

1. The Sun's surface temperature is 5700 K.
 - (i) How much power is radiated by the Sun?
 - (ii) Given that the distance to Earth is about 200 Sun radii, what is the maximum power possible from one square kilometer solar energy installation?
 - (iii) What is the wavelength of maximum intensity of solar radiation?

Data:

$$T = 5700\text{K}$$

$$\text{power is radiated by sun} = \frac{P}{A} = ?$$

$$P_{\text{maximum}} = ?$$

$$\lambda_{\text{maximum}} = ?$$

SOLUTION:

$$P = \sigma A T^4$$

$$\frac{P}{A} = \sigma T^4$$

$$P = (5.67 \times 10^{-8}) (5700)^4$$

$$P = (5.67 \times 10^{-8})(1.055 \times 10^{15})$$

$$P = 5.98 \times 10^7 \text{ W}$$

The intensity of radiation on Earth

$$\text{intensity} = \frac{\text{power}}{\text{area}}$$

$$I = \frac{P}{(4 \pi R'^2)} \quad \{R' = 200 R_S\}$$

$$I = \frac{3.64 \times 10^{26}}{\{4 \pi (200 \times 6.96 \times 10^8)^2\}}$$

$$I = 1.49 \times 10^3 \text{ W/m}^2$$

The maximum power is given by

$$P_{\text{max}} = \text{intensity} \times \text{area of intalation}$$

$$P_{\text{max}} = I \times (1 \text{ km})^2$$

$$P_{\text{max}} = (1.49 \times 10^3) \times (1 \times 10^6)$$

$$P_{\text{max}} = 1.49 \times 10^9 \text{ Watt}$$

The maximum wavelength is given by

$$\lambda_{\text{maximum}} T = \text{constant}$$

$$\lambda_{\text{maximum}} 5700 = 2.897 \times 10^{-3}$$

$$\lambda_{\text{maximum}} = \frac{2.897 \times 10^{-3}}{5700}$$

$$\lambda_{\text{maximum}} = 5.08 \times 10^{-7} \text{ m}$$

2. The temperature of your skin is approximately 32 °C. What is the wavelength at which the peak occurs in the radiation emitted from your skin? (9.05×10^{-5} m).

Data:

$$T = 32\text{ }^{\circ}\text{C}$$

$$T = 32 + 273 = 305\text{ K}$$

$$\lambda_{\text{maximum}} = ?$$

SOLUTION:

The maximum wavelength is given by

$$\lambda_{\text{maximum}} T = \text{constant}$$

$$\lambda_{\text{maximum}} (305) = 2.897 \times 10^{-3}$$

$$\lambda_{\text{maximum}} = \frac{2.897 \times 10^{-3}}{305}$$

$$\lambda_{\text{maximum}} = 9.49 \times 10^{-6}\text{ m}$$

3. An FM radio transmitter has a power output of 100 kW and operates at a frequency of 94 MHz, How many photons per second does the transmitter emit?

Data:

$$P = 100\text{ KW} = 100000\text{ W}$$

$$f = 94\text{ MHz} = 94 \times 10^6\text{ Hz}$$

$$\frac{n}{t} = ?$$

SOLUTION:

$$P = \frac{E}{t} \quad \{E = n h f\}$$

$$P = \frac{n h f}{t}$$

$$\frac{n}{t} = \frac{P}{h f}$$

$$\frac{n}{t} = \frac{100\ 000}{(6.629 \times 10^{-34})(94 \times 10^6)}$$

$$\frac{n}{t} = 1.605 \times 10^{30}\text{ photons/second}$$

- 4 A light source of wavelength λ illuminates a metal and ejects photoelectrons with a maximum kinetic energy of 1.0 eV. A second light source with half the wavelength of the first ejects photoelectrons with a maximum kinetic energy of 4.0 eV. Determine the work function of the metal.

