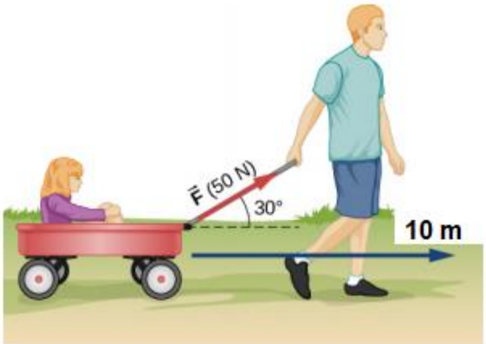
CHAPTER = 5 WORK ENERGY AND POWER

BOOK NUMERICAL

1 A man pulls a trolley through a distance of 10 m by applying a force of 50N which makes an angle of 60^0 with the horizontal. Calculate the work done by the man.

DATA

F = 50 N
S = 10 m
$\theta = 30^0$
W = ?



SOLUTIONS

$$W = F\,S\,cos\theta$$

$$W = (50)(10)\,cos30^0$$

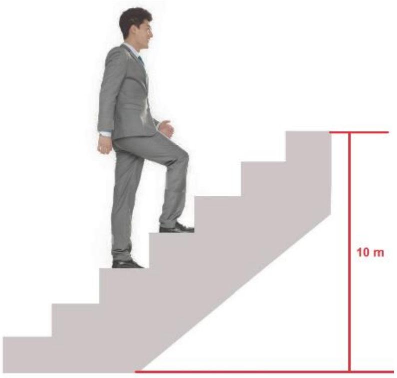
$$W = (50)(10)\,(0.866)$$

$$W = 433\,J$$

2 A 100 kg man runs up a long flight of stairs in 9.8 seconds. The vertical height of the stair is 10m Calculate its power.

DATA

m = 100 kg
t = 9.8 sec
h = 10 m
P = ?



SOLUTIONS

$$P = \frac{m\,g\,h}{t}$$

$$P = \frac{100 \times 9.8 \times 10}{9.8}$$

$$P = 1000\,W$$

$$1\,hp = 746\,W$$

$$P = \frac{1000\,W}{746}$$

$$P = 1.34\,hp$$

3 When an object is thrown upwards. It rises to a height 'h'. How high is the object in terms? of h, when it has lost one-third of its original kinetic energy?

DATA Height = h ; H = ? for which K.E Lost = $\frac{1}{3} \times initial \;\; K.E$ **SOLUTION:** Initial K.E at the ground = $\frac{1}{2} mv^2$ but P.E = 0 At max height 'h' i.e. at point P. Final K.E = 0 P.E. = mgh $\Rightarrow$ Loss in K.E = Gain in P.E .... (i)	or $\frac{1}{2} mv^2 = mgh$ Let 's' be the height of an object when it has lost $\frac{1}{3}$ of its K.E then Eq. (i) $\Rightarrow$ $\frac{1}{3}$ loss in K.E = $\frac{1}{3}$(Gain in P.E) $\frac{1}{3}\left(\frac{1}{2}mv^2\right) = mgH$ $\frac{1}{3}(mgh) = mgH$ $\Rightarrow$ $H = \frac{1}{3}$ h

4 A 70 kg man runs up a hill through a height of 3m in 2 seconds.

(a) How much work does she do against the gravitational field?

(b) What is the average power output?

DATA m = 70 kg h = 3 m t = 2 sec W =? P = ? **SOLUTIONS** $W = F S$ $W = (m g)(h)$ $W = (70)(9.8)(3)$ $W = 2058 \; J$	$P = \frac{W}{t}$ $P = \frac{2058}{2}$ $P = 1029 \, W$ $1 \, hp = 746 \, W$ $P = \frac{1029 \, W}{746}$ $P = 1.379 \, hp$

5 A neutron travels a distance of 12 m in a time interval of 3.6 x 10^{-4} sec. Assuming its speed was constant, Find its kinetic energy. Take the mass of neutron 1.7 x 10^{-27} kg.

DATA

S = 12 m

t = 3.6 x 10^{-4} sec

m = 1.7 x 10^{-27} kg

K.E = ?

