UNIT = 24 RELATIVITY

BOOK NUMERICAL

1. A rod 1 meter long is moving along its length with a velocity 0.6c. Calculate its length as it appears to an observer (a) on the earth (b) moving with the rod itself.

Data:

$L_0 = 1\ m$

$v = 0.6\ c$

$L = ?$ (on the Earth)

$L = ?$ (moving with the rod)

SOLUTION:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$L = 1 \times \sqrt{1 - \frac{(0.6\ c)^2}{c^2}}$$

$$L = 1 \times \sqrt{1 - \frac{0.36\ c^2}{c^2}}$$

$$L = 1 \times \sqrt{1 - 0.36} = 1 \times \sqrt{0.64}$$

$$L = 0.8\ m$$

When the observer moving with the rod itself, observed the same length

$L = 1\ m$

2 How fast would a rocket have to go relative to an observer for its length to be contracted to 99% of its length at rest?

Data:

$L = 99\%\ L_0$

$L = 0.99\ L_0$

$v = ?$

SOLUTION:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$0.99\ L_0 = L_0 \times \sqrt{1 - \frac{(0.6\ c)^2}{c^2}}$$

$$0.99 = 1 \times \sqrt{1 - \frac{v^2}{c^2}}$$

squaring both the side

$$(0.99)^2 = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^2$$

$$0.9801 = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - 0.9801$$

$$\frac{v^2}{c^2} = 0.0199$$

$$v^2 = 0.0199\ c^2$$

$$v = \sqrt{0.0199}\sqrt{c^2}$$

$$v = 0.141 \times c = 0.141 \times 3 \times 10^8$$

$$v = 4.23 \times 10^7\ m/s$$

3 A particle with a proper lifetime of 1 μ s moves through the laboratory at 2.7 x 10^8 m/s (a) What is its lifetime, as measured by observers in the laboratory? (b) What will be the distance traversed by it before disintegrating?

Data:

$\Delta t_0 = 1\ \mu s = 1 \times 10^{-6}\ s$

$v = 2.7 \times 10^8\ m/s$

$\Delta t = ?$

SOLUTION:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \frac{1 \times 10^{-6}}{\sqrt{1 - \frac{(2.7 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$\Delta t = \frac{1 \times 10^{-6}}{\sqrt{1 - 0.81}} = \frac{1 \times 10^{-6}}{\sqrt{0.19}}$$

$$\Delta t = \frac{1 \times 10^{-6}}{0.43588}$$

$$\Delta t = 2.294 \times 10^{-6}\ s$$

$$S = v\ \Delta t$$

$$S = (2.7 \times 10^8)\ (2.294 \times 10^{-6})$$

$$S = 619.38\ m$$

4. At what speed is a particle moving if the mass is equal to three times its rest mass?

Data:

$v = ?$

$m = 3\ m_0$

SOLUTION:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3\ m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring both sides

$$(3)^2 = \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right)^2$$

$$9 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$v^2 = \frac{8}{9}\ C^2$$

$$\sqrt{v^2} = \sqrt{\frac{8}{9}}\ \sqrt{C^2}$$

$$v = \sqrt{\frac{8}{9}}\ c$$

$$v = \frac{2\sqrt{2}}{3}\ c$$

5. **How much energy is produced if 4 kg of a substance is fully converted into energy?**

Data: $m = 4$ kg $C = 3 \times 10^8\ m/s$ $E = ?$ **SOLUTION:** $E = m\,c^2$	$E = (4)\,(3 \times 10^8)^2$ $E = (4)\,(9 \times 10^{16})$ $E = 36 \times 10^{16}$ $E = 3.6 \times 10^{17}\,J$