Data:

$$E_k = 1.0 \text{ eV}$$

$$E'_k = 4.0 \text{ eV}$$

$$\lambda' = \frac{\lambda}{2}$$

$$\Phi = ?$$

SOLUTION:

For the case of first light source

$$\frac{hc}{\lambda} = \Phi_0 + E_k$$

$$\frac{hc}{\lambda} = \Phi_0 + 1.0 \text{ eV} \dots\dots\dots (i)$$

For the case of second light source

$$\frac{hc}{\lambda'} = \Phi_0 + E'_k$$

$$\frac{hc}{\frac{\lambda}{2}} = \Phi_0 + 4.0 \text{ eV}$$

$$\frac{hc}{\lambda} \times 2 = \Phi_0 + 4.0 \text{ eV}$$

$$\frac{hc}{\lambda} = \frac{1}{2}(\Phi_0 + 4.0 \text{ eV}) \dots\dots\dots (ii)$$

Comparing equation (i) and (ii), we get

$$\Phi_0 + 1.0 \text{ eV} = \frac{1}{2}(\Phi_0 + 4.0 \text{ eV})$$

$$2\Phi_0 + 2.0 \text{ eV} = \Phi_0 + 4.0 \text{ eV}$$

$$2\Phi_0 - \Phi_0 = 4.0 \text{ eV} - 2.0 \text{ eV}$$

$$\Phi_0 = 2.0 \text{ eV}$$

5. A 430 nm violet light is an incident on a calcium photo electrode with a work function of 2.71 eV. Find the energy of the incident photons and the maximum kinetic energy of ejected electrons.

Data:

$$\lambda = 430 \text{ nm} = 430 \times 10^{-9} \text{ m}$$

$$\Phi_0 = 2.71 \text{ eV}$$

$$E = ? , \quad E_k = ?$$

SOLUTION:

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{430 \times 10^{-9}}$$

$$E = 4.6227 \times 10^{-19} \text{ J}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$E = \frac{4.6227 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$E = 2.88 \text{ eV}$$

$$E = \Phi_0 + E_k$$

$$2.88 \text{ eV} = 2.71 \text{ eV} + E_k$$

$$2.88 \text{ eV} - 2.71 \text{ eV} = E_k$$

$$0.17 \text{ eV} = E_k$$

6. Cut-off frequency for the photoelectric effect in some materials is $8 \times 10^{13} \text{ Hz}$. When the incident light has a frequency of $1.2 \times 10^{14} \text{ Hz}$, the stopping potential is measured as 0.16 V . Estimate a value of Planck's constant from these data and determine the percentage error of your estimation.

Data:

$$f_0 = 8 \times 10^{13} \text{ Hz}$$

$$f = 1.2 \times 10^{14}$$

$$V_0 = 0.16 \text{ V}$$

$$k = ? , \text{ percentage error } (k) = ?$$

SOLUTION:

$$hf = hf_0 + eV_0$$

$$hf - hf_0 = eV_0$$

$$h(f - f_0) = eV_0$$

$$h(1.2 \times 10^{14} - 8 \times 10^{13}) = (1.6 \times 10^{-19})(0.16)$$

$$h \times 4 \times 10^{13} = 2.56 \times 10^{-20}$$

$$h = \frac{2.56 \times 10^{-20}}{4 \times 10^{13}}$$

$$h = 6.4 \times 10^{-34} \text{ Js}$$

Percentage error in Planck constant

$$\text{Percentage error} = \frac{|h - h_{\text{calculated}}|}{h}$$

$$\text{Percentage error}$$

$$= \frac{|6.626 \times 10^{-34} - 6.4 \times 10^{-34}|}{6.626 \times 10^{-34}} \times 100$$

$$\text{Percentage error} = \frac{|2.26 \times 10^{-34}|}{6.626 \times 10^{-34}} \times 100$$

$$\text{Percentage error} = 3.4 \%$$

7. The work function of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is:

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
ϕ in eV	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

DATA

$$\lambda_0 = 300 \text{ nm} = 300 \times 10^{-9} \text{ m}$$

$$\Phi_0 = ?$$

The work function is given by

$$\Phi_0 = \frac{hc}{\lambda_0}$$

$$\Phi_0 = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{300 \times 10^{-9}}$$

$$\Phi_0 = 6.626 \times 10^{-19}$$

$$\Phi_0 = \frac{6.626 \times 10^{-19}}{1.6 \times 10^{-19}}$$

$$\Phi_0 = 4.14 \text{ eV}$$

This is the value of the work function we have calculated (4.14 eV), so there are only five materials in the table whose work function is greater than 4.14 eV , so they will produce the photoelectric effect.

