CHAPTER = 17 SECOND LAW OF THERMOGYNAICS

BOOK NUMERICAL

1. 1. A Carnot engine takes 2000J of heat from a reservoir at 500 K does some work, and discards some heat to a reservoir at 350K. How much heat is discarded, how much work does the engine do, and what is the efficiency?

Data: $Q_1 = 2000\ J$ $T_1 = 500\ K$ $T_2 = 350\ K$ $Q_2 = ?$ $\Delta W = ?$ $\eta = ?$ **SOLUTION:** **DISCARDED HEAT** $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ $\frac{2000}{Q_2} = \frac{500}{350}$ $500 \times Q_2 = 350 \times 2000$ $Q_2 = \frac{350 \times 2000}{500}$ $Q_2 = 1400\ J$	**WORK DONE** $\Delta W = Q_1 - Q_2$ $\Delta W = 2000 - 1400$ $\Delta W = 600\ J$ **EFFICIENCY OF CARNOT ENGINE** $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$ $\eta = \left(1 - \frac{350}{500}\right) \times 100$ $\eta = (1 - 0.7) \times 100$ $\eta = 30\ \%$

2 One kilogram of ice at 0°C is melted and converted to water at 0°C. Compute its change in entropy.

Data: $m = 1\ kg$ $T = 0\ ℃ = 273\ K$ $L_f = 3.36 \times 10^5\ J/kg$ $\Delta Q = ?$ $\Delta S = ?$ **SOLUTION:** $\Delta Q = m\, L_f$	$\Delta Q = 1 \times 3.36 \times 10^5$ $\Delta Q = 3.36 \times 10^5\ J$ **Chang in entropy** $\Delta S = \frac{\Delta Q}{T}$ $\Delta S = \frac{3.36 \times 10^5}{273} = 1230\ J/kg$

3. In a high-pressure steam turbine engine, the steam is heated to 600°C and exhausted at about 90°C. What is the highest possible efficiency of any engine that operates between these two temperatures?

Data:	SOLUTION:
$T_1 = 600\ ℃ = 600 + 273 = 873\ K$ $T_2 = 90\ ℃ = 90 + 273 = 363\ K$ $\eta = ?$	EFFICIENCY OF CARNOT ENGINE $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$ $\eta = \left(1 - \frac{363}{873}\right) \times 100$ $\eta = (1 - 0.4158) \times 100$ $\eta = 0.584 \times 100$ $\eta = 58.4\ \%$

4. The temperature difference between the surface water and bottom water in Manchester Lake might be 5°C. Assuming the surface water to be at 20°C. What is the highest efficiency of a steam engine if it operates between these two temperatures?

Data:	SOLUTION:
$\Delta T = 5\ ℃$ $T_1 = 20\ ℃ = 20 + 273 = 293\ K$ $\Delta T = T_1 - T_2$ $T_2 = T_1 - \Delta T$ $T_2 = 20 - 5 = 15\ ℃$ $T_2 = 15 + 273 = 288\ K$ $\eta = ?$	*EFFICIENCY OF STEAM ENGINE* $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$ $\eta = \left(1 - \frac{288}{293}\right) \times 100$ $\eta = (1 - 0.9829) \times 100$ $\eta = 0.0171 \times 100$ $\eta = 1.71\ \%$

5. **A heat engine works at the rate of 500kW. The efficiency of the engine is 30%. Calculate the loss of heat per hour.**

Data:	***EFFICIENCY OF STEAM ENGINE***
$P = 500\ KW = 500\ \times 1000\ W$	$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$
$P = 500\,000\ W = 5.0\ \times\ 10^5\ W$	$\eta = \left(\frac{\Delta Q}{Q_1}\right) \times 100$
$\eta = 30\ \%$	$30 = \left(\frac{1.8\ \times\ 10^9}{Q_1}\right) \times 100$
$t = 1\ h = 1\ \times 3600\ = 3600\ s$	$Q_1 = \left(\frac{1.8\ \times\ 10^9}{30}\right) \times 100 = 6\ \times 10^9\ J$
$Q_2 = ?$	$Q_2 = Q_1\ - \Delta Q$
SOLUTION:	$Q_2 = 6\ \times\ 10^9\ -\ 1.8\ \times\ 10^9$
$\Delta Q = P\ \times t$	$Q_2 = 4.2\ \times\ \ 10^9\ J$
$\Delta Q = 5.0\ \times\ 10^5\ \times 3600$	
$\Delta Q = 1.8\ \times\ 10^9\ J$	

