UNIT-21 SOLID STATE PHYSICS

1. **The 'lead' in pencils is a graphite composition with Young's modulus of $1.0 \times 10^{9}\ \mathrm{N/m^2}$. Calculate the change in length of the lead in an automatic pencil if you tap it straight into the pencil with a force of $4.0\ \mathrm{N}$. The lead is $0.50\ \mathrm{mm}$ in diameter and $60\ \mathrm{mm}$ long. $(1.0\ \mathrm{mm})$**

Data:

$Y = 1.0 \times 10^{9}\ \mathrm{N/m^2}$

Original length $= L = 60\mathrm{mm} = 0.060\mathrm{m}$

$F = 4.0\mathrm{N}$

The diameter of the lead $= D = 0.50\mathrm{mm} = 5 \times 10^{-4}\mathrm{m}$

$r = \frac{D}{2} = \frac{5 \times 10^{-4}}{2} = 2.5 \times 10^{-4}\mathrm{m}$

The change in length $= \Delta L = ?$

SOLUTION:

The area of the cross-section of the lead is given as:

$$A = \pi r^2$$

$$A = 3.14(2.5 \times 10^{-4})^2 = 1.96 \times 10^{-7}\mathrm{m^2}$$

According to Hook's law:

$$\Delta L = \frac{FL}{AY}$$

$$\Delta L = \frac{4.0 \times 0.060}{1.96 \times 10^{-7} \times 1.0 \times 10^{9}} = 0.00122\mathrm{m}$$

$$\Delta L = 1.22\mathrm{mm}$$

2. **A wire of 2.2 m long and 2.25 mm in diameter, when stretched by a weight of 8.8 kg, its length has been increased by 0.25 mm. Find the stress, strain, and Young's modulus of the material of the wire. Given $g = 9.8\ \mathrm{m/s^2}$.**

$(2.2 \times 10^{7} \frac{N}{m^2}, 1.14 \times 10^{-4}, 2 \times 10^{11} \frac{N}{m^2})$

Data:

$\mathbf{L = 2.2\ m}$

$\mathbf{D = 2.25\ mm = 2.25 \times 10^{-3} m}$

$\mathbf{r = \frac{D}{2} = \frac{2.25 \times 10^{-3}}{2} = 1.125 \times 10^{-3} m}$

$\mathbf{m = 8.8\ kg}$

$\mathbf{\Delta L = 0.25mm = 2.5 \times 10^{-4} m}$

SOLUTION:

The force acting on the wire is given as:

$$F = W = mg$$

$$F = 8.8 \times 9.8 = 86.24N$$

The area of the cross-section of the lead is given as:

$$A = \pi r^2$$

$$A = 3.14(1.125 \times 10^{-3})^2 = 3.97 \times 10^{-6} m^2$$

The stress on the wire:

$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress} = \frac{86.24}{3.97 \times 10^{-6}} = 2.17 \times 10^7 N/m^2$$

The strain in the wire:

$$\text{Strain} = \frac{\Delta L}{L}$$

$$\text{Strain} = \frac{2.5 \times 10^{-4}}{2.2} = 1.136 \times 10^{-4}$$

The Young's modulus is given as:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{2.17 \times 10^7}{1.136 \times 10^{-4}} = 1.9 \times 10^{11} N/m^2$$

3. A farmer making juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (i.e., $\frac{\Delta V}{V} = 2 \times 10^{-3}$) relative to the space available. Calculate the normal force exerted by the juice per square centimeter, if its bulk modulus is $1.8 \times 10^9\ N/m^2$. Assuming that the bottle does not break. ($432\ N/cm^2$)

Data:

$$\frac{\Delta V}{V} = 2 \times 10^{-3}$$

$$B = 1.8 \times 10^9\ N/m^2$$

B =?

SOLUTION:

$$B = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\Delta P = B \times \frac{\Delta V}{V}$$

$$\Delta P = 1.8 \times 10^9 \times 2 \times 10^{-3}$$

$$\Delta P = 3.6 \times 10^6 N/m^2$$

$$\Delta P = \frac{3.6 \times 10^6}{10^4} = 360 N/cm^2$$

4. **The elastic limit of copper is 1.5×10^8 N/m^2. It is to be stretched by a load of 10 kg. Find the diameter of the wire if the elastic limit is not to be exceeded.** (**0.912 mm**)

Data:

Stress or the elastic limit $= 1.5 \times 10^8$ N/m^2

Mass of the load $= m = 10$kg

Diameter of the wire =?

SOLUTION:

The stretching force acting on the copper wire:

$$F = W = mg$$

$$F = 10 \times 9.8 = 98N$$

$$\text{Stress} = \frac{F}{A}$$

$$1.5 \times 10^8 = \frac{98}{A}$$

$$A = \frac{98}{1.5 \times 10^8} = 6.53 \times 10^{-7} m^2$$

Area of the cross-section of the wire:

$$A = \pi r^2$$

$$r^2 = \frac{A}{\pi} = \frac{6.53 \times 10^{-7}}{3.14} = 2.08 \times 10^{-7}$$

$$r = \sqrt{2.08 \times 10^{-7}} = 4.56 \times 10^{-4} m$$

$$\text{Diameter of the wire} = 2r = 2 \times 4.56 \times 10^{-4} = 0.000912m$$

$$\text{Diameter of the wire} = 0.912mm$$

5. **What would be the greatest length of a steel wire fixed at one end, and can it be hung freely without breaking? The breaking stress of steel is $7.8 \times 10^8\ \mathrm{N/m^2}$, and the density of steel is $7800\ \mathrm{kg/m^3}$. $(1.02 \times 10^4\mathrm{m})$**

Data:

The breaking stress $= 7.8 \times 10^8\ \mathrm{N/m^2}$

$\rho = 7800\ \mathrm{kg/m^3}$

The greatest length of the wire = L =?