Cu	Ag	Fe	Pt	W
4.8	4.3	4.7	6.3	4.75

8. X-rays with an energy of 300 keV undergo Compton scattering with a target. If the scattered X-rays are detected at 30° relative to the incident X-rays, determine the Compton shift at this angle, the energy of the scattered X-rays, and the energy of the recoiling electron.

DATA

$$E_1 = 300 \text{ KeV} = 300 \times 10^3 \text{ eV}$$

$$E_1 = 300 \times 10^3 (1.6 \times 10^{-19})$$

$$E_1 = 4.8 \times 10^{-14} \text{ J}$$

$$\theta = 30^\circ$$

$$\Delta\lambda = ?$$

$$\Delta\lambda = ?$$

$$E_2 = ?$$

$$E_k = ?$$

The Compton shift is given by

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos \theta)$$

$$\Delta\lambda = \frac{(6.626 \times 10^{-34})}{(1.6 \times 10^{19})(3 \times 10^8)} (1 - \cos 30)$$

$$\Delta\lambda = \frac{(6.626 \times 10^{-34})}{(1.6 \times 10^{19})(3 \times 10^8)} (0.1339)$$

$$\Delta\lambda = 3.25 \times 10^{-13} \text{ m}$$

Wavelength of incident photon

$$\lambda_1 = \frac{hc}{E_1}$$

$$\lambda_1 = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{4.8 \times 10^{-14}}$$

$$\lambda_1 = 4.14 \times 10^{-12} \text{ m}$$

Wavelength of scattered photon

$$\Delta\lambda = \lambda_1 - \lambda_2$$

$$\lambda_2 = \Delta\lambda + \lambda_1$$

$$\lambda_2 = 3.25 \times 10^{-13} + 4.14 \times 10^{-12}$$

$$\lambda_2 = 4.465 \times 10^{-12} \text{ m}$$

Energy of scattered photon

$$E_2 = \frac{hc}{\lambda_2}$$

$$E_2 = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{4.465 \times 10^{-12}}$$

$$E_2 = 4.45 \times 10^{-14} \text{ J}$$

$$E_2 = \frac{4.45 \times 10^{-14}}{1.6 \times 10^{-19}}$$

$$E_2 = 278 \times 10^3 \text{ eV}$$

$$E_2 = 278 \text{ KeV}$$

energy of the recoiling electron

$$E = E_1 - E_2$$

$$E = 300 \text{ KeV} - 278 \text{ KeV}$$

$$E = 22 \text{ KeV}$$

9. A photon with a wavelength of $6.0 \times 10^{-12} \text{ m}$ collides with an electron. After the collision the photon wavelength is found to have been changed by exactly one (Compton Wavelength is $2.43 \times 10^{-12} \text{ m}$).
- What is the photon's wavelength after collision?
 - Through what angle has been deflected in this collision?
 - What is the angle for the electron after the collision?
 - What is the electron's kinetic energy, in eV, after collision?

DATA

$$\lambda_1 = 6.0 \times 10^{-12} \text{ m}$$

$$\Delta\lambda = 2.43 \times 10^{-12} \text{ m}$$

$$\lambda_2 = ? \quad \theta = ? \quad \Phi = ? \quad E_k = ?$$

SOLUTION

The wavelength of the scattered photon

$$\Delta\lambda = \lambda_1 - \lambda_2$$

$$\lambda_2 = \Delta\lambda + \lambda_1$$

$$\lambda_2 = 2.43 \times 10^{-12} + 6.0 \times 10^{-12}$$

$$\lambda_2 = 8.43 \times 10^{-12} \text{ m}$$

The Compton shift is given by

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos \theta)$$

$$2.43 \times 10^{-12} = \frac{(6.626 \times 10^{-34})}{(1.6 \times 10^{19})(3 \times 10^8)} (1 - \cos \theta)$$

$$2.43 \times 10^{-12} = 2.43 \times 10^{-12} (1 - \cos \theta)$$

$$1 = 1 - \cos \theta$$

$$\cos \theta = 1 - 1$$

$$\theta = \cos^{-1}(0)$$

$$\theta = 90^\circ$$

For scattered angles of electron Φ

Law of conservation of momentum along x-axis

$$\frac{h}{\lambda_1} \cos(\theta) + 0 = \frac{h}{\lambda_2} \cos(\theta) + P \cos \Phi$$