SOLUTIONS

$$v = \frac{s}{t}$$

$$v = \frac{12}{3.6 \text{ x } 10-4}$$

$$v = \frac{12}{3.6 \text{ x } 10-4}$$

$$v = 3.33 \times 10^4 \ m/s$$

$$K.E = \frac{1}{2} m v^2$$

$$K.E = \frac{1}{2}(1.7 \text{ x } 10^{-27})\ (3.33 \times 10^4)^2$$

$$K.E = 9.425 \times 10^{-19}$$

$$1\ eV = 1.6 \times 10^{-19}$$

$$K.E = \frac{9.425 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$K.E = 5.89\ eV$$

6 A stone is thrown vertically upwards and can reach a height of 10 m, find the speed of the stone when it is just 2 m above ground.

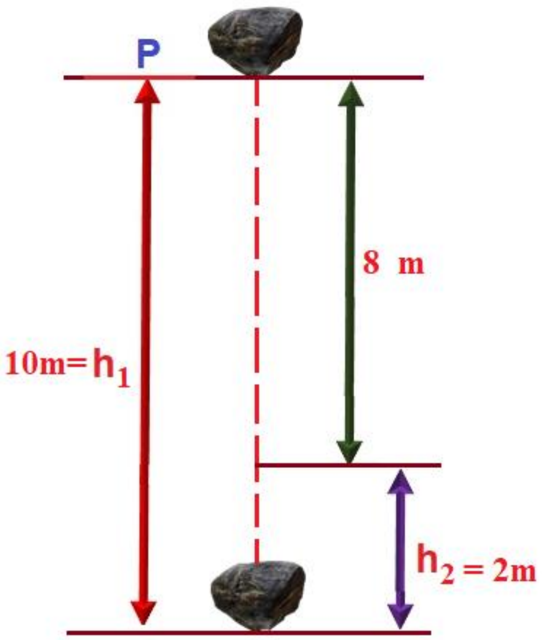
DATA

h_1 = 10 m

h_2 = 2 m

V = ?

SOLUTIONS



$$\Delta(K.E) = \Delta(P.E)$$

$$\frac{1}{2} m\left({v_f}^2 - {v_i}^2\right) = mg(h_1 - h_2)$$

$$\frac{1}{2}\left({v_f}^2 - {v_i}^2\right) = g(h_1 - h_2)$$

$$\frac{1}{2}\left({v_f}^2 - 0\right) = (9.8)(10 - 2)$$

$${v_f}^2 = (9.8)(8)(2)$$

$${v_f}^2 = 156.8$$

Taking square root on both the sides

$$\sqrt{{v_f}^2} = \sqrt{156.8}$$

$$v_f = 12.52\ m/s$$

7 The potential energy of a body at the top of a building is 200 Joules when it is dropped its kinetic energy just before striking the ground is 160 Joules, find the work done against the air resistance.

DATA **P.E = 200 J** **K.E = 160 J** **W = ?** **SOLUTIONS** Loss of P.E $= \text{Gain in K.E}$ $+ \text{work against air resistance}$	$\Delta P.E = \Delta K.E + W$ $\Delta P.E - \Delta K.E = W$ $200 - 160 = W$ $40\ J = W$

8 Find the energy equivalent of 1 gram?

DATA **m = 1 g** $= 1 \times 10^{-3}\ kg$ **C = 3** $\times 10^{8}\ m/s$ **E = ?** **SOLUTIONS** $E = m C^2$	$E = m C^2$ $E = (1 \times 10^{-3})(3 \times 10^{8})^2$ $E = 9 \times 10^{13}\ J$

9 A Kilowatt motor pump, pumps the water from the ground to a height of 10 m Find, how much liters of water it can pump in one hour?