SOLUTION:

$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress} = \frac{mg}{A}$$

$$\text{Stress} = \frac{mgL}{AL}$$

$$\text{Stress} = \rho g L \quad \left(\frac{m}{AL} = \rho\right)$$

$$L = \frac{\text{Stress}}{\rho g} = \frac{7.8 \times 10^8}{7800 \times 9.8} = 1.02 \times 10^4\mathrm{m}$$

6. **A mild steel wire of radius 0.55 mm and length 3.5 m is stretched by a force of 52 N. Calculate (a) Longitudinal stress, (b) Longitudinal strain, and (c) Elongation produced in the wire if Young's modulus is $2.1 \times 10^{11}\ \mathrm{N/m^2}$.**

$(5.47 \times 10^{11}\ \mathrm{N/m^2}, 2.6 \times 10^{-4}, 0.91\ \mathrm{mm})$

Data:

The radius of the wire $= 0.55\ \mathrm{mm} = 0.55 \times 10^{-3}\mathrm{m}$

$L = 3.5\mathrm{m}$

$F = 52\mathrm{N}$

Longitudinal stress =?

Longitudinal strain =?

$Y = 2.1 \times 10^{11}\ \mathrm{N/m^2}$

ΔL =?

SOLUTION:

Area of the cross-section of the wire:

$$A = \pi r^2$$

$$A = 3.14 \times (0.55 \times 10^{-3})^2 = 9.5 \times 10^{-7}\text{m}^2$$

The longitudinal stress:

$$\text{Longitudinal stress} = \frac{F}{A} = \frac{52}{9.5 \times 10^{-7}}$$

$$\text{Longitudinal stress} = 5.47 \times 10^{7}\text{N/m}^2$$

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

$$\text{Strain} = \frac{5.47 \times 10^{7}}{2.1 \times 10^{11}} = 2.6 \times 10^{-4}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

$$2.6 \times 10^{-4} = \frac{\Delta L}{3.5}$$

$$\Delta L = 2.6 \times 10^{-4} \times 3.5 = 9.124 \times 10^{-4}\text{m}$$

$$\Delta L = 0.912\text{mm}$$

7. Calculate the change in volume of a lead block of volume $1.3\ \text{m}^3$ subjected to a pressure of 12 atm. Also, calculate the compressibility of lead. Given the bulk modulus as $B = 80 \times 10^9\ \text{N/m}^2$.
$(1.97 \times 10^{-5}\ \text{m}^3, 1.25 \times 10^{-11}\text{m}^2/\text{N})$

Data:

$\mathbf{V = 1.3\ m^3}$

$\mathbf{\Delta P = 12\ atm = 12 \times 1.01 \times 10^5 Pa = 1.21 \times 10^6 Pa}$

$\mathbf{B = 80 \times 10^9\ N/m^2}$

$\Delta \mathbf{V} = ?$

The compressibility of lead =?

SOLUTION:

$$B = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\Delta V = \frac{V \times \Delta P}{B} = \frac{1.3 \times 1.21 \times 10^{6}}{80 \times 10^{9}}$$

$$\Delta V = 1.96 \times 10^{-5} m^3$$

The compressibility of lead is the reciprocal of its bulk modulus.

$$\beta = \frac{1}{B} = \frac{1}{80 \times 10^9} = 1.25 \times 10^{-11} m^2/N$$

8. **The thickness of a metal plate is $\mathbf{0.35}$ inches. It's drilled to have a hole of radius $\mathbf{0.08}$ inches on the plate. If the shear strength is $\mathbf{4 \times 10^4 lbs/in^2}$, Determine the force needed to make that hole.** $(\mathbf{0.176\ in^2, 7 \times 10^3\ lbs})$

Data:

The thickness of a metal plate $= t = 0.35$ inches

The radius of the hole $= r = 0.08$ inches

The shear strength is $= \tau = 4 \times 10^4 lbs/in^2$

The force needed to make the hole $= F = ?$

SOLUTION:

The circumference of the circular hole is given by:

$$C = 2\pi r$$

$$C = 2 \times 3.14 \times 0.08 = 0.5024 \text{ inches}$$

The shear area (A) is the product of the circumference of the hole and the thickness of the plate:

$$A = C \times t$$

$$A = 0.5024 \times 0.35 = 0.176 \text{ in}^2$$

The force required to make the hole is the product of the shear area and the shear strength:

$$F = A \times \tau$$

$$F = 0.176 \times (4 \times 10^4) = 7.0 \times 10^3 \text{ lbs}$$