***: Lorentz introduced the concept that time is not absolute but relative. He proposed that time can differ for observers in different inertial frames,

***2*** ***Lorentz Transformation Equations***: Lorentz derived a new set of transformation equations that account for the effects of relative motion at high speeds.The Lorentz Transformation equations are given by

$$t' = \gamma\left(t - \frac{v\,x}{c^2}\right)$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

where $\gamma$ is the Lorentz factor for the object's motion, which approaches infinity as $v$ approaches speed of light

$$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

**3** ***Constancy of the Speed of Light:***

Lorentz's transformations ensure that the speed of light remains constant (c) for all observers, regardless of their relative motion.

**7. Calculate the value of y (Lorentz factor) if the object is moving at the speed of light.**

Ans where $\gamma$ is the Lorentz factor for the object's motion

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If the relative velocity $v = c$

$$\gamma = \frac{1}{\sqrt{1 - \frac{c^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - 1}}$$

$$\gamma = \frac{1}{0}$$

$$\gamma = \infty$$

This means that the Lorentz factor γ approaches infinity ($\gamma \rightarrow \infty$) when an object moves at the speed of light.