

(i) What is the ideal banking angle for a gentle turn of 1.20 km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

DATA

$$r = 1.20 \text{ km}$$

$$r = 1200 \text{ m}$$

$$v = 105 \text{ km/h}$$

$$v = \frac{105 \times 1000}{3600} = 29.16 \text{ m/s}$$

$$\theta = ?$$

SOLUTION:

$$\theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

$$\theta = \tan^{-1} \left\{ \frac{(29.16)^2}{(1200) \times (9.8)} \right\}$$

$$\theta = \tan^{-1} \left\{ \frac{850.305}{11760} \right\}$$

$$\theta = \tan^{-1} (0.0723)$$

$$\theta = 4.13^\circ$$

(ii) Show that the following equations are dimensionally correct

(i) $F = \frac{mv^2}{r}$

SOLUTION

$$[F] = \frac{[m] [v^2]}{[r]}$$

$$[MLT^{-2}] = \frac{[M] [(LT^{-1})^2]}{[L]}$$

$$[MLT^{-2}] = \frac{[M] [L^2 T^{-2}]}{[L]}$$

$$[MLT^{-2}] = \frac{[M] [L L T^{-2}]}{[L]}$$

$$[MLT^{-2}] = [M] [L T^{-2}]$$

$$[MLT^{-2}] = [MLT^{-2}]$$

we have [L.H.S] = [R.H.S]

Hence, the given equation is dimensionally correct.

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(ii) $2aS = v_f^2 - v_i^2$

SOLUTION

$$2 [a] [S] = [v_f^2] - [v_i^2]$$

2 has no dimension

$$[LT^{-2}] [L] = [(LT^{-1})^2] - [(LT^{-1})^2]$$

$$[L T^{-2}] [L] = [L^2 T^{-2}] - [L^2 T^{-2}]$$

Dimension of acceleration minus Dimension of acceleration is the Dimension of acceleration

$$[L^2 T^{-2}] = [L^2 T^{-2}]$$

we have [L.H.S] = [R.H.S]

Hence, the given equation is dimensionally correct

(iii) Two tugboats are towing a ship, each exerts a force of 6000 N and the angle between the ropes is 60°. Calculate the resultant force on the ship.

Data:

$$F_1 = 6000 \text{ N}, F_2 = 6000 \text{ N}$$

$$\theta = 60^\circ, F = ?$$

Solution:

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$F = \sqrt{(6000)^2 + (6000)^2 + 2(6000)(6000)\cos 60^\circ}$$

$$F = \sqrt{36000000 + 36000000 + 72000000 \times 0.5}$$

$$F = \sqrt{36000000 + 36000000 + 36000000}$$

$$F = 10392.305 \text{ N}$$

(iv) A 50 gm bullet is fired into a 10 kg block that is suspended by a long cord so that it can swing as a pendulum. If the block is displaced so that its centre of gravity rises by 10cm. What was the speed of the bullet?

Data:

$$m_1 = 50 \text{ g} = \frac{50}{1000} = 0.05 \text{ kg}$$

$$m_2 = 10 \text{ kg}$$

$$\text{Initial velocity of the bullet } U_1 = ?$$

$$\text{Initial velocity of the gun } U_2 = 0$$

$$\text{Final velocity of the bullet } V_1 = V$$

$$\text{Final velocity of the gun } V_2 = V$$

$$\text{Centre of gravity rise } h = 10 \text{ cm} = 0.1 \text{ m}$$

SOLUTION:

Applying the Law of conservation of momentum

$$m_1 U_1 + m_2 U_2 = m_1 V_1 + m_2 V_2$$

$$0.05 U_1 + 10(0) = 0.05 V + 10V$$

$$0.05 U_1 = 10.05 V \dots \dots \dots \text{(i)}$$

After the collision block-bullet system moves like a pendulum \therefore loss of K.E = P.E gained

$$\frac{1}{2} (m_1 + m_2) V^2 = (m_1 + m_2) gh$$

$$\frac{1}{2} V^2 = g h$$

$$V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1}$$

$$V = \sqrt{1.96} = 1.4 \text{ m/s}$$

Putting the value of $V = 1.4 \text{ m/s}$ in eq (i)

$$0.05 U_1 = 10.05 \times 1.4$$

$$U_1 = \frac{10.05 \times 1.4}{0.05} = 281.4 \text{ m/s}$$

(v) A car starts from rest and moves with a constant acceleration. During the 5th second of its motion it covers a distance of 36 m, calculate the acceleration of the car.