6. **A heat engine performs work of 0.4166 watts in one hour and rejects 4500J of heat to the sink. What is the efficiency of engine?**

Data:	Heat absorbed
$P = 0.4166\ W$	$\Delta Q = Q_1\ - Q_2$
$t = 1\ h = 1\ \times 3600\ = 3600\ s$	$Q_1 = \Delta Q + Q_2$
$Q_2 = 4500\ W$	$Q_1 = 1499.76\ +\ 4500$
$\eta = ?$	$Q_1 = 5999.76$
$Q_1 = ?$	***EFFICIENCY OF STEAM ENGINE***
SOLUTION:	$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$
$\Delta Q = P\ \times t$	$\eta = \left(1 - \frac{4500}{5999.76}\right) \times 100$
$\Delta Q = 0.4166\ \times 3600$	$\eta = (1 -\ 0.75) \times 100\ = 0.25 \times 100$
$\Delta Q = 1499.76\ J$	$\eta = \ 25\ \%$

7. **A Carnot engine operates between the temperatures 850K and 300K. the engine performs 1200J of work in each cycle, which takes 0.25 sec**
(a) What is the efficiency of this engine?
(b) What is the average power of this engine?
(c) How much energy is extracted as heat from the high-temperature reservoir?
(d) How much energy is delivered as heat to the low-temperature reservoir?

Data: $T_1 = 850\ K$ $T_2 = 300\ K$ $W = 1200\ J$ $t = 0.25\ s$ $\eta = ?$ $Q_1 = ?$ **SOLUTION:** ***EFFICIENCY OF STEAM ENGINE*** $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$ $\eta = \left(1 - \frac{300}{850}\right) \times 100$ $\eta = (1 - 0.353) \times 100$ $\eta = 0.647 \times 100$ $\eta = 64.7\%$	**(b) Average power** $P_{av} = \frac{W}{t}$ $P_{av} = \frac{1200}{0.25} = 4800\ J = 4.8\ K\ J$ (c) **Heat absorbed** $Q_1 = \left(\frac{W}{\eta}\right) \times 100$ $Q_1 = \left(\frac{1200}{64.7}\right) \times 100$ $Q_1 = 1854.71\ J$ **(d) energy to sink** $W = Q_1 - Q_2$ $Q_2 = Q_1 - W$ $Q_2 = 1854.71 - 1200$ $Q_2 = 654.71\ J$

8. **A Carnot engine absorbs 52kJ as heat and exhausts 36kJ as heat in each cycle.**
Calculate: (a) The engine efficiency
(b) The work done per cycle in kilojoules.

Data: $Q_1 = 52\ KJ = 52 \times 10^3\ J$ $Q_2 = 32\ KJ = 36 \times 10^3\ J$ SOLUTION ***EFFICIENCY OF STEAM ENGINE*** $\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$	$\eta = \left(1 - \frac{36 \times 10^3}{52 \times 10^3}\right) \times 100$ $\eta = (1 - 0.6923) \times 100$ $\eta = 0.3077 \times 100$ $\eta = 30.77\%$ $W = Q_1 - Q_2$ $W = 52000 - 36000 = 16000\ J$ $W = 16\ KJ$

UNIT 17 SECOND LAW OF THERMODYNAMICS
WORKED EXAMPLES

1 A heat engine operates between two reservoirs at temperatures of 25^0C and 300 ^{0}C. What is the maximum efficiency of this engine?

Data:	SOLUTION:
$T_2 = 25\ ℃$ $T_2 = 25 + 273 = 298\ K$ $T_1 = 300\ ℃$ $T_1 = 300 + 273 = 573\ K$ $\eta = ?$	$\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$ $\eta = \left(1 - \frac{298}{573}\right) \times 100$ $\eta = (1 - 0.52) \times 100$ $\eta = 048 \times 100$ $\eta = 48\ \%$

2 The low-temperature reservoir of a Carnot engine is at 7 ^{0}C and has an efficiency of 40%. How much the temperature of the high-temperature reservoir is increased to increase the efficiency to 50%?

