

UNIT-27 NUCLEAR PHYSICS

1. In 9.0 days, the number of radioactive nuclei decreases to one-eighth the number present initially. What is the half-life (in days) of the material?

Data:

$$t = 9.0 \text{ days}$$

$$N = \frac{1}{8} N_0$$

$$T_{1/2} = ?$$

SOLUTION:

The number of radioactive nuclei decays according to the formula:

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{8} N_0 = N_0 e^{-\lambda(9.0)}$$

$$\frac{1}{8} = e^{-\lambda(9.0)}$$

$$8^{-1} = e^{-\lambda(9.0)}$$

Take the natural logarithm of both sides:

$$\ln(8^{-1}) = \ln[e^{-\lambda(9.0)}]$$

$$-1(\ln 8) = -\lambda(9.0) \ln e$$

$$2.079 = \lambda(9.0) (1)$$

$$\lambda = 0.231 \text{ per day}$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$T_{1/2} = \frac{0.693}{0.231}$$

$$T_{1/2} = 3.0 \text{ days}$$

2. The $^{32}_{15}\text{P}$ isotope of phosphorus has a half-life of 14.28 days. What is its decay constant in units of s^{-1} ?

Data:

$$T_{1/2} = 14.28 \text{ days}$$

$$T_{1/2} = 14.28 \times 24 \times 3600$$

$$T_{1/2} = 1.23 \times 10^6 \text{ s}$$

$$\lambda = ?$$

SOLUTION:

We can use the relationship between the half-life $T_{1/2}$ and the decay constant

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{1.23 \times 10^6}$$

$$T_{1/2} = 5.63 \times 10^{-7} \text{ s}^{-1}$$

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3. Find the binding energy (in MeV) for lithium ${}^7_3\text{Li}$ (atomic mass = 7.016 003 u).

Data:

$$M_{\text{Li}} = 7.016003 \text{ u}$$

$$m_p = 1.007276 \text{ u}$$

$$m_n = 1.008665 \text{ u}$$

For ${}^7_3\text{Li}$

Number of protons $Z = 3$

Number of neutrons $n = 4$

To determine

$$B.E = ?$$

SOLUTION:

the binding energy of the lithium isotope

$$B.E = \Delta m \times 931.5 \text{ MeV} \dots (i)$$

the mass defect is given as

$$\Delta m = [(Z m_p + n m_n) - M_{\text{Li}}]$$

$$\Delta m = \{[(3 \times 1.007276) + (4 \times 1.008665)] - 7.016033 \text{ u}\}$$

$$\Delta m = (3.021828 + 4.03466 - 7.016003 \text{ u})$$

$$\Delta m = (7.056488 - 7.016003 \text{ u})$$

$$\Delta m = 0.040485$$

Putting the value of mass defect in the binding energy equation

$$B.E = 0.040485 \times 931.5 \text{ MeV}$$

$$B.E = 37.71 \text{ MeV}$$

4. The binding energy of a nucleus is 225.0 MeV. What is the mass defect of the nucleus in atomic mass units?

Data:

To determine

$$B.E = 225.0 \text{ MeV}$$

SOLUTION:

the binding energy of the lithium isotope

$$B.E = \Delta m \times 931.5 \text{ MeV}$$

$$\Delta m = \frac{B.E}{931.5 \text{ MeV}}$$

$$\Delta m = \frac{225.0 \text{ MeV}}{931.5 \text{ MeV}}$$

$$\Delta m = 0.241545 \text{ u}$$

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5. A copper penny has a mass of 3.0 g. Determine the energy (in MeV) that would be required to break all the copper nuclei into their constituent protons and neutrons. Ignore the energy that binds the electrons to the nucleus and the energy that binds one atom to another in the structure of the metal. For simplicity, assume that all the copper nuclei are ${}_{29}^{63}\text{Cu}$ (atomic mass 62.939 598 u).