Data:

Initial velocity $v_i = 0$

Distance covered during 5th second

$S_{5\text{th sec}} = 36 \text{ m}$

Acceleration $a = ?$

Total distance covered $S = ?$

SOLUTION:

Let the distance covered by the car in 5 seconds be S_5 given by:

$$S_5 = v_i t + \frac{1}{2} a t^2$$

$$S_5 = 0 + \frac{1}{2} a (5)^2$$

$$S_5 = 12.5 a \dots \dots \dots \text{(i)}$$

Distance covered by the car in 4 sec. is given by:

$$S_4 = v_i t + \frac{1}{2} a t^2$$

$$S_4 = 0 + \frac{1}{2} a (4)^2$$

$$S_4 = 8a \dots \dots \dots \text{(ii)}$$

Distance covered during 5th sec. will be:

$$S_5 - S_4 = 36$$

$$12.5a - 8a = 36$$

$$4.5a = 36$$

$$a = \frac{36}{4.5}$$

$$a = 8 \text{ m/s}^2$$

(vi) A 70 kg man runs up a hill through a height of 3m in 2 seconds. Calculate the work done and power output?

DATA

$m = 70 \text{ kg}$

$h = 3 \text{ m}$

$t = 2 \text{ sec}$

$W = ?$

$P = ?$

SOLUTIONS

$$W = F S$$

$$W = (m g)(h)$$

$$W = (70)(9.8)(3)$$

$$W = 2058 \text{ J}$$

$$P = \frac{W}{t}$$

$$P = \frac{2058}{2}$$

$$P = 1029 \text{ W}$$

$$1 \text{ h p} = 746 \text{ W}$$

$$P = \frac{1029 \text{ W}}{746}$$

$$P = 1.379 \text{ h p}$$

OR

OR A 20-meter length of cable has a cross-sectional area of 1mm^2 and a resistance of $5\ \Omega$. Calculate the conductance of the material of the wire.

Data:

$$L = 200\ \text{m}$$

$$A = 1\ \text{mm}^2 = 1 \times 10^{-6}\ \text{m}$$

$$R = 5\ \Omega$$

$$\rho = ?$$

SOLUTION:

Conductivity is given by

$$\sigma = \frac{L}{R \cdot A}$$

$$\sigma = \frac{200}{(5) \cdot (1 \times 10^{-6})}$$

$$\sigma = 4 \times 10^7\ \text{S/m}$$

(vii) A particle having charge $2 \times 10^{-19}\text{C}$, is held in an electric field between two parallel metal plates 4cm apart, is acted upon by a force of $10^4\ \text{N}$. What is the intensity of the electric field?

Data:

$$q = 2 \times 10^{-19}\ \text{C}$$

$$d = 4\ \text{cm} = 4 \times 10^{-2}\ \text{m}$$

$$F = 10^4\ \text{N}$$

$$E = ?$$

SOLUTION:

The electric field intensity is given by

$$E = \frac{F}{q}$$

$$E = \frac{10^4}{2 \times 10^{-19}}$$

$$E = 5 \times 10^{22}\ \text{N/C}$$

OR The period of oscillation of an object in an ideal spring and mass system is 0.50 s and the amplitude is 5.0 cm. what is the speed at the equilibrium point? and the acceleration at the point of maximum extension of the spring.

DATA

$$T = 0.50\ \text{s}$$

$$x_0 = 5.0\ \text{cm} = 0.05\ \text{m}$$

$$v = ?$$

$$a = ?$$

SOLUTIONS

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2(3.142)}{0.50} = 12.56\ \text{s}^{-1}$$

$$v = x_0 \omega$$

$$v = 0.05 \times 12.56$$

$$v = 0.05 \times 12.56$$

$$v = 6.28\ \text{m/s}$$

$$a = x_0 \omega^2$$

$$a = (0.05)(12.56)^2$$

$$a = 7.88\ \text{m/s}^2$$

(viii) If the velocity of sound in air at 27°C is 345 ms⁻¹. Find the velocity at 127°C

DATA

$$T_1 = 27^\circ\text{C}$$

$$T_1 = 27 + 273 = 300 \text{ K}$$

$$T_2 = 127^\circ\text{C}$$

$$T_2 = 127 + 273 = 400 \text{ K}$$

$$v_1 = 345 \text{ m/s}$$

$$v_2 = ?$$

SOLUTIONS

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{345}{v_2} = \sqrt{\frac{300}{400}}$$

$$\frac{345}{v_2} = 0.866$$

$$398.38 \text{ m/s} = v_2$$

OR A source of sound and a listener are moving towards each other with velocities, which are 0.5 times and 0.2 times the speed of sound, respectively. If the source is emitting a 2000 Hz tone. Calculate the frequency heard by the listener.

