1. Calculate the energy of an electron in the n = 2 orbit of a hydrogen atom according to the Bohr model.

Data:	SOLUTION:
$n = 2$ $E_2 = ?$	The energy of an electron in the nth orbit of a hydrogen atom is given as $E_n = \frac{-13.6\ eV}{n^2}$ Now n = 2 $E_n = \frac{-13.6\ eV}{(2)^2}$ $E_n = \frac{-13.6\ eV}{4}$ $E_n = -3.4\ eV$

2. Calculate the speed of the electron if it orbits in(a) the smallest allowed orbit and (b) the second smallest orbit. (c)If the electron moves to larger orbits, does its speed increase, decrease, or stay the same?

Data:	
(a) $n = 1$ $v_1 = ?$ **SOLUTION:** The speed of the electron in the smallest allowed orbit, n=1 $v_n = \frac{n\hbar}{m r_n}$ $v_n = \frac{(n)\hbar}{m(n^2 \times r)}$ $v_1 = \frac{(1)(1.05 \times 10^{-34})}{(9.1 \times 10^{-31})(1 \times 0.53 \times 10^{-10})}$ $v_1 = \frac{1.05 \times 10^{-34}}{4.823 \times 10^{-41}}$ $v_1 = 2.177 \times 10^6\ m/s$	(b) Speed of the electron in the second smallest orbit n=2 $v_n = \frac{n\hbar}{m r_n}$ $v_n = \frac{(n)\hbar}{m(n^2 \times r)}$ $v_2 = \frac{(2)(1.05 \times 10^{-34})}{(9.1 \times 10^{-31})(4 \times 0.53 \times 10^{-10})}$ $v_2 = \frac{1.05 \times 10^{-34}}{2 \times 1.9292 \times 10^{-41}}$ $v_2 = 1.09 \times 10^6\ m/s$ (c) *As the principal quantum number n increases, the electron's speed decreases because the speed is inversely proportional to n. Therefore, as the electron moves to larger orbits with higher values of n, its speed decreases.*

3. What are the (a) energy, (b) magnitude of the momentum, and (c) wavelength of the photon emitted when a hydrogen atom undergoes a transition from a state with n = 3 to a state with n = 1?

Data:

$n_3 = 3$

$n_1 = 1$

(a) $\Delta E = ?$

(b) $P = ?$

(c) $\lambda = ?$

SOLUTION:

For the transition from n=3 to n = 1, the energy of the emitted photon is:

$$\Delta E = E_3 - E_1$$

$$\Delta E = \frac{-13.6\ eV}{n_3^2} - \left(\frac{-13.6\ eV}{n_1^2}\right)$$

$$\Delta E = \frac{-13.6\ eV}{(3)^2} + \frac{13.6\ eV}{(1)^2}$$

$$\Delta E = \frac{-13.6\ eV}{9} + \frac{13.6\ eV}{1}$$

$$\Delta E = \frac{-(1)13.6\ eV + (9)13.6\ eV}{9}$$

$$\Delta E = \frac{8 \times 13.6\ eV}{9}$$

$$\Delta E = 12.088\ eV$$

(b) wavelength of the photon emitted when a hydrogen atom undergoes a transition from a state with n = 3 to a state with n = 1

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_3^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{(1)^2} - \frac{1}{(3)^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{9-8}{9}\right)$$

$$\frac{1}{\lambda} = \frac{1.097 \times 10^7 \times 8}{9}$$

$$\frac{1}{\lambda} = \frac{9}{1.097 \times 10^7 \times 8}$$

$$\lambda = 1.025 \times 10^{-7}\ m$$

(c) Magnitude of the momentum of the photon

$$P = \frac{h}{\lambda}$$

$$P = \frac{6.63 \times 10^{-34}}{1.025 \times 10^{-7}}$$

$$P = 6.47 \times 10^{-27}\ kg\ m/s$$

4. What is the energy of the photon emitted by a hydrogen atom when the hydrogen atom changes directly from the n=5 state to the 2 state?