Data:

To determine

$$m = 3.0 \text{ g}$$

$$M = 62.939 \text{ 598 u}$$

$$M = 62.939 \text{ 598 g/mole}$$

$$m_p = 1.07276 \text{ u}$$

$$m_n = 1.008665 \text{ u}$$

SOLUTION:

The number of moles of copper in 3.0 g is:

$$n = \frac{\text{mass in gram}}{\text{Atomic mass} \left(\frac{\text{gram}}{\text{mole}} \right)}$$

$$n = \frac{3.0}{62.939598}$$

$$n = 0.0476647 \text{ mol}$$

$$\text{Number nuclei} = (0.0476647 \text{ mol})(6.022 \times 10^{23})$$

$$\text{Number of nuclei (N)} = 2.87036 \times 10^{22}$$

The mass defect is given as

$$\Delta m = [(Z m_p + n m_n) - M_{Li}]$$

$$\Delta m = [(29 \times 1.007276) + (34 \times 1.008665) - 62.939 \text{ 598}]$$

$$\Delta m = (29.211004 + 34.29461 - 62.939 \text{ 598})$$

$$\Delta m = (63.505614 - 62.939 \text{ 598})$$

$$\Delta m = 0.566016 \text{ u}$$

Total mass defect of N atoms

$$\Delta m' = (0.566016)(2.87036 \times 10^{22})$$

$$\Delta m' = 1.624669 \times 10^{22}$$

$$B.E = (\Delta m')(931.5 \text{ MeV})$$

$$B.E = (1.624669 \times 10^{22})(931.5 \text{ MeV})$$

$$B.E = 1.513 \times 10^{22} \text{ MeV}$$

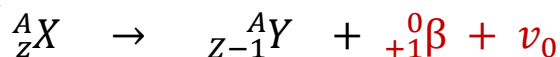
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6. Write the β^+ decay process for each of the following nuclei with their proper chemical symbols, including Z and A for each daughter nucleus: (a) $^{18}_9F$ (b) $^{15}_8O$

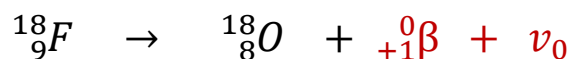
SOLUTION:

$$\text{For } = {}^{18}_9F$$

The general equation of positron emission is:



Now



$$\text{For } = {}^{15}_8O$$

The general equation of positron emission is:



Now



7. A device used in radiation therapy for cancer contains 0.50 g of cobalt $^{60}_{27}Co$ (59.933 819 u). The half-life of $^{60}_{27}Co$ is 5.27 years. Determine the activity of the radioactive material.

Data:

To determine

$$m = 0.5 \text{ g}$$

$$M = 59.933 \text{ 819 u}$$

$$M = 59.933 \text{ 819 g/mole}$$

SOLUTION:

The number of moles of copper in 0.5 g is:

$$n = \frac{\text{mass in gram}}{\text{Atomic mass } \left(\frac{\text{gram}}{\text{mole}}\right)}$$

$$n = \frac{0.5}{59.933 \text{ 819}}$$

$$n = 8.34535 \times 10^{-3} \text{ mol}$$

$$\text{Number nuclei} = (8.34535 \times 10^{-3} \text{ mol})(6.022 \times 10^{23})$$

$$\text{Number of nuclei (N)} = 5.02557 \times 10^{21}$$

$$T_{1/2} = 5.27 \text{ years}$$

$$T_{1/2} = 5.27 \times 3.156 \times 10^7 \text{ seconds}$$

$$T_{1/2} = 1.66 \times 10^8 \text{ seconds}$$

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.694}{1.66 \times 10^8} = 4.1746 \times 10^{-9} \text{ s}^{-1}$$

We calculate the activity using the formula

$$A = N \lambda = (5.02557 \times 10^{21})(4.1746 \times 10^{-9})$$

$$A = 2.0979 \times 10^{13} \text{ Bq}$$