DATA **P = 1 KW= 1000 W** **h = 10 m** **t = 1 h = 3600 sec** **m =?** **SOLUTIONS** $P = \frac{W}{t}$ $P = \frac{mgh}{t}$	$1000 = \frac{m \times 9.8 \times 10}{3600}$ $1000 \times 3600 = m \times 9.8 \times 10$ $3600000 = m \times 98$ $m = \frac{3600000}{98}$ $m = 3.673 \times 10^{4}\ kg$ $1\ kg = 1\ Litre$ $m = 3.673 \times 10^{4}\ litre$

10 A rocket of mass 2kg is launched into the air, and when it attains a height of 15 m, the 400 Joules of its chemical fuel burn. Find the speed of the rocket at maximum height.

DATA **m = 2 kg** **Δh = 15 m** **W = 400 J** **V_f =?** **SOLUTIONS** **Work done = change in K.E + change inP.E** $W = \frac{1}{2}mV_f^2 - \frac{1}{2}mV_i^2 + mgh_2 - mgh_1$ $W = \frac{1}{2}m(V_f^2 - V_i^2) + mg(h_2 - h_1)$	$W = \frac{1}{2}m(V_f^2 - V_i^2) + mg\Delta h$ $400 = \frac{1}{2} \times 2(V_f^2 - 0) + 2 \times 9.8 \times 15$ $400 = V_f^2 + 294$ $400 - 294 = V_f^2$ $106 = V_f^2$ *Taking square root on both the side* $\sqrt{106} = \sqrt{V_f^2}$ $10.29\ m/s = v_f$

11 A motor pumps the water at the rate of 500 gram/min to the height of 120 m If the motor is 50% efficient then how much input electric power is needed.

DATA $\frac{m}{t} = 500\ g/min$ $\frac{m}{t} = \frac{\frac{500}{1000}}{60}\ kg/s$ $\frac{m}{t} = 8.33 \times 10^{-3}\ kg/s$ **h = 120 m** $\eta = 50\ \%$ P_{in} =? **SOLUTIONS** $P_{out} = \frac{W}{t}$ $P_{out} = \frac{mgh}{t}$	$P_{out} = \frac{m}{t}(g\ h)$ $P_{out} = (8.33 \times 10^{-3})(9.8 \times 120)$ $P_{out} = 9.79\ Watt$ $\eta = \frac{P_{out}}{P_{in}} \times 100$ $P_{in} = \frac{9.79}{50} \times 100$ $P_{in} = 19.58\ Watt$

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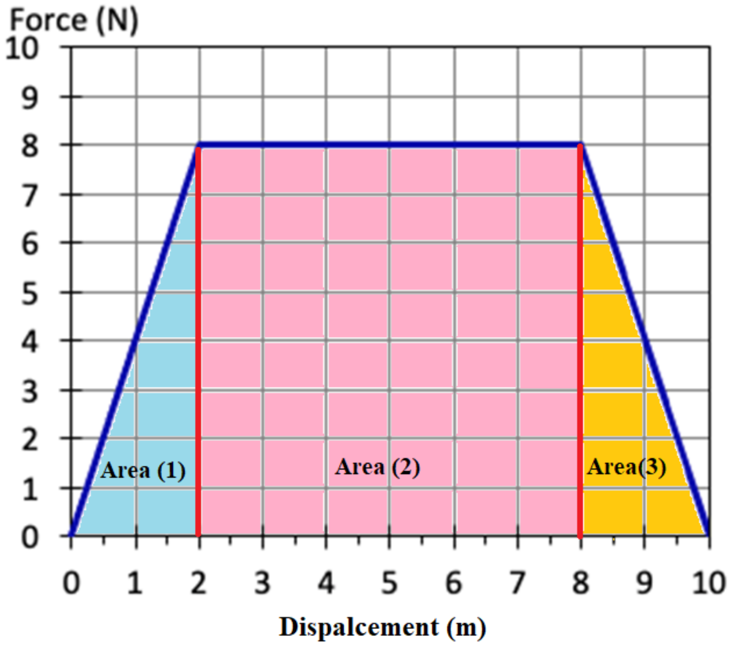
ADDITIONAL NUMERICAL

1 A variable force is applied on an object during the distance of 10 m as shown below. Find the work done by this force over the 10 m distance.