6. **Calculate the rest energy of an electron in joules and in electron volts.**

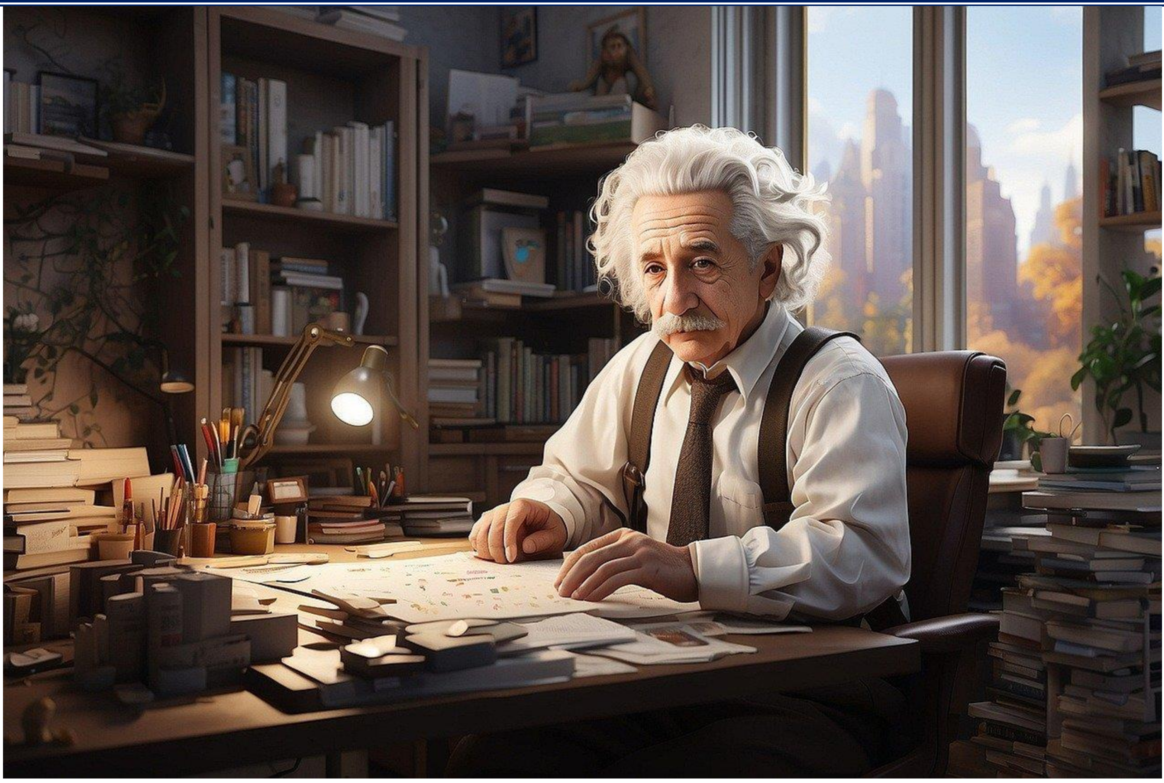
Data: $m = 9.1 \times 10^{-31}$ kg $C = 3 \times 10^8\ m/s$ $E = ?$ **SOLUTION:** $E = m\,c^2$ $E = (9.1 \times 10^{-31})\,(3 \times 10^8)^2$ $E = (9.1 \times 10^{-31})\,(9 \times 10^{16})$	$E = 8.19 \times 10^{-14}\,J$ *Energy in electron volt* $1\,eV = 1.6 \times 10^{-19}\,J$ $E = \dfrac{8.19 \times 10^{-14}}{1.6 \times 10^{-19}}$ $E = 511 \times 10^3$ $E = 0.511 \times 10^6\,eV$ $E = 0.511\ MeV$

7. **Calculate the K.E. of an electron moving with a velocity of 0.98 times the velocity of light in the laboratory system.**