DATA:

$$V_s = 0.5 \text{ V}$$

$$V_0 = 0.2 \text{ V}$$

$$\nu = 2000 \text{ Hz}$$

$$\nu' = ?$$

SOLUTION:

When both the source and the listener are moving towards each other, the apparent frequency is:

$$\nu' = \left(\frac{V + V_0}{V - V_s} \right) \nu$$

$$\nu' = \left(\frac{V + 0.2 V}{V - 0.5 V} \right) 2000$$

$$\nu' = \frac{V}{V} \left(\frac{1 + 0.2}{1 - 0.5} \right) 2000$$

$$\nu' = \left(\frac{1.2}{0.5} \right) 2000$$

$$\nu' = \left(\frac{1200}{0.5} \right)$$

$$\nu' = 2400 \text{ Hz}$$

$$\nu' = 2.4 \times 1000 \text{ Hz}$$

$$\nu' = 2.4 \text{ K Hz}$$

(ix) Derive an equation for a balanced Wheatstone bridge

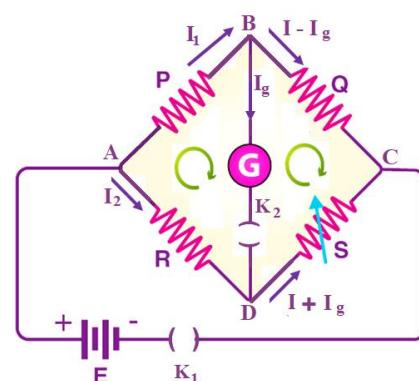
Ans. Wheatstone Bridge Principle states that if four resistances P, Q, R, and S are arranged to form a bridge as shown in the figure with a cell.

If we apply Kirchhoff's 2nd law in ABDA, we get

$$I_1 P + I_g R_g - I_2 R = 0$$

If value of R is such that the galvanometer shows no deflection that is $I_g = 0$. Putting this value in above equation

$$I_1 P + (0) R_g - I_2 R = 0$$



$$I_1 P - I_2 R = 0$$

$$I_1 P = I_2 R \dots \dots \dots (i)$$

If we apply Kirchhoff's 2nd law in BCDB, we get

$$(I_1 - I_g) Q + I_g R_g - (I_2 - I_g) S = 0$$

If value of R is such that the galvanometer shows no deflection that is $I_g = 0$. Putting this value in above equation

$$(I_1 - 0) Q + (0) R_g - (I_2 - 0) S = 0$$

$$I_1 Q - I_2 S = 0$$

$$I_1 Q = I_2 S \dots \dots \dots (ii)$$

Dividing equation (i) by (ii)

$$\frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}$$

$$\frac{P}{Q} = \frac{R}{S}$$

OR

Define Electric flux, write its SI unit. Under what conditions the electric flux through a surface will be:

* Maximum * Minimum

ELECTRIC FLUX

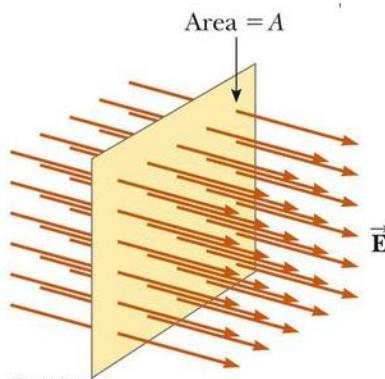
The word "flux" comes from a Latin word meaning "to flow", and its symbol is Φ (uppercase Greek phi).

DEFINITION-

Electric flux can be defined as : the total number of electric lines of force crossing a square area normally to the field is called Electric flux.

$$\Delta\Phi = EA$$

If the area is not perpendicular to \mathbf{E} ,



DEFINITION-2

Electric flux is also defined as the dot product of the electric field and vector area.

Mathematically

$$\Phi = \vec{E} \cdot \vec{A}$$

$$\Phi = E A \cos \theta$$

Flux is a scalar quantity.

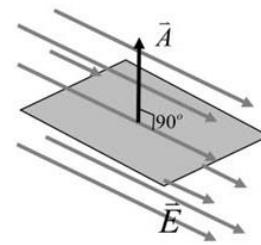
UNIT:

SI unit of electric flux is $\text{N.m}^2/\text{C}$

SPECIAL CASES MINIMUM FLUX

When $\theta = 90^\circ$ & $\cos 90^\circ = 0$

$$\begin{aligned}\Phi &= E A \cos \theta \\ &= E A \cos 90^\circ \\ &= E A (0) \\ \Phi &= 0\end{aligned}$$

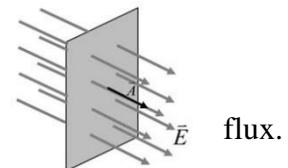


MAXIMUM FLUX

When $\theta = 0^\circ$ & $\cos 0^\circ = 1$

$$\begin{aligned}\Phi &= E A \cos \theta \\ &= E A \cos 0^\circ \\ &= E A (1) \\ \Phi &= E A\end{aligned}$$

The maximum number of lines will cross the area normally, hence maximum



(viii) **Prove that the gravitational field of earth is conservative.**

In a gravitational field, the net work done by the force on an object moving around a closed path is zero; a gravitational field is a conservative field.