The total work done is given by the shaded area.

$$Total\ work\ done = Area(1) + Area(2) + Area(3)$$

$$Total\ work\ done = \frac{1}{2}(force)(displacement) + (force \times displacement) + \frac{1}{2}(force)(displacement)$$

$$Total\ work\ done = \frac{1}{2}(8)(2) + (8 \times 6) + \frac{1}{2}(8)(2)$$

$$W = 8 + 48 + 8$$

$$W = 64\ J$$

2 The mass of the earth is 5.98 x 10^{24} kg and the mass of the sun is 1.99 x 10^{30} kg, and the earth is 160 million km away from the sun, Calculate the GPE of the earth

DATA	SOLUTIONS
$M_e = 5.98 \times 10^{24}\ kg$	$U = -\frac{G\ M_e\ M_s}{R}$
$M_s = 1.99 \times 10^{30}\ kg$	$U = -\frac{6.67 \times 10^{-11}\ (5.98 \times 10^{24})\ (1.99 \times 10^{30})}{1.60 \times 10^{11}}$
$R = 160\ million\ km$	$U = -\frac{7.937 \times 10^{44}}{1.60 \times 10^{11}}$
$R = 160 \times 10^{6} \times 10^{3}\ m$	$U = -\ 4.96 \times 10^{33}\ J$
$R = 1.60 \times 10^{11}\ m$	
$GPE\ (U) = ?$	

3 A person riding their bike has a mass of 120 kg; they are riding at 10 m/s. suddenly a dog crosses the road and to avoid hitting the dog the bicyclist brakes applying a braking force of 500 N for a distance of 10 meters. What is the final velocity of the bicyclist when they stop braking?

DATA $m = 120\ kg$ $v_i = 10\ m/s$ $F = 500\ N$ $S = 10\ m$ $v_f = ?$ **SOLUTIONS** $Total\ work\ done =$ $change\ in\ kinetic\ energy$ $W = \frac{1}{2} m\,V_f^2 - \frac{1}{2} m\,V_i^2$ $F\ \ S = \frac{1}{2} m\,V_f^2 - \frac{1}{2} m\,V_i^2$	$(-500)\ (10) = \frac{1}{2}(120)\,V_f^2 - \frac{1}{2}(120)(10)^2$ $-5000 = 60 \times V_f^2 - 6000$ $6000 - 5000 = 60 \times V_f^2$ $\frac{1000}{60} = V_f^2$ $16.66 = V_f^2$ $taking\ square\ root\ on\ both\ the\ side$ $\sqrt{16.66} = \sqrt{V_f^2}$ $4.08\ m/s = V_f$

4 The kinetic energy of a car is 7000 J as it travels along a horizontal road. How much work is required to stop the car in 20 s?

DATA $(K.E)_i = 700\ J$ $(K.E)_f = 0\ J$ $t = 20\ sec$ $W = ?$	**SOLUTIONS** $work\ done\ to\ stop\ the\ car = change\ in\ kinetic\ energy$ $W = (K.E)_f - (K.E)_i$ $W = 0 - 7000$ $W = -7000\ J$ **negative sign means work is done against car**

5 A motorboat moves at a steady speed of 4 $m\ s^{-1}$. Water resistance acting on it is 4000 N. Calculate the power of its engine.

DATA $v = 4\ m/s$ $F = 4000\ N$ $P = ?$	**SOLUTIONS** $Power = (force).(velocity)$ $P = F\,v$ $P = (4000)\ (4)$ $P = 16000\ W$