Data: $E_k = ?$ $v = 0.98\,c$ $E = ?$ **SOLUTION:** $E = m\,c^2$ $E = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \times c^2$ $E = \dfrac{m_0\,c^2}{\sqrt{1 - \frac{(0.98\,c)^2}{c^2}}}$	$E = \dfrac{m_0\,c^2}{\sqrt{1 - \frac{0.9604 \times c^2}{c^2}}} = \dfrac{m_0\,c^2}{\sqrt{1 - 0.9604}}$ $E = 5\ m_0\,c^2$ Now $E = m_0\,c^2 + E_k$ $5\,m_0\,c^2 = m_0\,c^2 + E_k$ $5\,m_0\,c^2 - m_0\,c^2 = E_k$ $E_k = 4\,m_0\,c^2 = 4(9.1 \times 10^{-31})(3 \times 10^8)^2$ $E_k = 3.27 \times 10^{-13}\,J$

8. At what velocity does the K.E. of a particle equal its rest energy?

Data:	**Squaring both sides**
$v = ?$ $E_k = m_0c^2$ **SOLUTION:** $E = m_0\,c^2 + E_k$ $E = m_0\,c^2 + m_0\,c^2$ $E = 2\,m_0\,c^2$ $E = m\,c^2$ $E = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \times c^2$ $2\,m_0\,c^2 = \dfrac{m_0\,c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ $2 = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	$(2)^2 = \left(\dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2$ $4 = \dfrac{1}{1-\frac{v^2}{c^2}}$ $1 - \dfrac{v^2}{c^2} = \dfrac{1}{4}$ $\dfrac{v^2}{c^2} = 1 - \dfrac{1}{4} = \dfrac{3}{4}$ $v^2 = \dfrac{3}{4}\,C^2$ $\sqrt{v^2} = \sqrt{\dfrac{3}{4}}\,\sqrt{C^2}$ $v = \sqrt{\dfrac{3}{4}}\,c$ $v = \dfrac{\sqrt{3}}{2}\,c$



9. **A particle of rest mass m_o moves with speed $\frac{c}{\sqrt{2}}$. Calculate its mass, momentum, total energy and kinetic energy.**

Data: $v = \frac{c}{\sqrt{2}}\ m/s$ $m = ?$ $P = ?$ $E = ?$ **SOLUTION:** $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $m = \frac{m_0}{\sqrt{1 - \frac{\left(\frac{c}{\sqrt{2}}\right)^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{c^2}{2\,c^2}}}$ $m = \frac{m_0}{\sqrt{1 - \frac{1}{2}}} = \frac{m_0}{\sqrt{\frac{1}{2}}}$ $m = \sqrt{2}\ m_0$	**Relativistic momentum** $P = m\,v$ $P = \sqrt{2}\ m_0 \times \frac{c}{\sqrt{2}}$ **Total energy** $P = m_0\,c$ $E = m\,c^2$ $E = \sqrt{2}\ m_0\ c^2$ **Kinetic energy** $E = m_0\ c^2 + E_k$ $\sqrt{2}\ m_0\ c^2 = m_0\ c^2 + E_k$ $1.4142\ m_0\ c^2 - m_0\ c^2 = E_k$ $0.4142\ m_0\ c^2 = E_k$

10. **The nearest star to Earth is Proximal Centauri, 4.3 light years away. A spaceship with a constant speed of 0.800c relative to the Earth travels toward the star.**

(a) **How much time would elapse on a clock as measured by travelers on the spacecraft?**

(b) **How long does the trip take according to Earth observers?**

Data:

$S = 4.3 \text{ light} - \text{ year}$

$1 \; light \; year = 9.461 \times 10^{15} \; m$

$S = 4.3 \times 9.461 \times 10^{15}$

$S = 4.06823 \times 10^{16} \; m$

$v = 0.800 \; c$

$\Delta t = ?$

SOLUTION:

$S = v \, t$

$\Delta t_{Earth} = \frac{S}{v} = \frac{S}{0.800 \; C}$

$\Delta t_{Earth} = \frac{4.06823 \times 10^{16}}{0.800 \times (3 \times 10^{8})}$