Data:

$n_5 = 5$

$n_2 = 2$

$\Delta E = ?$

SOLUTION:

For the transition from n=3 to n = 1, the energy of the emitted photon is:

$$\Delta E = E_5 - E_2$$

$$\Delta E = \frac{-13.6\ eV}{n_5^2} - \left(\frac{-13.6\ eV}{n_2^2}\right)$$

$$\Delta E = \frac{-13.6\ eV}{(5)^2} + \frac{13.6\ eV}{(2)^2}$$

$$\Delta E = \frac{-13.6\ eV}{4} + \frac{13.6\ eV}{25}$$

$$\Delta E = \frac{-13.6\ eV(4) + 13.6\ eV(25)}{100}$$

$$\Delta E = \frac{21 \times 13.6\ eV}{100}$$

$$\Delta E = 2.856\ eV$$

5. How much work must be done to pull apart the electron and the proton that make up the hydrogen atom if the atom is initially in (a) its ground state and (b) the state with n =3?

SOLUTION: Atom in the Ground State (n=1),For the ground state: $\Delta E = E_\infty - E_1$ $\Delta E = \frac{-13.6\ eV}{n_\infty^2} - \left(\frac{-13.6\ eV}{n_1^2}\right)$ $\Delta E = \frac{-13.6\ eV}{(\infty)^2} + \frac{13.6\ eV}{(1)^2}$ $\Delta E = 0 + \frac{13.6\ eV}{1}$ $\Delta E = 13.6\ eV$ The work required to separate the electron and proton is the absolute value of this energy $\Delta W = \lvert\Delta E\rvert = 13.6\ eV$	Atom in the Ground State (n=3),For the ground state: $\Delta E = E_\infty - E_3$ $\Delta E = \frac{-13.6\ eV}{n_\infty^2} - \left(\frac{-13.6\ eV}{n_3^2}\right)$ $\Delta E = \frac{-13.6\ eV}{(\infty)^2} + \frac{13.6\ eV}{(3)^2}$ $\Delta E = 0 + \frac{13.6\ eV}{9}$ $\Delta E = 1.51\ eV$ The work required to separate the electron and proton is the absolute value of this energy $\Delta W = \lvert\Delta E\rvert = 1.51\ eV$

6. (a) What is the wavelength of light for the least energetic photon emitted in the Balmer series of the hydrogen atom spectrum lines?
(b) What is the wavelength of the series limit?

SOLUTIONS wavelength of the photon emitted when a hydrogen atom undergoes a transition from a state with n = 2 to a state with n = 1 $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_3^2}\right)$ $\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{(1)^2} - \frac{1}{(2)^2}\right)$ $\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{3}{4}\right)$ $\lambda = \frac{4}{1.097 \times 10^7 \times 3}$ $\lambda = 6.59 \times 10^{-7}\ m$ $\lambda = 659 \times 10^{-9} = 659\ nm$	(b) wavelength of the photon emitted when a hydrogen atom undergoes a transition from a state with $n = \infty$ to a state with n = 2 $\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_3^2}\right)$ $\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{(2)^2} - \frac{1}{(\infty)^2}\right)$ $\frac{1}{\lambda} = 1.097 \times 10^7\left(\frac{1}{4} - 0\right)$ $\lambda = \frac{4}{1.097 \times 10^7}$ $\lambda = 3.65 \times 10^{-7}\ m$ $\lambda = 365 \times 10^{-9} = 365\ nm$

7. **A laser emits light with a wavelength of 632.8 nm and has a power output of 55 mW. Calculate the energy of one photon emitted by this laser.**

Data:

$\lambda = 632.8\, nm = 632.8 \times 10^{-9} m$

$P = 55 \text{ mW} = 55 \times 10^{3} \text{ W}$

SOLUTION:

To calculate the energy of one photon emitted by the laser

$$E = \frac{h\,c}{\lambda}$$

$$E_n = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{632.8 \times 10^{-9}}$$

$$E_n = \frac{1.989 \times 10^{-25}}{632.8 \times 10^{-9}}$$

$$E_n = 3.143 \times 10^{-19}\ J$$

8. **Calculate the wavelength of X-rays if the energy of one photon emitted by the X-ray machine is 1.9878x10^{-15} Joules.**

Data:

$E = 1.9878 \times 10^{-15}\, J$

$\lambda = ?$

SOLUTION:

To calculate the energy of one photon emitted by the laser

$$E = \frac{h\,c}{\lambda}$$

$$\lambda = \frac{h\,c}{E}$$

$$\lambda = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{1.9878 \times 10^{-15}}$$

$$\lambda = \frac{1.989 \times 10^{-25}}{1.9878 \times 10^{-15}}$$

$$\lambda = 1.006 \times 10^{-10}\ m$$

$$\lambda = 0.1006 \times 10^{-9}\ m$$

$$\lambda = 0.1006\, nm$$