To prove this statement, consider a closed triangular path ABCA as shown. Let us calculate the work done around a closed path of mass 'm' in the gravitational field of earth.

work done along AB = $W_{A \rightarrow B}$

$$W_{A \rightarrow B} = \vec{F}_g \cdot \vec{S}_1$$

$$W_{A \rightarrow B} = F S_1 \cos \alpha$$

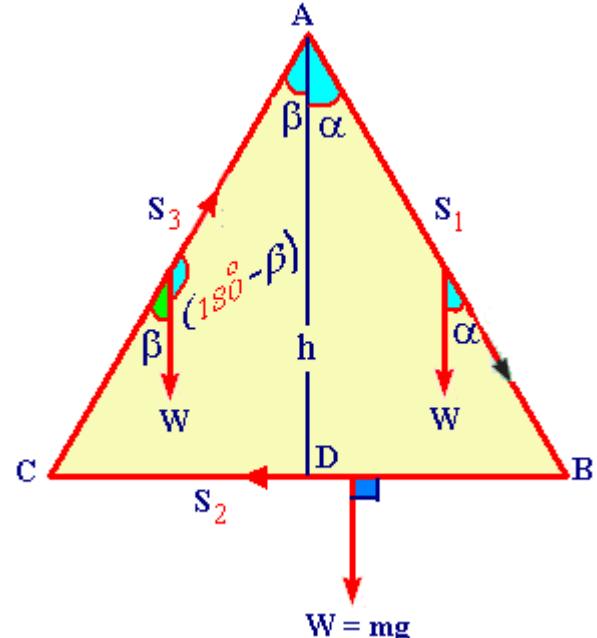
$$W_{A \rightarrow B} = W S_1 \cos \alpha \quad \dots \dots \dots \text{(i)}$$

From the triangle ACD

$$\cos \alpha = \frac{AD}{AC}$$

$$\cos \alpha = \frac{h}{S_1}$$

$$h = S_3 \cos \alpha \quad \dots \dots \dots \text{(ii)}$$



Substituting the value of h from equation (ii) in equation (i), we get

$$W_{A \rightarrow B} = mg h \quad \dots \dots \dots \text{(iii)}$$

work done from B to C

$$W_{B \rightarrow C} = \vec{F}_g \cdot \vec{S}_2$$

$$W_{B \rightarrow C} = F S_2 \cos 90^\circ \quad \alpha = 90^\circ$$

$$W_{B \rightarrow C} = W S_2 \cos 90^\circ \quad W = mg \quad \cos 90^\circ = 0$$

$$W_{B \rightarrow C} = mg S_2 (0) \quad \dots \dots \dots \text{(iv)}$$

work done along CA = $W_{C \rightarrow A}$

$$W_{C \rightarrow A} = \vec{F}_g \cdot \vec{S}_3$$

$$W_{C \rightarrow A} = F S_3 \cos \theta$$

$$W_{C \rightarrow A} = W S_3 \cos(180 - \beta) \quad \theta = 180 - \beta$$

$$W_{C \rightarrow A} = W S_3 (-\cos \beta) \quad \cos(180 - \beta) = -\cos \beta$$

$$W_{C \rightarrow A} = -mg S_3 \cos \beta$$

$$W_{C \rightarrow A} = -mg h \quad (h = S_3 \cos \beta)$$

$$\text{Total work} = W_{A \rightarrow B} + W_{B \rightarrow C} + W_{C \rightarrow A}$$

$$\text{Total work} = mgh + 0 + (-mgh)$$

$$\text{Total work} = 0$$

Thus, the gravitational field is conservative

OR

What is interference of light? Write three conditions of interference of light.

Ans INTERFERENCE OF LIGHT

DEFINITION

When two waves of light superpose each other, then under suitable conditions, they enhance the effect of each other at some points and cancel their effects at other points. This is called interference.

CONDITIONS OF INTERFERENCE

Interference occurs whenever two waves come together, but certain conditions need to be fulfilled to produce an observable interference.

1. The light waves must have an identical wavelength.
2. The distance between the screen and the source must be large.
3. The sources must be coherent—that is, they must maintain a constant phase to each other.