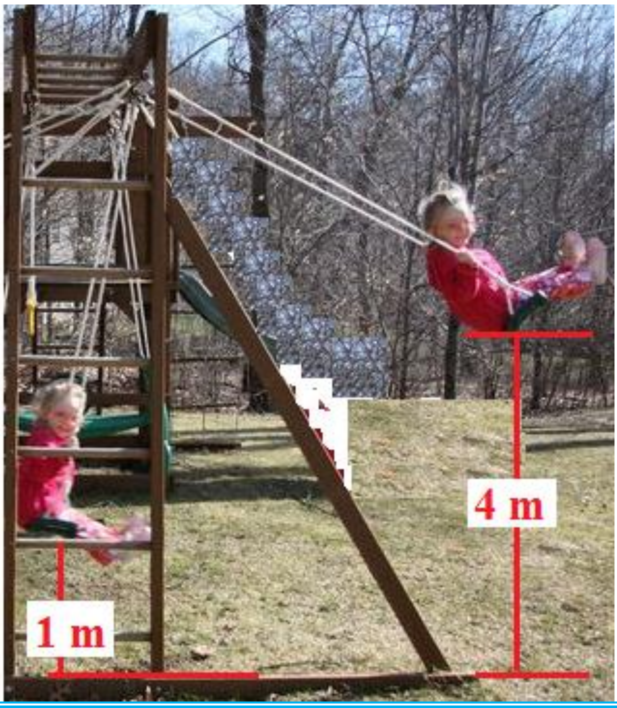
6 A 75 kg man moved 65 steps up in 20 seconds. Find his power, if each step is 15 cm high.

DATA	SOLUTIONS
$m = 75\,kg$ $total\ step = 65$ $t = 20\,sec$ Height of each step= 15 cm = 0.15 m $h =$ $(total\ step)(height\ of\ each\ step)$ $h = (65)(0.15) = 9.75\,m$ $P_{in} =?$	$P = \frac{W}{t}$ $P = \frac{mgh}{t}$ $p = \frac{75 \times 9.8 \times 9.75}{20}$ $P = \frac{7.166 \times 10^3}{20}$ $P = 358\,W$

7 An electric motor of 1hp is used to run the water pump. The water pump takes 10 minutes to fill an overhead tank. The tank has a capacity of 800 liters and a height of 15 m. Find the actual work done by the electric motor to fill the tank. Also, find the efficiency of the system. (Mass of 1 liter of water = 1 kg)

DATA	
$P = 1\,hp = 746\,W$ $t = 10\,min = 10 \times 60 = 600\,sec$ $V = 800\,Litre$ $m = 800\,Kg$ $\{1\ litre\ of\ water = 1\,kg\}$ h = 15 m W = ? ɳ =? **SOLUTIONS** $P = \frac{W}{t}$ $W = P \times t$	$W = 746 \times 600$ $W_{input} = 447600\,J$ $W_{out} = m\,g\,h$ $W_{out} = 800 \times 9.8 \times 15$ $W_{out} = 117600\,J$ $\eta = \frac{W_{out}}{W_{in}} \times 100$ $\eta = \frac{117600}{447600} \times 100$ $\eta = 26.27\,\%$

8 A boy in a swing is 4 meters from the ground at his highest point and 1 meter from it at his lowest point. What is his maximum speed?



DATA

$h_2 = 4$ m

$h_1 = 1$ m

$\Delta h = 3\ m$

SOL

$$\textit{loss of kinetics energy} = \textit{gain of potential energy}$$

$$mg\Delta h = \frac{1}{2} m V^2$$

$$g\Delta h = \frac{1}{2} V^2$$

$$2g\Delta h = V^2$$

$$\sqrt{2g\Delta h} = V$$

$$\sqrt{2 \times 9.8 \times 3} = V$$

$$7.66\ m/s = V$$

9 A water pump is needed to lift water through a height of 2.5m at the rate of 500gm/minute. What will be the minimum horsepower of the pump?

Data:

$g = 9.8\ m/sec^2$

$h = 2.5$ m

$$\frac{m}{t} = 500 \frac{gm}{min}$$

$$\frac{m}{t} = \frac{500}{1000} \frac{kg}{min}$$

$$\frac{m}{t} = \frac{0.50}{60} \frac{kg}{s}$$

P = ? in hp

Solution:

$$P = \frac{\text{work done}}{\text{Time}}$$

$$P = \frac{P.E}{t}$$

$$P = \frac{mgh}{t}$$

$$P = \left(\frac{m}{t}\right) gh$$

$$P = \frac{0.5}{60} \times 9.8 \times 2.5$$

$$P = 0.204\ W$$

$$1hp = 746\ watt$$

$$P = \frac{0.204}{746}$$

$$P = 2.7 \times 10^{-4}\ hp$$