Data:	
$T_2 = 7\ ℃$ $T_2 = 7 + 273 = 280\ K$ $\eta = 40\ \%$ $T_1 = ?$ $T_2 = 7\ ℃$ $T_2 = 7 + 273 = 280\ K$ $\eta = 50\ \%$ $T_1' = ?$ SOLUTION: $\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$ $40 = \left(1 - \frac{280}{T_1}\right) \times 100$ $0.4 = 1 - \frac{280}{T_1}$	$\frac{280}{T_1} = 1 - 0.4$ $T_1 = \frac{280}{0.6} = 466.67\ K$ now $\eta = \left(1 - \frac{T_2}{T_1'}\right) \times 100$ $50 = \left(1 - \frac{280}{T_1'}\right) \times 100$ $0.5 = 1 - \frac{280}{T_1'} \Rightarrow \frac{280}{T_1'} = 1 - 0.5$ $T_1' = \frac{280}{0.5} = 560\ K$ Increase the temperature of the hot reservoir $T = T_1' - T_1 = 560 - 466.67$ $T = 93.33\ K$

3 **What is the change in entropy of 30g water at 0 ^{0}C as it is changed into ice at, 0 ^{0}C?**

Data:	SOLUTION:
$m = 30\ g = 0.03\ Kg$ $T = 0\ ℃ = 0 + 273 = 273\ K$ $L_f = 3.36 \times 10^5\ J/kg\ K$ $\Delta Q = ?$ $\Delta S = ?$	$Q = m\ L_f$ $Q = 0.03 \times 3.36 \times 10^5$ $Q = 1.008 \times 10^4\ J$ $\Delta S = \frac{\Delta Q}{T}$ $\Delta S = \frac{1.008 \times 10^4}{273} = 36.92\ J/Kg$

4 **A refrigerator has a coefficient of performance of 3.25. How much work must be supplied to this refrigerator in order to remove 261 J of heat from its interior?**

Data: $COP = 3.25$ $W = ?$ $Q_C = 261\ J$ **SOLUTION:** $COP = \frac{Q_c}{W}$ $W = \frac{Q_c}{COP}$	$W = \frac{261}{3.25}$ $W = 80.3\ J$ Thus, 80.3 J of work removes 261 J of heat from the refrigerator, and exhausts 80.3 J + 261 J = 341 J of heat into the kitchen.

5 **A heat engine performs 200 J work in each cycle with an efficiency of 20%. For each cycle of operation (a) how much heat is absorbed and (b) how much heat is expelled?**

Data: $W = 200\ J$ $\eta = 20\%$ $Q_1 = ?$ $Q_2 = ?$ **SOLUTION:** $\eta = \frac{W}{Q_1} \times 100$	$Q_1 = \frac{W}{\eta} \times 100$ $Q_1 = \frac{200}{20} \times 100$ $Q_1 = 1000\ J$ $W = Q_1 - Q_2$ $Q_2 = Q_1 - W$ $Q_2 = 1000 - 200 = 800\ J$

6 A hot reservoir at 576 K transfers 1050 J of heat irreversibly to a cold reservoir at 305 K. Find the change in entropy of the universe.

Data:

$T_1 = 576\ K$

$\Delta Q = 1050\ J$

$T_2 = 305\ K$

$\Delta Q = ?$

$\Delta S_{universe} = ?$

SOLUTION:

Calculate the entropy change of the hot reservoir.
This entropy change is negative:

$$\Delta S_{hot\ resrvoir} = -\frac{\Delta Q}{T}$$

$$\Delta S_{hot\ resrvoir} = -\frac{1050}{576}$$

$$\Delta S_{hot\ resrvoir} = -1.82\ J/K$$

Calculate the entropy change of the cold reservoir.
This entropy change is positive:

$$\Delta S_{cold\ resrvoir} = \frac{\Delta Q}{T}$$

$$\Delta S_{cold\ resrvoir} = \frac{1050}{305}$$

$$\Delta S_{cold\ resrvoir} = 3.44\ J/K$$

change of the universe:

$$\Delta S_{universe} = \Delta S_{hot} + \Delta S_{cold}$$

$$\Delta S_{universe} = -1.82 + 3.44$$

$$\Delta S_{universe} = 1.62\ J/kg$$