$$\frac{h}{\lambda_1} \cos(0) + 0 = \frac{h}{\lambda_2} \cos(90) + P \cos \Phi$$

$$\frac{h}{\lambda_1} = \frac{h}{\lambda_2} (0) + P \cos \Phi$$

$$\frac{h}{\lambda_1} = P \cos \Phi \dots \dots \dots (i)$$

Law of conservation of momentum along the y-axis

$$\frac{h}{\lambda_1} \sin(\theta) + 0 = \frac{h}{\lambda_2} \sin(\theta) - P \sin \Phi$$

$$\frac{h}{\lambda_1} \sin(0) + 0 = \frac{h}{\lambda_2} \sin(90) - P \cos \Phi$$

$$0 = \frac{h}{\lambda_2} (1) - P \sin \Phi$$

$$\frac{h}{\lambda_2} = P \sin \Phi \dots \dots \dots (ii)$$

Dividing equation (ii) by (i)

$$\frac{\frac{h}{\lambda_2}}{\frac{h}{\lambda_1}} = \frac{P \sin \Phi}{P \cos \Phi}$$

$$\frac{\lambda_1}{\lambda_2} = \tan \Phi$$

$$\frac{6.0 \times 10^{-12}}{8.43 \times 10^{-12}} = \tan \Phi$$

$$0.711 = \tan \Phi$$

$$\Phi = \tan^{-1}(0.711)$$

$$\Phi = 35.41^\circ$$

$$\Phi = 35.45^\circ, \text{ which is smaller than } 90^\circ$$

Electron's kinetic energy after collision

$$E_k = E_1 - E_2$$

$$E_k = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$E_k = hc \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \{hc = 1.987 \times 10^{-25}\}$$

$$E_k = 1.987 \times 10^{-25} \left(\frac{1}{6.0 \times 10^{-12}} - \frac{1}{8.43 \times 10^{-12}} \right)$$

$$E_k = 9.55 \times 10^{-15} \text{ J}$$

$$E_2 = \frac{9.55 \times 10^{-15}}{1.6 \times 10^{-19}}$$

$$E_2 = 5.96 \times 10^4 \text{ eV}$$

10. Find the de Broglie wavelength of an electron in the ground state of hydrogen.

DATA

$$r = 0.53 \times 10^{-10} \text{ m}$$

$$n = 1$$

$$\lambda = ?$$

The angular momentum of electron is given by

$$mvr = n \left(\frac{h}{2\pi} \right)$$

$$mv = \frac{nh}{2\pi r}$$

$$mv = \frac{(1)(6.626 \times 10^{-34})}{2\pi (0.53 \times 10^{-10})}$$

$$mv = 1.9897 \times 10^{-24} \text{ kg m/s}$$

The de Broglie wavelength is given by

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.9897 \times 10^{-24}}$$

$$\lambda = 3.33 \times 10^{-10} \text{ m}$$

11. Determine the minimum uncertainties in the positions of the following objects if their speeds are known with a precision of $1.0 \times 10^{-3} \text{ m/s}$: (a) an electron and (b) a bowling ball of mass 6.0 kg.

DATA

$$\Delta x = ?$$

$$v = 1.0 \times 10^{-3} \text{ m/s}$$

According to the uncertainty principle

$$\Delta x \Delta P \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi \Delta P}$$

$$\Delta x \geq \frac{h}{4\pi m v}$$

$$\Delta x \geq \frac{6.626 \times 10^{-34}}{4\pi (9.1 \times 10^{-31}) (1.0 \times 10^{-3})}$$

$$\Delta x \geq 0.0579 \text{ m}$$

$$\Delta x \Delta P \geq \frac{h}{4\pi}$$

$$\Delta x \geq \frac{h}{4\pi \Delta P}$$

$$\Delta x \geq \frac{h}{4\pi m v}$$

$$\Delta x \geq \frac{6.626 \times 10^{-34}}{4\pi (6.0) (1.0 \times 10^{-3})}$$

$$\Delta x \geq 8.78 \times 10^{-33} \text{ m}$$