$\Delta t_{Earth} = 1.695 \times 10^{8} \; s$

Time in year

$\Delta t_{Earth} = \frac{1.695 \times 10^{8} \; s}{3.156 \times 10^{7}}$

$\Delta t_{Earth} = 5.37 \; years$

$\Delta t_{Earth} = \frac{\Delta t_{ship}}{\sqrt{1 - \frac{v^2}{c^2}}}$

$1.695 \times 10^{8} = \frac{\Delta t_{ship}}{\sqrt{1 - \frac{(0.800 \; c)^2}{c^2}}}$

$1.695 \times 10^{8} = \frac{\Delta t_{ship}}{\sqrt{1 - \frac{(0.800 \; c)^2}{c^2}}}$

$1.695 \times 10^{8} = \frac{\Delta t_{ship}}{\sqrt{1 - 0.640}}$

$1.695 \times 10^{8} = \frac{\Delta t_{ship}}{0.600}$

$\Delta t_{ship} = 1.695 \times 10^{8} \times 0.600$

$\Delta t_{ship} = 1.017 \times 10^{8} \text{ sec}$

$\Delta t_{ship} = \frac{1.017 \times 10^{8}}{3.156 \times 10^{7}}$

$\Delta t_{ship} = 3.225 \text{ years}$

WORKED EXAMPLES

1 The pendulum's period is measured to be 3.00 s in the inertial frame of the pendulum. What is the period as measured by an observer moving at a speed of 0.950 c with respect to the pendulum?

Data: $\Delta t_0 = 3.00\ s$ $v = 0.95\ c$ $\Delta t = ?$ **SOLUTION:** $\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$	$\Delta t = \dfrac{3.00}{\sqrt{1 - \frac{(0.95\ c)^2}{(c)^2}}}$ $\Delta t = \dfrac{3.00}{\sqrt{1 - 0.9025}}$ $\Delta t = \dfrac{3.00}{0.3122}$ $\Delta t = 9.61\ s$

2 A starship is measured to be 125 m long while at rest with respect to an observer. If this starship flies past the observer at a speed of 0.99 c, what length will the observer measure for the starship?

Data: $L_0 = 125\ m$ $v = 0.99\ c$ $L = ?$ **SOLUTION:** $L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$ $L = 125 \times \sqrt{1 - \frac{(0.99\ c)^2}{c^2}}$	$L = 125 \times \sqrt{1 - (0.99)^2}$ $L = 125 \times \sqrt{0.0199}$ $L = 3.52\ m$

3 At what speed will an object's relativistic mass be twice its rest mass?

Data: $v = ?$ $m = 2\,m_0$ **SOLUTION:** $m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $2\,m_0 = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ $2 = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ **Squaring both sides** $(2)^2 = \left(\dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\right)^2$	$4 = \dfrac{1}{1 - \frac{v^2}{c^2}}$ $1 - \dfrac{v^2}{c^2} = \dfrac{1}{4}$ $\dfrac{v^2}{c^2} = 1 - \dfrac{1}{4} = \dfrac{3}{4}$ $v^2 = \dfrac{3}{4}\,C^2$ $\sqrt{v^2} = \sqrt{\dfrac{3}{4}}\,\sqrt{C^2}$ $v = \sqrt{\dfrac{3}{4}}\,c$ $v = \dfrac{\sqrt{3}}{2}\,c$

4 An electron has a velocity of 0.990 c. Calculate the relativistic factor γ using the Relativity Formula.

Data: $v = 0.99\,c$ $\gamma = ?$ **SOLUTION:** $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\gamma = \dfrac{1}{\sqrt{1 - \frac{(0.99\,c)^2}{c^2}}}$	$\gamma = \dfrac{1}{\sqrt{1 - (0.99)^2}}$ $\gamma = \dfrac{1}{\sqrt{0.0199}}$ $\gamma = \dfrac{1}{0.141}$ $\gamma = 7.09$ *Therefore the relativistic factor will be 7.09.*